Reverse reaction magnetic field in two-wire high current busduct

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Work has shown how a reverse reaction magnetic field influences the whole magnetic field within the conductor and its vicinity. A description of this is presented in formulae for relative field values and parameters taking into account frequency, conductivity and diameter of the conductor. This has shown the field to be an elliptical field.

1. Introduction

Unshielded double-wire high current busducts with tubular conductors (Fig. 1) can be installed in switching stations NN and WN [1-3].

Magnetic field $\mathbf{H}^w(r, \Theta)$ of current $I_1$ in the first tubular conductor induces on the second neighboring parallel tubular conductor eddy currents $J_{21}(r, \Theta) = I_z J_{21}(r, \Theta)$ (fig.1), which in external area generate reverse reaction magnetic field $\mathbf{H}^w(r, \Theta)$ [4-6].
2. Magnetic field in the external area of the tubular conductor

Magnetic field $H_{\text{ext}}^r(r,\Theta)$ in the external area ($r \geq R_2$) of the second tubular conductor

$$H_{\text{ext}}^r(r,\Theta) = H_w^r(r,\Theta) + H_{\text{tr}}^r(r,\Theta)$$

(1)

where $H_{\text{tr}}^r(r,\Theta)$ is the reverse reaction magnetic field outside of the conductor.

The electric field strength $E_{\text{tr}}^r(r,\Theta)$, accompanying the magnetic field $H_{\text{tr}}^r(r,\Theta)$ in the external area of the conductor ($r > R_2$), fulfills the scalar Laplace’s equation

$$\nabla^2 E_{\text{tr}}^r(r,\Theta) = 0$$

(2)

whose solution, the separation of variables method, has the form

$$E_{\text{tr}}^r(r,\Theta) = \sum_{n=1}^{\infty} E_{n}^r(r,\Theta) = \sum_{n=1}^{\infty} \frac{B_n}{r^n} \cos n\Theta$$

(2a)

where $B_n$ is a constant, which will be calculated from boundary conditions.

Applying the second Maxwell’s equation to formula (2a), we obtain the complex form of the vector of the reverse reaction magnetic field strength outside the conductor

$$H_{\text{tr}}^r(r,\Theta) = 1_r H_r^r(r,\Theta) + 1_\Theta H_\Theta^r(r,\Theta)$$

(3)

where

$$H_r^r(r,\Theta) = \sum_{n=1}^{\infty} \frac{n B_n}{j \omega \mu_0 r^{n+1}} \sin n\Theta$$

(3a)

and

$$H_\Theta^r(r,\Theta) = -\sum_{n=1}^{\infty} \frac{n B_n}{j \omega \mu_0 r^{n+1}} \cos n\Theta$$

(3b)

Constant $B_n$ is [7]

$$B_n = \frac{j \omega \mu_0 I_1 R_2}{2 n \pi} \frac{R_2}{d} \left( \frac{n}{d} \right)^n \frac{s_{an}}{d_{an}}$$

(4)

where

$$s_{an} = -n \beta_c K_n(\Gamma_1 R_2) [I_{n-1}(\Gamma_1 R_1) + I_{n+1}(\Gamma_1 R_1)] +$$

$$+ n \{2 I_{n+1}(\Gamma_1 R_2) K_n(\Gamma_1 R_1) + I_n(\Gamma_1 R_1) \left[ K_{n-1}(\Gamma_1 R_2) + K_{n+1}(\Gamma_1 R_2) \right] +$$

$$+ \Gamma_1 R_1 [I_{n+1}(\Gamma_1 R_2) K_{n-1}(\Gamma_1 R_1) - I_{n-1}(\Gamma_1 R_1) K_{n+1}(\Gamma_1 R_2)] \}$$

(4a)
and

\[ d_{cn} = I_{n-1}(\Gamma_1 R_2) K_{n+1}(\Gamma_1 R_1) - I_{n+1}(\Gamma_1 R_1) K_{n-1}(\Gamma_1 R_2) \]  

(4b)

where functions \( I_n(\Gamma_1 R_1), \ K_n(\Gamma_1 R_1), \ K_n(\Gamma_1 R_2), \ I_{n-1}(\Gamma_1 R_1), \ K_{n-1}(\Gamma_1 R_1), \ I_{n-1}(\Gamma_1 R_2), \ K_{n-1}(\Gamma_1 R_2), \ I_{n+1}(\Gamma_1 R_1), \ K_{n+1}(\Gamma_1 R_1) \) and \( K_{n+1}(\Gamma_1 R_2) \) are the modified Bessel’s functions of the firsts and second kind respectively and of \( n, n-1 \) and \( n+1 \) order [8].

In the above formulas

\[ \Gamma_j = \sqrt{j \omega \mu \gamma} = \sqrt{\omega \mu \gamma} \exp[j \frac{\pi}{4}] = k + jk = \sqrt{2} j k \]

(5)

in which attenuation constant

\[ k_1 = \sqrt{\frac{\omega \mu \gamma}{2}} = \frac{1}{\delta} \]

(5a)

where \( \delta \) is the electrical skin depth, \( \omega \) in an angular frequency, \( \gamma \) means electrical conductivity of conductor, and permeability of free space \( \mu_0 = 4 \pi \times 10^{-7} \ T \cdot m^{-1} \) [9-10].

Therefore the reverse reaction magnetic field in the external area of the second tubular busbar is determined by the formula (3), where its components, after replacing the \( B_n \) constant, are

\[ H_{r}^{\alpha}(r, \Theta) = -\frac{L_1}{2\pi \ell_1 R_1} \sum_{n=1}^{\infty} \left( \frac{R_2}{r} \right)^n \left( \frac{R_2}{d} \right)^n \frac{\mathcal{S}_{cn}}{d_{cn}} \sin n\Theta \]

(6)

and

\[ H_{\theta}^{\alpha}(r, \Theta) = -\frac{L_1}{2\pi \ell_1 R_1} \sum_{n=1}^{\infty} \left( \frac{R_2}{r} \right)^n \left( \frac{R_2}{d} \right)^n \frac{\mathcal{S}_{cn}}{d_{cn}} \cos n\Theta \]

(6b)

If the above formulas refer to value

\[ H_0 = \frac{L_1}{2\pi R_2} \]

(7)

and following introduction of the relative distance between the conductors

\[ \lambda_c = \frac{d}{R_2} \geq 1 \]

(8)

relative variable

\[ \zeta = \frac{r}{R_2} \]

(9)

and parameter
\[ \beta_c = \frac{R_1}{R_2} \quad \text{przy czym} \quad 0 \leq \beta_c \leq 1 \]  

(10)

Hence relative value components for the reverse reaction magnetic field are defined by the formulae

\[
\hat{h}_r^{\text{nr}}(\zeta, \Theta) = \frac{1}{\sqrt{2} j \alpha_c \beta_c \zeta} \sum_{n=1}^{\infty} \left( \frac{1}{\zeta} \right)^n \left( \frac{1}{\lambda_c} \right)^n \frac{s_{cn}}{d_{cn}} \sin n\Theta
\]

(11)

and

\[
\hat{h}_r^{\text{nr}}(\zeta, \Theta) = -\frac{1}{\sqrt{2} j \alpha_c \beta_c \zeta} \sum_{n=1}^{\infty} \left( \frac{1}{\zeta} \right)^n \left( \frac{1}{\lambda_c} \right)^n \frac{s_{cn}}{d_{cn}} \cos n\Theta
\]

(11a)

where \( \zeta \geq 1 \) and \( 0 \leq \Theta \leq 2\pi \).

The distribution of the above components within the function of parameter \( \alpha_c \) are presented in Figures 2 and 3.

a)

![Graph a](image-a)

b)

![Graph b](image-b)

Fig. 2. The distribution of relative radial component values reverse reaction magnetic field:
a) the modulus, b) the argument
3. Distribution of the reverse reaction magnetic field modulus in external area

The set of arguments for the radial and tangential field components are different and therefore at each point the study area the reverse reaction magnetic field of the conductor is elliptic field. The relative value of this field modulus, relative value of the longer ellipsis semi axis expressed by the formula

\[ h_r^{rv}(\zeta, \Theta) = h_1(\zeta, \Theta) + h_2(\zeta, \Theta) \]  

(12)

where

\[ h_1(\zeta, \Theta) = \frac{1}{2} | h_r^{rv}(\zeta, \Theta) + j h_\phi^{rv}(\zeta, \Theta) | \]  

(12a)

and

\[ h_2(\zeta, \Theta) = \frac{1}{2} | h_r^{rv*}(\zeta, \Theta) + j h_\phi^{rv*}(\zeta, \Theta) | \]  

(12b)

The distribution of these values on the external surface of the second tubular busbar for various values of the \( \alpha_e \), parameter versus \( \Theta \) angle is shown in Figure 4.
For first conductor (fig. 1) the reverse reaction magnetic field has components

$$H_r^{rev} (r, \Theta) = \frac{I_2}{2\pi \int_1^R r r} \sum_{n=1}^{\infty} (-1)^n \left( \frac{R_2}{r} \right)^n \left( \frac{R_2}{d} \right)^n \frac{s_{c_n}}{d_{c_n}} \sin n \Theta \quad (13)$$

and

$$H_\theta^{rev} (r, \Theta) = -\frac{I_2}{2\pi \int_1^R r r} \sum_{n=1}^{\infty} (-1)^n \left( \frac{R_2}{r} \right)^n \left( \frac{R_2}{d} \right)^n \frac{s_{c_n}}{d_{c_n}} \cos n \Theta \quad (13a)$$

Then, for relative values we have respectively

$$h_r^{\tau} (\zeta, \Theta) = -\frac{1}{\sqrt{2j \alpha \beta_c \zeta}} \sum_{n=1}^{\infty} (-1)^n \left( \frac{1}{\zeta} \right)^n \left( \frac{1}{\lambda_c} \right) \frac{s_{c_n}}{d_{c_n}} \sin n \Theta \quad (14)$$

and

$$h_\theta^{\tau} (\zeta, \Theta) = -\frac{1}{\sqrt{2j \alpha \beta_c \zeta}} \sum_{n=1}^{\infty} (-1)^n \left( \frac{1}{\zeta} \right)^n \left( \frac{1}{\lambda_c} \right) \frac{s_{c_n}}{d_{c_n}} \cos n \Theta \quad (14a)$$

The distribution of these values on the external surface of the first tubular busbar for various values of the $\alpha_c$ parameter versus $\Theta$ angle is shown in Figure 5.

In the external area of the tubular busbar the magnetic field of reverse reaction is a field fading quickly, what is demonstrated in Figure 6.
Fig. 5. The distribution of relative quantity the reverse reaction magnetic field modulus in the external area of the first busbar

Fig. 6. The module of the magnetic field of reverse reaction within the tubular busbar external area

4. Conclusions

For a two-wire non-screened busducts the magnetic field distribution in busbars and in both internal and external area of tubular busbars is irregular, caused by the skin effect, but first of all by the proximity effect.

Figures 2 and 3 show, that the distribution of the magnetic field of reverse reaction in the external area of the busbar depends on the \( \alpha_e \) parameter and is an irregular distribution with regard to the \( \Theta \) angle. Consequently, the total magnetic
field in the busbar and around it is irregular. Figures 4 and 5 show that the reverse reaction magnetic field assumes the highest values in the nearest point of the external source of the magnetic field.

The figures presented above show that the reverse reaction magnetic field, i.e. the external proximity effect and the skin effect should be taken into consideration when the magnetic field of high-current busducts is analysed, also for power frequency applications.

References