Research of Work of a Rotor Crush Machine on Elastic Foundation with the Use of Graphs

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Abstract
The use of graphs for the research of a rotor crush machine work on elastic foundation is offered. For machines the structure graph of its design is being written. The graph of levels structure of the generalized coordinates of mathematical model is being formed on the basis of structure graph of machine design. It allows building of the mathematical model, which describe the physical processes of the machine work quickly and qualitatively.

Keywords: mathematical model, rotor crush machine, elastic foundation.

1. Introduction
The process of solid bodies crushing is used in many industries in which this or that material has to be crushed. The costs of crushing within of total costs can reach 70% [1-2].

2. The analysis of recent studies
Analyzing the state of the crushing theory and practice [3-7], it is necessary to conclude that the efficiency of modern crushing machines is rather low. The existent types of these machines do not contain elements, the improvement of which would radically increase the effectiveness of crushing process. One of the possible ways of the crushing process improvement can be the realization of the idea of combining several methods of crushing, for example, the impact with vibration [8-10]. In such circumstances, the study of mechanisms of crushing machines, in which several methods of crushing are used simultaneously, is particularly expedient. Thus, the body vibration of crushing machine of percussion action would help to destroy the established layer of material, which is crushed. In addition, the elastically fixed rotor crusher beaters would have a greater amplitudes of relative fluctuations, that would facilitate better offtaking of the crushing products. Accordingly, it is expedient to elaborate new machine schemes, in which two physical phenomena: impact and vibration, would be used simultaneously, in order to increase the efficiency of the rotor crushing machine.

In the process of new machine models of creation, it is expedient to use computer experiments that allows developers to use efficiently their resources and time. At the same time, to carry out the computer experiments it is necessary to have the designed
mathematical model of the sufficient quality level. The elaboration of the construction methods of such mathematical models is an urgent problem.

3. General regulations

As an example of the graphs usage in the study of crushing machines work, let’s consider the rotor crushing machine work with the crusher on the elastic foundation and rigidly fixed gear and beaters.

Subsequently, generally accepted assumptions in studies of the machines dynamics will be used [11,12]: – the body of crusher, which is in the plane-parallel movement in the vertical plane, the rotor with imbalances and beaters which are in a compound motion in the vertical plane, are considered to be hard inert bodies; – elastic elements of the body crusher bearer – inertialess bodies with tensile stiffness and shearing rigidity; – ignoring elasticity of rotor crushing machines drive components it is considered to be rigid inertial body, the rotation of which is caused by the driving torque of the drive engine; it is changed according to the external static mechanical characteristic.

Accepted third assumption leads to a change of graphs structures of constructive schemes of rotor crushing machine [13]. In this case it takes the form (Fig. 1). Based on the constructive scheme graph structure (Fig. 1), the design diagram of the rotor crushing machine was formed (Fig. 2) with discretely distributed elastic inertialess and rigid inertial elements. This design diagram allows to investigate the crusher body dynamics of plane-parallel motion, compound motion of its rotor with imbalance and beaters, rotational motion of the crush machine drive members, operation of crushing and, consequently, to study the impact of crusher vibration on the crushing process, to find out the peculiarities of the interaction of elements of the "energy source – vibration exciter – working body – technological loading" in various operation modes of the machine with stable or variable masses of technological loading.

Figure 1. Graph of constructive scheme structure of rotor crushing machine with the crusher on the elastic foundation and and rigidiy fixed positive drive
Based on the first assumption was adopted the plane coordinate system \(XOY\), which is rigidly joined with the land and is located in the vertical plane. In addition, \(X_1O_1Y_1\), system was rigidly joined with the crusher body of the rotor crushing machines, the axes of which form a plane parallel to the coordinate plane \(XOY\). Subsequently, it is more convenient to use the term "free state of the mechanical system". In the case of rotor crushing machine – it is a condition of state of rest of its inertial elements that are beyond the gravitational and electromagnetic forces (machine engine drive is disconnected from electric energy sources).

![Design diagram of rotor crushing machine with the crusher on the elastic foundation and rigidly fixed positive drive and beaters](image1)

**Figure 2.** Design diagram of rotor crushing machine with the crusher on the elastic foundation and rigidly fixed positive drive and beaters a); accepted coordinate system and generalized coordinates b)

Taking into consideration the research tasks, based on constructive scheme structure graph (Fig. 3) and designed diagram (Fig. 4) through the modernization of the graph 2 a, which is represented in the table 1, the graph structure of generalized coordinates level couplings (Fig. 3) of rotor crush machine work mathematical model on elastic foundation and rigidly fixed positive drive, was recorded, where as the generalized coordinates were taken:

![The graph structure of generalized coordinates level couplings](image2)

**Figure 3.** The graph structure of generalized coordinates level couplings (Fig. 3) of rotor crush machine work mathematical model on elastic foundation and rigidly fixed positive drive
$q_1, q_2, q_3$ – coordinates of point $O_1$ (Fig. 4) of the coordinate system $X_1O_1Y_1$, beginning, in a fixed coordinate system $XOY$ and the angle of its rotation relatively to the fixed system, that is: $q_1 = x_{01}$; $q_2 = y_{01}$; $q_3 = \phi$ – angle of the rotor crusher rotation with imbalance and beaters relatively to the variable coordinate system $X_1O_1Y_1$; $q_4 = \psi$ – angle of shaft rotation of the leading semicoupling of machine gear; the angles of bodies rotation counterclockwise were considered as additional.

Using the general equation of dynamics [14] of a discrete mechanical systems in generalized coordinates (1), based on the graph of level relations structure of generalized coordinates (Fig. 3) further, a mathematical model of rotor crush machine work on elastic foundation and rigidly fixed positive drive was being built.

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_s} \right) - \frac{\partial T}{\partial q_s} = Q_s, \quad s = 1, \ldots, n;$$

(1)

where $T$ – the total kinetic energy of the movable inertial elements of rotor crush machine; $\dot{q}_s$ – the time derivative of the generalized coordinates; $n$ – the number of freedom degrees of rotary crush machine elements; $Q_s$ – generalized force, which corresponds to the generalized coordinate $q_s$.

The total kinetic energy $T$ of the entire mechanical system equals:

$$T = T_k + T_e + T_i + T_d,$$

where $T_k = \frac{1}{2} [m_x \cdot (\dot{x}_k^2 + \dot{y}_k^2) + I_x \cdot \dot{\phi}_k^2]$ – kinetic energy of the crusher body; $m_x, I_x$ – mass of the body and its central moment of inertia; $x_k, y_k$ – coordinates of the center of the body weight in a coordinate system XOY; $T_e = \frac{1}{2} [m_e \cdot (\dot{x}_e^2 + \dot{y}_e^2) + I_e (\dot{\phi} + \dot{\psi})^2]$ – kinetic energy of the clusher rotor shaft with the beaters and semicoupling; $m_e, I_e$ – correspondingly, the mass and moment of inertia of the shaft with the beaters and semicoupling of elastical joining coupling relatively to the axis of their rotation in the crusher body; $x_e, y_e$ – coordinates of the rotation axis of the shaft with beaters in a coordinate system XOY; $T_i = \frac{1}{2} [m_i \cdot (\dot{x}_i^2 + \dot{y}_i^2) + I_i (\dot{\phi} + \dot{\psi})^2]$ – kinetic energy of the imbalances; $m_i, I_i$ – the mass and moment of inertia of the imbalances relatively to the axis of shaft rotation in the crusher body; $x_i, y_i$ – coordinates of the imbalances center of gravity in a coordinate system XOY; $T_d = \frac{1}{2} I_{\omega d} \cdot \dot{\beta}^2$ – kinetic energy of the machine drive elements that are in rotational motion; $I_{\omega d}$ – total moment of inertia of these elements is brought to the axis of the leading semicoupling shaft of the drive gear; $\dot{\beta}$ – the angular velocity of the leading semicoupling shaft.
\[
q_1 = x_{01}; \\
d \left( \frac{\partial T}{\partial \varepsilon} \right)_{01} - \frac{\partial T}{\partial \varepsilon} = m_k \left[ v_{01} - \dot{\phi} \cdot L_1(\phi) - (\dot{\phi})^2 \cdot L_2(\phi) \right] + m_a \left[ v_{01} - \dot{\phi} \cdot l_1(\phi) - (\dot{\phi})^2 \cdot l_2(\phi) \right] + m_a \left[ v_{01} - \dot{\phi} \cdot l_1(\phi) - (\dot{\phi})^2 \cdot l_2(\phi) - (\dot{\phi} + \ddot{\phi})^2 \cdot l_4(\phi, \psi) \right] + m_a \left[ v_{01} + \dot{\phi} \cdot l_2(\phi) - (\dot{\phi})^2 \cdot l_1(\phi) \right] + m_a \left[ v_{01} + \dot{\phi} \cdot l_2(\phi) - (\dot{\phi})^2 \cdot l_1(\phi) - (\dot{\phi} + \ddot{\phi})^2 \cdot l_4(\phi, \psi) \right] + m_a \left[ v_{01} + \dot{\phi} \cdot l_2(\phi) \right] \] \\
q_2 = y_{01}; \\
d \left( \frac{\partial T}{\partial \varepsilon} \right)_{01} - \frac{\partial T}{\partial \varepsilon} = m_k \left[ \ddot{v}_{01} + \dot{\phi} \cdot L_2(\phi) - (\dot{\phi})^2 \cdot L_1(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) - (\dot{\phi})^2 \cdot l_1(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) + (\dot{\phi} + \ddot{\phi}) \cdot l_4(\phi, \psi) - (\dot{\phi})^2 \cdot l_1(\phi) - (\dot{\phi} + \ddot{\phi})^2 \cdot l_4(\phi, \psi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) \right] \\
q_3 = \psi; \\
d \left( \frac{\partial T}{\partial \varepsilon} \right)_{01} - \frac{\partial T}{\partial \varepsilon} = m_k \left[ \ddot{v}_{01} + \dot{\phi} \cdot L_2(\phi) - (\dot{\phi})^2 \cdot L_1(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) - (\dot{\phi})^2 \cdot l_1(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) + (\dot{\phi} + \ddot{\phi}) \cdot l_4(\phi, \psi) - (\dot{\phi})^2 \cdot l_1(\phi) - (\dot{\phi} + \ddot{\phi})^2 \cdot l_4(\phi, \psi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) + (\dot{\phi} + \ddot{\phi}) \cdot l_4(\phi, \psi) - (\dot{\phi})^2 \cdot l_1(\phi) - (\dot{\phi} + \ddot{\phi})^2 \cdot l_4(\phi, \psi) \right] \right] + m_a \left[ \ddot{v}_{01} + \dot{\phi} \cdot l_2(\phi) \right] \] \\
(2)

\[
q_1 = x_{01}; \\
Q_{\alpha_{m1}} = -\left( F_{3\beta}^+ + F_{4\beta}^+ \right) \\
q_2 = y_{01}; \\
Q_{\alpha_{m2}} = -\left( F_{3\beta}^+ + F_{4\beta}^+ + G_k + G_B + G_M + G_H \right) \\
q_3 = \dot{\phi}; \\
Q_{\psi} = +M \left( F_{3\beta}^+ \right) + M \left( F_{4\beta}^+ \right) - M \left( F_{3\beta}^- \right) - M \left( F_{4\beta}^- \right) - M G_k - M G_B - M G_M - M G_H \\
q_4 = \psi; \\
Q_{\psi} = c_w \cdot (\beta - \psi - \varphi) - \left[ m_k \cdot g \cdot \cos(\varphi + \psi) + \int_{g_0}^{g} \alpha_m \cdot L : \Delta \varphi^2(r) \cdot \rho(r) \cdot dr \right] \\
q_5 = \beta; \\
Q_{\beta} = M H P(\dot{\beta}) - c_m \cdot (\beta - \psi - \varphi). \\
(3)

The left parts of the equation (1) have the form of (2). The right parts of equation (1) have the form of (3).

Dependencies (2) and (3) are the sets of left and right parts of the equations of the rotor crush machine work mathematical model on elastic foundation and rigidly fixed
positive drive, which is based on the graph of level relations structure of generalized coordinates (Fig. 5). The comparison of the results of the study of the machine obtained by means of physical and mathematical model experiment shows its good matches.

As a result of the research the following conclusions can be made:

1. The character of engine’s drive machine operation does not change by the presence of vibrating crusher body. The reaction of the engine on the change of the necessary productivity values, type of crushed grain and its moisture content is similar to the machines with rigidly fixed crusher body.

2. Movement of the crusher body doesn’t alternate with a change of the material quantity in it within a wide range of changes in the estimated productivity, before the reduction of the drive motor rotation. Amplitude of drum vibration decreases at the moment of initial loading at the sudden gate opening.

3. The drum installation on the elastic foundation leads to the appearance of three additional frequencies, the values of which are determined by the ratio between the mass and moment of the drum inertia, rigidity in the horizontal and vertical directions of its elastic bearing, place of its fixation to the drum.

Amplitude of the drum body vibration, the form of its values field in a vertical plane considerably depends on the ratio between the values of the vibration exciter mass imbalance, distance from the point of suspension to the center of mass, angular velocity of the rotor rotation, drum machines mass and rigidity of elastic bearings.

Value of the amplitudes of the drum forced vibrations along the coordinate axes \( x \), \( y \) by the dependency:

\[
\Lambda = \frac{m_r \varepsilon \left( y_{scm} \right)^2}{\left( m_c + m_m + m_p \right) \left[ \frac{C_x^2 + C_y^2}{m_c + m_m + m_p} - \left( y_{scm} \right)^2 \right]},
\]

where \( \sum m_i \) – the total mass of all drum elements; \( \psi_{nom} \) – the nominal angular velocity of the machine rotor. It is necessary to replace "\( C_x \)" by "\( C_y \)" in order to estimate the amplitude along the axis "\( y_m \)" to the dependency (8).

The values, obtained theoretically and experimentally at different intervals of adjustable parameters varying are presented on the diagrams (Fig. 4).

Studying the relationship of crusher productivity with its structural and dynamic parameters one cleared out the following:

1. Oscillograms of beaters acceleration in the crushing mode have non-sinusoidal shape (Fig. 4, a) with amplitude factor \( kA = 4.1 \). Boundary frequency of the signal spectrum on the 10% amplitude criterion is \( f_{hr} = 300 \) Hz, harmonic factor \( k_h = 0.58 \), the distortion factor \( k_y = 0.87 \). Considering the allocation of the normalized power of the spectrum harmonics (Fig. 6, b) it appears that more than 80% of power is transferred by the first harmonic.

2. The crusher productivity \( Q_{TEOP} \) at the constant access rate \( n \) and width of discharge gap \( \delta \) with increasing bias of imbalance \( h \) is decreased.
Figure 4. Oscillogram of acceleration (a) and the allocation of the normalized power of the spectrum harmonics (b)

4. Conclusions

The proposed methodology of the creation of mathematical models using graphs of necessary complexity allows to describe the physical processes of the machine work quickly and qualitatively.

References

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