The use of the Circular Hough Transform for counting coins

Abstract

The paper presents the circular Hough transform in the process of detecting and counting coins. The issues of linear and circular Hough transform are discussed. An algorithm for counting coins in a three-dimensional image using the discrete Hough space is demonstrated. Moreover, the results of the application for detecting and counting coins in a static image and video stream are presented. Observations for histograms for HSV and RGB color palettes, for different camera resolutions and various parameters of image segmentation, edge detection and smoothing filter have been made.

Keywords: Hough transform, image detection, segmentation, digital image processing, edge detection.

1. Introduction

The Hough Transform (HT) is used in multiple scientific fields. The Hough transform was initially invented to determine the set of parameters that would describe a trajectory of the particles obtained in experimental research conducted at CERN. At first, the HT was only cited in literature and used as a method for detecting straight lines in images. After that the transform for analytical curves was used. Then, the Hough Transform was developed in terms of its application for solving many problems that appeared in computational technology (e.g. Hough Transform For Particle Track Finder) together with technology advancement. As a result of studies, a number of variations of computational technology have been invented. They are:

- fast Hough transform
- fuzzy Hough Transform
- Hough transform with varying resolution
- hierarchical and randomized Hough transform.

The Hough transform is most commonly used in image processing as a method for extracting straight lines, circles, polygons and arcs.

2. Linear Hough Transform

The linear Hough transform is the simplest case of transform detecting straight lines. In the image space it is described by the equation of line $k$:

$$y = ax + b$$

where there are two highlighted points $(x_1, y_1)$ and $(x_2, y_2)$.

Using the dependence:

$$B = -x_i A + y_i,$$

the lines can be presented for each point on the straight in the AB parameter space. The points of line $(x_i, y_i)$ and $(x_2, y_2)$ correspond to the streamers in the AB parameter space (also referred to as Hough space or Hough accumulator) which intersect at the point $(A_0, B_0)$. The line, in turn, is represented by $n$ points that do not need to be located exactly on this straight. Using the equation (2), in the AB parameter space there will be $n$ straights, defined by the points which can intersect at the points $\frac{n(n-1)}{2}$. Then one needs to choose such a point that will represent the straight with the biggest number of collinear points in the original image. When using the dependence (1), there arises a problem of discontinuity of parameters $a$ and $b$, when $\phi \pm 90^\circ$ which means that the line is perpendicular. The solution of this problem is to write down the equation of the straight in the canonical equation form. The straight which is located in the $xy$ parameter space is shown in Fig.1, using parameters $\alpha, \beta, \phi, r, b$.

![Fig. 1. Equation of straight $k$ with parameters describing the straight](image)

Therefore, $\sin \phi$ is the ratio of the straight located opposite the angle $\phi$ to the hypotenuse:

$$\sin \phi = \frac{r}{b} \Rightarrow r = b \cdot \sin \phi$$

And the direction line coefficient is given by:

$$a = \tan \alpha$$

$$\tan \alpha = \tan(90^\circ + \phi) = -\cot(\phi) \Rightarrow \frac{\cos \phi}{\sin \phi}$$

which allows obtaining the normal equation:

$$r = x \cdot \cos \phi + y \cdot \sin \phi$$

where $i = 1, 2, \ldots, n$ ($n$ - number of transform points), $x, y$ - point coordinates, $\phi$, $r$ - variable parameters.

Taking the above into consideration, the straight $k$ will correspond to one point in the $\phi, r$ parameter plane. As a result, the equation (5) defines the relationship between the curve in the parameter space and the point in an image, where $\phi$, $r$ are represented as variables and coordinates $x, y$ as given values. The point in an image corresponds to a sinusoid in the parameter plane (the so called butterfly pattern). The discrete Hough space is treated as a two-dimensional array of accumulators. According to the algorithm of Richard Duda and Peter Hart, straight line extraction based on the equation (5) is made for each point $(x_i, y_i)$ in the $\phi, r$ parameter space determining the value $r$ for $\phi \in [0, 2\pi]$. In practice, the search might be reduced accordingly to a studied phenomenon.
3. Circular Hough Transform

The circular Hough transform is based on the same procedure as the linear Hough transform, with the difference that a circle point on plane $x, y$ is transformed to a 3-dimensional array of parameters. The circle equation is:

$$r^2 = (x-a)^2 + (y-b)^2$$  \hspace{1cm} (6)

where: $a, b$– coordinates of the center circle, $x, y$– coordinates of the circle edge, $r$– circle radius.

In the Cartesian coordinate system, a circle can be described by the parametric circle equation (Fig. 2).

The above results in:

$$
\cos(\phi) = \frac{x}{r} \Rightarrow x = r \cdot \cos(\phi)
$$  \hspace{1cm} (7)

$$
\sin(\phi) = \frac{y}{r} \Rightarrow y = r \cdot \sin(\phi)
$$  \hspace{1cm} (8)

Therefore, in the $xy$ Cartesian space, the parametric circle equation is:

$$
\begin{cases}
  x = a + r \cdot \cos(\phi) \\
  y = b + r \cdot \sin(\phi)
\end{cases}
$$  \hspace{1cm} (9)

$$
\phi = (0, 2\pi)
$$  \hspace{1cm} (10)

The dependencies (6, 9, 10) were used in the simulation part of the investigations in the process of coin detection and plotting detected circles along the contour lines of particular coins.

4. Counting coins by the Circular Hough Transform

Assuming that there is a circle in the image coordinates $x, y$ (the circle edge points are known), then a circle in the $a, b$ parameter space corresponds to each point of the edge of the circle. The $a, b$ parameter space will be circle accumulation of the input image for a given radius $r$, as illustrated in Fig. 3.

The most important parameter while detecting a circle is its radius. It decides on the size of the circles plotted in the $a, b$ parameter space. When the radius is smaller or larger than the normal radius of a circle, the circles drawn in the $a, b$ parameter space will never intersect at one point. The properly selected radius will cause that the drawn circles will intersect at one point which will be the center circle located in the coordinates of the image.

In the case of the circular transform, the discrete transform parameter space $[a, b, r]$ is a 3-dimensional matrix. Circle detection in the input image is reduced to searching the biggest number of circle intersections. Figure 4 shows an example of the 3-dimensional discrete Hough space.

To extract the coins in the image, an algorithm of selective search of the 3-dimensional Hough space was used. In order to guarantee stabilized results, a low-pass Gaussian filter was applied. Then, a multi-criteria segmentation was performed using the dependency:

$$
g(x, y) = \begin{cases} 
1 & \text{for } I(x, y) \geq T \land I(x, y) \leq T \\
0 & \text{for } I(x, y) \leq T \land I(x, y) \geq T
\end{cases}
$$  \hspace{1cm} (11)

where: $T=(0,1)$ – threshold segmentation, $I(x, y)$ – point value in the input image, $g$ – binary output image.

In the next part of the algorithm work, for each point $(x_i, y_j)$ belonging to the contour line of a particular circle, the circles for a particular radius $r_j$ were determined in the parameter space according to the following formula:

$$
r_j^2 = (x_i - a)^2 + (y_j - b)^2
$$  \hspace{1cm} (12)

where: $i=1, 2, \ldots, N_i$ and $N_i$ – number of transform points.

The coins of the National Bank of Poland were the recognizable objects. For them, the accumulator array whose third dimension was the number of all vector elements $r$ (9 elements for a particular coin) was created. The number of the detected coins was the result of searching. Below there are the example results presenting accumulator Hough arrays (where the maximum is being searched) in the 3D space for a coin of 1 grosz value.
The preliminary observation confirms that the circular Hough transform can be an effective tool for counting coins. The segmentation is crucial to the effectiveness of the recognition stage. Therefore in the experimental part, a conversion of the source signal from RGB to HSV color space was used. We used only V component, which allowed reducing the influence of uneven lighting of the scene on efficient image segmentation and edge detection. Such an assumption affects the accuracy of determining the radii of the coins recognized in the parameter space. By assuming the resolution of the registered stream equal to 2338x1700 px, there was obtained the detection accuracy which allowed for correct identification of all the analyzed coins. Moreover, at the stage of the preliminary measurement, it was observed that searching the accumulator array for a higher vector resolution of the successive radius of particular coins was a key impediment in the application of this transform in real time (recorded times are 12-15 s). Coin detection and counting in the video stream were made in a parallel way by cropping coins into individual frames, then for each of them the Hough transform was made and the accumulator array was searched.

5. Application to coin detection and counting in an image

Using the Matlab environment and the circular Hough transform, there was developed an application which recognized and counted the coins of NBP in a static image or a video stream. Observations of histograms were made for images in HSV and RGB palettes, for different camera resolutions and various parameters of image segmentation, edge detection and smoothing filter. The application was accessorized with a functionality allowing calibrating a coin image and a coin radius when raising the arm supporting the camera. The designed program automatically checks for accessibility in the camera system allowing carrying out calculations in a static image or a video stream after appropriate adjusting of settings. After loading an image file or an image frame from a camera to the program, one can display their histograms for both RGB and HSV image.

The experimental version is extended by a calibration module which allows for the selection of settings for position and coin radius – the option is available only for images captured from the camera. An example result of segmentation and edge detection of coins is shown in Fig. 6.

A series of experiments of the designed application for identification and counting of coins were conducted. Fig. 7 shows the result of the circular Hough transform in the process of coin detection.

A calibration module was designed. It allows for proper selection of settings for position and coin radius (this option is available only for images captured from the camera). In Figs. 8 and 9, the process of framing a scene and calibration of the station for the captured image for the video stream is presented.

6. Summary

The Circular Hough Transform is a powerful tool in the process of image detection even at the presence of high roar levels in the image associated e.g. with changing scene lighting conditions determining the effectiveness of circle detection. While attempting to identify coins in a static image, i.e. under the same lighting conditions (for example, a scanned image), it was found that the transform flawlessly copes with the task of coin counting. The NBP coins are approximately of the same size, that is 1 grosz is roughly the same size as 10 grosz, 2gr- 20gr, 5gr- 50gr and 1zl-5zl, so in the case of lighting condition disorders the effectiveness of the algorithm of counting coins decreases. On the basis of the experimental results, the additional module recognizing the texture of individual coins (obverse and reverse) is going to be introduced in further versions. It will have a stabilizing effect on the effectiveness of work of the whole application.
7. References


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