Maintaining the water consumption, in an urban system: A probabilistic approach is applied

Karima SELMANI BOUAYOUNE A,B,C,D,E,F, El Mostapha BOUDI A, Aziz BACHIR A

University Mohamed V Agdal, Mohammadia Engineering School, Laboratory of Quality Security and Maintenance, Avenue des Nations Unies, Rabat 10000, Morocco; e-mail: karimaselmani@yahoo.fr, elmostapha.boudi7@gmail.com, a.bachir@emi.ac.ma


Abstract

An urban system is influenced by many disruptions that may cause failures for it, in the end. In order to maintain continuity of its operations, analysis of its components operation becomes very necessary. To do this, water infrastructure is chosen from its components to analyze the evolution of the water flow, when the population consumes the drinking water. This infrastructure is essential for the urban system and it is used daily by the population. For examining how to maintain water consumption, the evolution of the discharge head (the maximum height reached by the pipe after the pump) is analyzed and monitored. This height is strongly linked to the drinking water rate. Using water is estimated by a Markov model and the futures heights are prevented. This prevention requires the calculation of the transition probability of the water flow used by the population. An example is provided, where it is determined the level of risk. Under this one, the urban system operates in security, against failures.

Key words: maintaining, risk, transition probability, urban stability, water flow

INTRODUCTION

An urban system (or system of the city) is a set of components that are essential for the life in the city. It is organized in order to ensure that the city can continue to be in life. According to the modeling of an urban system presented by BARRERE-LUTOFF [2000] and MOINE [2007], these components are: urban infrastructure, urban areas, population, activities and powers. We mention also the interaction of this system with the economic, social, physical and urban networks. Most papers that are focused to maintain the continuity of urban system to be in life, have discussed in their works the sustainability [ANDERSSON 2006; CASTAN, BULKELEY 2013; CHILDERST et al. 2014] and resilience [BERAUD et al. 2011; ERNSTON et al. 2010; LIM, LIM 2016].

FALCO et al. [2015] develop a framework for microgrids application to promote water resilience in the face of changing climate impacting water infrastructure. For management of water system, several papers (BRDYS, ULANICKI [1994], CEMBRANO et al. [2000; 2004; 2011], MAYS [2004], OCMANO et al. [2009]) have presented models of Drinking Water Networks (DWN), in order to supply water to consumers. These models, as some others that have been used for prediction, utilize stochastic models for analysis and preventing. HOSSEINI and EMMAJOMEH [2014] have worked on Water Distribution Networks, to evaluate its serviceability level. MOOSAVIAN and JAEFARZADEH [2014] applied hydraulic analysis of water net-
work using Hardy-Cross method, they have used a mathematic modeling. A stochastic approach was used by [YUNG et al. 2011] to quantify water supply system risk in terms of reliability, resiliency, and vulnerability (RRV), under the influence of population. In the same way, we search to incorporate a probabilistic model to evaluate the risk that can be reach the urban system, when the use of the water infrastructure by the population increases.

Based on researches of BERAUD et al. [2012], the urban infrastructure has a crucial role in the ability of the urban system to maintain continuity of its operations. LAVIGNE [1988] has discussed about “urban failure”. The importance of the urban infrastructure appears in its ability to recover from disaster [SANDERS 1992] and the ability to continue [CAMPANELLA 2006; PELLING 2003; SANDERS 1992]. Thus, maintaining an urban system to be in life can be understood as maintaining its urban infrastructure to continue in operation.

From all of this, it becomes very important to work on an urban infrastructure to cope our problem, that is: “how to maintain the continuity of operations for the urban system, despite disruptions that may influence its components and achieve, finally, to a failure of this system”. We can name this by “Maintaining in Operational Conditions”. It can be defined as: “All the means and procedures that a system will need to remain in operation, throughout its duration of use and despite the occurrence of disruptions and failures”.

More the continuity of the system operation is maintained, more the system is stable. In this article, we choose to analyze the stability of the urban system and to conclude if it is maintained in operational conditions, using the infrastructure of water pumps facilities. In fact, “Human life depends on water daily, especially for drinking and food production” [VAN OVERLOOP 2006], so, it is best to work on this infrastructure, because, it is used by the entire population and all activities and all these factors make it an essential infrastructure in the urban system.

The aim of this purpose is to seek means to maintain in operational conditions the infrastructure of water pumps facilities, despite the increase of population in cities. Our idea is to study the use of drinking water from population. To do this, we will discuss the behavior of the discharge head (the maximum height reached by the pipe after the pump), which increases whenever there is pulling water from the reservoir. The discharge head is a decreasing function on the water flow. A probabilistic approach, based on a Markov model is used to estimate this height after a long time of drinking water consumption from the population.

This article includes four parts. The first present our study and describe how residents use water from its resource to the reservoir, considering the installation of the drinking water as a network. In the second part, we present the considered Markov model for this study, in which the repression height evolves, after the consumption of the drinking water. We seek to estimate future heights in the third part, in which we assume that the flow of water drawn from the tank follows a pattern of [NASH 1957]. The fourth part is an application of the obtained results, especially those in the third section.

In this way, we can deduce if the system remains in operation despite the changes that influence it, during the time of its operation. If not, on what parameter we must act to maintain its stability?

**ANALYSIS OF THE PHENOMENON**

Every year, the population increases in cities, due to economic and social conditions. This growth is owing to increased demand for all goods and services, including the use of drinking water resources.

For studying this phenomenon, we choose to monitor the channels of water, which are in the form of networks. They are usually called: Water Supply Network (WSN). A network is modeled by a graph $\Gamma = (\Phi, \Pi)$, where $\Phi$ is the set of nodes and $\Pi$ is the set of paths.

Each Water Supply Network (WSN) is comprised of underground conducts, with diameters and different materials. A conduct is a segment delimited by two water consumption nodes. The water flow is made from the node with the highest pressure to the node whose pressure is lower. We can associate to each node a set of pumps, characterized by a power and a characteristic curve, that describes the relationship between the discharge head and the flow rate, supplied with a function $H = f(Q)$. People use water from channels associated to local cisterns. The distribution of water from the reservoir into the cistern is presented in the following figure (Fig. 1).

![Diagram of water supply network](source: own elaboration)

In this figure, the discharge head is well illustrated in the diagram. This height is increased whenever there is pulling water from the reservoir. In our work, we consider that the nodes are the water reservoirs. We seek a relation between the water level in the reservoir and the water flow consumed by the population, using the water pump associated with this reservoir.
ESTABLISHING THE MODEL FOR THE WATER FLOW

CONSTRUCTING THE TRANSITION PROBABILITY OF THE WATER CONSUMPTION

We seek to build a transition probability matrix. We use a similar method to the one built in the article of drinking water conduits [LARGE et al. 2015]:

- we decompose the time in tranche \([T_k, T_{k+1}]), k \in \mathbb{N},\)
- we consider the network node is the reservoir of water and paths are pumps, associated with these reservoirs,
- each reservoir or node is characterized by:

\[
\begin{align*}
\delta_i &= 1 \text{ if there is pulling water at } T_{k+1} \\
&= 0 \text{ else}
\end{align*}
\]

We define the probability of pulling water from reservoir, before the time \(t\) by the following equation,

\[
Q(h) = P(H > h)
\]  \(2\)

\(H\) is a random variable representing the discharge head after a pulling water.

Building the Markov transition matrix

We are seeking now to model the evolution of the height of the water, for a given node (reservoir). The probability \(p\) represents the height of discharge head change from one value to another higher. However, \(q\) is the probability representing the transitions of water flow used by residents.

We consider \(m\) heights of head discharge and we associate to every height a state of the system. So, we have \(m\) states \([h_1, \ldots, h_n, \ldots, h_m],\) where \(h_1 < \ldots < h_n < \ldots < h_m, 1 < n < m.\) A simple calculation of \((p_n, n \geq 1)\) (the probability that the height changes from state \(n\) to state \(n + 1\)) is given by the equation (3).

\[
\begin{align*}
p_n &= P(H > h_{n+1} | H > h_n) \\
&= \frac{P(H > h_{n+1}, H > h_n)}{P(H > h_n)} \\
&= \frac{Q(h_{n+1})}{Q(h_n)}
\end{align*}
\]  \(3\)

Then, the Markov chain associated to this probability can be represented as the Figure 2 shows.

And the transition matrix, in which the height of head discharge passes from a height \(h_k\) to the height \(h_{k+1},\) for \(k \in \mathbb{N},\) is given by the equation (4).

\[
P_{ii} = \begin{bmatrix}
1 - \frac{Q(h_2)}{Q(h_1)} & \frac{Q(h_2)}{Q(h_1)} & 0 & 0 & 0 & 0 \\
0 & 1 - \frac{Q(h_2)}{Q(h_1)} & \frac{Q(h_3)}{Q(h_1)} & 0 & 0 & 0 \\
0 & 0 & \ddots & \ddots & 0 & 0 \\
0 & 0 & 0 & \ddots & \ddots & 0 \\
0 & 0 & 0 & 0 & \ddots & \ddots \\
0 & 0 & 0 & 0 & 0 & \frac{Q(h_{m+1})}{Q(h_m)} & 1 - \frac{Q(h_{m+1})}{Q(h_m)}
\end{bmatrix}
\]  \(4\)

Estimation of future heights

In the water pump, the discharge head is a bijective and decreasing function, depending on the flow (Fig. 3).

NASH [1957] supposes that the flow trajectory, within a basin with rainfall, is equivalent to the flow through a linear succession of reservoirs. The flow out of a reservoir becomes the input for the next. Thus, the flow of the \(n^{th}\) reservoir is given by the equation (5)
where \( T_n \) is his distribution function. Because this function is increasing, we can conclude for \( t = F_{Tn}^{-1}(q) \), that \( T_n \leq t \), when \( Q_n \leq q \).

This is the density function of the gamma law, with parameters \( \lambda \) and \( n \). Thus, \( Q_n = F_{Tn}(t) \), where \( (T_n, n \geq 0) \) is a random variable of \( \Gamma(\lambda, n) \) and \( F_{Tn} \) is his distribution function. Furthermore, as we can notice from the Fig. 3, the discharge head and the flow, with the function \( H = f(Q) \); source: own elaboration.

We apply our studies on the Moroccan barrage: “Al Wahda”, which is a part of the Sebou basin and contains 20 large barrages, 44 small barrages and lakes [FILAII-MEKNASSI 2009]. It has a storage capacity of 3720 million \( m^3 \). The current overall storage capacity of 20 large barrages is more than 6020 million \( m^3 \).

We consider that the value of the storage reservoir coefficient is \( k = 3720 \over 6020 \approx 0.62 \), thus, \( \lambda \approx 1.6 \).

We get for 5 states, over a period of 40 months, the Figure 4, in which we can view the probability changes from a state to another. We can see, from this figure, that the system passes from a state to another, rapidly, during the five first months, and the probability to pass from a state

Thus,

\[
p_n = 1 - \frac{1}{n!} t^n e^{-\lambda t}
\]

We calculate firstly \( I_0 \):

\[
I_0 = \frac{1}{\lambda} \left(1 - e^{-\lambda t} \right)
\]

Then, by recurrence, we find \( I_{n-1} \):

\[
I_{n-1} = \frac{(n-1)!}{\lambda^n} - \frac{1}{\lambda} \left( \sum_{k=1}^{n-1} \frac{(n-1)!}{k!} \right) e^{-\lambda t}
\]

And,

\[
p_n = 1 - \frac{1}{\lambda^n} t^n e^{-\lambda t} - \frac{1}{\lambda} \left( \sum_{k=1}^{n-1} \frac{(n-1)!}{k!} \right) e^{-\lambda t}
\]

Finally,

\[
p_n = 1 - 1 \over n! \left( t^n e^{-\lambda t} - \frac{1}{\lambda} \left( \sum_{k=1}^{n-1} \frac{(n-1)!}{k!} \right) e^{-\lambda t} \right)
\]

**NUMERICAL APPLICATION**

The characteristic curve, describing the relationship between the discharge head and the flow, with the function \( H = f(Q) \); source: own elaboration

_**Fig. 3.** Variation of the transition probability of the states, for the discharge head (the coefficient of storage is \( k = 0.62 \)); \( p_n \) is calculated in the equation (16); source: own elaboration._

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**Fig. 4.** Variation of the transition probability of the states, for the discharge head (the coefficient of storage is \( k = 0.62 \)); \( p_n \) is calculated in the equation (16); source: own elaboration.

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to another is mostly high (more than 0.5). This is ex-
plain that the system is not stable and we can con-
clude that the system is not maintained in operational
conditions. So, we must maintain all conditions of
exploitation that the system needs to remain at the
same state possibly as we can. We search then to
change the coefficient of storage.

RESULTS AFTER CHANGING COEFFICIENT
OF STORAGE

The coefficient $k$ in our example is a value less
than 1. We can make some tests with changing the
values of $k$. In the Figure 5. In this figure, there are
some simulations of probabilities for various values of
the coefficient $k$. We can notice that when $k$ is
less than 1, we have the same cases of our example and
the system is not maintained. However, when $k$ is
more than 1, the transition probabilities from a state to
another is low for most of months and the system
changes slowly from a state to another and, conse-
quently, it is more stable. Therefore, the system is
strongly maintained in operational conditions. Hence,
it should keep these exploitation conditions. Special-
ly, the coefficient of storage must be in a high value.

![Fig. 5. Variation of the transition probability of the states, for the discharge head, for various coefficient of storage $k$; $p_n$ as in Fig. 4; source: own study](image-url)
THE BEST CHOICE OF THE COEFFICIENT OF STORAGE

As we have conclude in the previous paragraph that the coefficient of storage $k$ must be more than 1, and we can obtain from the equation (16), using $\lambda = 1/k$, that:

For $k > 1$:

$$p_n = 1 - \left[ \frac{k^n n! e^{\lambda t} - k \sum_{l=0}^{n} \frac{n!}{(n-l)!} k^{l-1} t^{l-n}}{k^n n! e^{\lambda t}} \right] < 0.5$$

(17)

So,

$$\left[ \frac{k^n n! e^{\lambda t} - k \sum_{l=0}^{n} \frac{n!}{(n-l)!} k^{l-1} t^{l-n}}{k^n n! e^{\lambda t}} \right] > 0.5$$

(18)

Then,

$$k^n n! e^{\lambda t} - k \sum_{l=0}^{n} \frac{n!}{(n-l)!} k^{l-1} t^{l-n} < 2$$

(19)

For a large $t$, we have

$$e^{\lambda t} > 1$$

(20)

For all $m > 0$, there exists $t(m, n)$ such that for $t > t(m, n)$ we have

$$k \sum_{l=0}^{n} \frac{n!}{(n-l)!} k^{l-1} t^{l-n} < m$$

(21)

We can conclude that for $t > t(m, n)$

$$\frac{k^n}{t^n} n! < 2 + m$$

(22)

So,

$$k^n \leq (2 + m) \frac{t^n}{n!}$$

(23)

Finally, for $t > t(m, n)$

$$k \leq \sqrt{(2 + m) \frac{t^n}{n!}}$$

(24)

We consider

$$k \leq M_{m,n} = \sqrt{(2 + m) \frac{t^n}{n!}}$$

(25)

In the Table 1, we present a test of $M_{m,n}$, for $m = 10^{-10}$, the states between 1 and 5, and for the time between 24 and 72 months. If we take for example $t = 42$ months, because we must choose a high value of $k$ (as we have noticed previously), we choose, then, the maximum value of $M_{m,n}$, so it must $k \leq 84$. We can choose, thus, $k = 84$. The Figure 6 shows the variations of probabilities for five states, for this coefficient. As we can conclude that transitions probabilities that the head discharge pass from a value to another more high are lows for all the duration. Consequently, the change of the system from a state to another can be maintained and the system will be more stable, in this case.

![Fig. 6. Variation of the transition probability of the states, for the discharge head (the coefficient of storage is $k = 84$); $p_n$ as in Fig. 4; source: own study](image)

CONCLUSIONS

In this article, the calculation which estimates the discharge head of the water reservoir, gives a new indicator of maintaining in operational conditions. This is because it informs on the ability or inability of the system to be stable in the future or not.

We have noticed the importance of maintaining in operational condition in the continuity of systems operation. We can maintain the system in process despite disruptions; we should just choose correctly parameters.

Our methodology helps to manage a population in an urban system, however it may be more general for other applications. Again, the obtained results can be generalized for other infrastructures that use water resources.

| Table 1. The $M_{m,n}$ obtained when states $n$ and months $t$ change |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| **State** $n$   | **24** | **30** | **36** | **42** | **48** | **54** | **60** | **66** | **72** |
| 1               | 48    | 60    | 72    | 84    | 96    | 108   | 120   | 132   | 144   |
| 2               | 24    | 30    | 36    | 42    | 48    | 54    | 60    | 66    | 72    |
| 3               | 16.640676 | 20.8008382 | 24.9610059 | 29.1211735 | 33.2813412 | 37.4415088 | 41.6016765 | 45.7618441 | 49.9220118 |

Explanation: $m = 10^{-10}$.

Source: own study.
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Karima SELMANI BOUAYOUNE, El Mostapha BOUDI, Aziz BACHIR

Zarządzanie zużyciem wody w systemie miejskim: podejście probabilistyczne

STRESZCZENIE


Słowa kluczowe: prawdopodobieństwo, przepływ wody, ryzyko, stabilność systemów miejskich, utrzymanie