A PROPOSAL OF THE PARAMETERS DETERMINATION OF VERTICAL AXIS INCLINATION OF TOWER BUILDINGS

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ABSTRACT

In architecture, towering structures are defined as structures whose lateral dimensions are much smaller than the height and they work as a support restrained in the foundation. One of the assessments of the spatial behavior of these structures relates to the determination of spatial variation of the vertical axis from the project position. Without getting into the technique of carrying out the measurements (the classic method of surveying, the GPS-RTK technique), it is assumed that on the object are coordinates $x_i, y_i, z_i$ determined for a sufficiently large number of points that describe solid body of such buildings and which define the center of the structure on selected levels (e.g. at the level of each building floor). Based on calculated in such a way horizontal cross-section centers, it is proposed to determine the vertical axis as the intersection edge of two orthogonal, approximate planes and the parameters determining its slope in the space of the structure.

INTRODUCTION

The structures that are determined as a tower include constructions in which transverse dimensions are much smaller than their height, they work as a bracket fixed into foundation and are charged with the wind in the horizontal direction and the gravity load. The structures that are determined as a tower include construction that the transverse dimensions are much smaller than their height, working as a bracket fixed into foundation and who is charged with the wind in the horizontal direction and the gravity load. Therefore in everyday language, we divide such structures into more slender, such as chimneys and more rigid, like church towers or pagodas. We also point out that these characteristics have influence upon the ways of studying their spatial
stability. It should be noted that these objects, which can be classified as slender, according to safety rules or designer decisions, may need to have a periodic check measurements of their spatial stability. The objects, for which periodic testing of the spatial structure behavior may be imposed by separate decisions, are church towers, lighthouses, skyscrapers, pagodas, etc. Due to the rare research of such constructions there are no specific recommendations made, not only how to measure, but first of all how to determine the subject of research. This statement, first of all concerns determination of the instantaneous vertical axis of structure (major axis) whose position is determined at the given moment of measurement. Conception of determining the instantaneous position of the vertical axis of structure. In surveying literature there are not many descriptions of tower structure studies with high rigidity. In particular, for periodic measurements of its bending. Existing descriptions relate mostly to the structures being in failure and for non typical structures such as an cooling towers. They describe the methods and accuracy of measurements relating to the external surfaces of buildings or their edges, limiting the task to the description of the course for main vertical axis of buildings in the form of a broken line connecting the axial points on the subsequent horizontal intersections. In consequence, the presentation of their results is mostly the image shown in Figure 3. In this study, we assumed, that the instantaneous main axis of the building at the time of measurement, is determined by the geometric centers of the main core structure determined by measuring details of its inner contour on consecutive horizontal sections.

![Photo 1. General view of the pagoda Zhenfeng for which it was required to determine the inclination of main axis.](image)

The concept of main vertical axis delimitation presented in this paper is referred to the work requirements imposed on inventory Zhenfeng pagoda’s in China (Fig. 1). To clarify the scope of the measurement it should be noted that the prescribed requirements to determine the state of the structure inclination related to two parts of buildings: the first - from the floor at the first floor level to the last floor (sixth) and the second part - which concerned to the spire pagoda’s (from the ceiling of last floor to the spherical bulge on spire). Exterior view of discussed pagoda is shown on Photo 1 and its horizontal dimensions in selected sections are shown in Fig. 1a and 1d. Division of tasks into two such fragments results from the building construction, which in the bottom part is a stone-brick monolith and in the upper part - spire - is an enclosed wooden structure. For such separated parts we determined the position of resultant axis as an
axis of rigid solids, and additionally the position of the axis of the entire tower (from the first floor level to the spherical bulge on spire). These measures are used to designate components of their own building loads.

Fig. 1. Dimensional sketches of selected Zhenfeng horizontal floor plans.

It was assumed that we will not consider the technology of points position measurement which describe the buildings cross-section geometry on each level (fig. 1 presents their variety on consecutive storey). It was only assumed that on their basis coordinates $x_i, y_i, z_i$ were determined for the geometric center of the tower shaft on each level of tower and that they determine the course of the main structure axis. Lateral misalignment in these centers, which represents the main axis of the building, result from inaccurate erection of the structure, its deformation over the period of exploitation and identification errors in the performed measurements. Drawing 2 presents the spatial distribution of these points presents a drawing 2. Its course in 3D space, as an edge of the intersection of two perpendicular planes, will be determined from these points by approximation with least square method (fig 3).
Determining the main axis course of the tower buildings

Based on the spatial position of the determined central points in the successive horizontal cross-sections of the tower buildings, its main axis will be presented as an intersection of two perpendicular to each other planes:

\[ \pi_1 : A_1 x + B_1 y + C_1 z + D_1 = 0, \]  
\[ \pi_2 : A_2 x + B_2 y + C_2 z + D_2 = 0, \]

for which the sum of squared distance from the determined points to these planes are the smallest:

\[ \sum_{i=1}^{n} |\pi_1 P_i|^2 = \sum_{i=1}^{n} (d_i')^2 \rightarrow \text{min}, \]  
\[ \sum_{i=1}^{n} |\pi_2 P_i|^2 = \sum_{i=1}^{n} (d_i'')^2 \rightarrow \text{min} \]

Solution to the problem will be divided into two stages:
- Stage 1, containing the determination of the plane equation \( \pi_1 \), for which function (3) is performed,
- Stage 2, containing the determination of the plane equation \( \pi_2 \) which is perpendicular to the plane \( \pi_1 \) and for which function (4) is performed.

Stage 1

For the distances of points \( P_i = \{P_1, P_2, ..., P_n\} \) from the plane \( \pi_1 \), expressed by formula:

\[ d_i' = \frac{|A_1 x_i + B_1 y_i + C_1 z_i + D_1|}{\sqrt{A_1^2 + B_1^2 + C_1^2}}. \]
and assuming that
\[ \sum_{i=1}^{n} |\pi_i P_i|^2 = \sum_{i=1}^{n} (d'_i)^2 \rightarrow \min, \]  
(6)

observation equation for each distances will become:
\[
dd'_i = \frac{\partial d'_i}{\partial A_1} dA_1 + \frac{\partial d'_i}{\partial B_1} dB_1 + \frac{\partial d'_i}{\partial C_1} dC_1 + \frac{\partial d'_i}{\partial D_1} dD_1 + d_{sorybl} \tag{7}
\]

where:
\[
\frac{\partial d'_i}{\partial A_1} = \pm x_i \cdot \left[ (A_i^0)^2 + (B_i^0)^2 + (C_i^0)^2 \right] - A_i^0 \cdot \left[ A_i^0 x_i + B_i^0 y_i + C_i^0 z_i + D_i^0 \right],
\]
\[
\frac{\partial d'_i}{\partial B_1} = \pm y_i \cdot \left[ (A_i^0)^2 + (B_i^0)^2 + (C_i^0)^2 \right] - B_i^0 \cdot \left[ A_i^0 x_i + B_i^0 y_i + C_i^0 z_i + D_i^0 \right],
\]
\[
\frac{\partial d'_i}{\partial C_1} = \pm z_i \cdot \left[ (A_i^0)^2 + (B_i^0)^2 + (C_i^0)^2 \right] - C_i^0 \cdot \left[ A_i^0 x_i + B_i^0 y_i + C_i^0 z_i + D_i^0 \right],
\]
\[
\frac{\partial d'_i}{\partial D_1} = \pm 1 \cdot \sqrt{(A_i^0)^2 + (B_i^0)^2 + (C_i^0)^2}.
\]

and the coefficients \((A_i^0); (B_i^0); (C_i^0); (D_i^0)\) are the approximate values of the coefficients for determining plane \(\pi_1\). These coefficients are determined by formula for the plane equation which passes through the three points \(K, L, M\) chosen at random:
\[
\begin{vmatrix}
 x - x_K & y - y_K & z - z_K \\
 x_L - x_K & y_L - y_K & z_L - z_K \\
 x_M - x_K & y_M - y_K & z_M - z_K \\
\end{vmatrix} = 0 \tag{8}
\]

Coefficients of the approximated plane \(\pi_1\) will be obtained by solving the system of observation equation (7) using the least squares method. It is obvious that this approximation makes sense for at least four points determining the course of the building axis.

Stage 2

After determining the coefficients \(A_1, B_1, C_1, D_1\) of the determined plane \(\pi_1\) we can determine the plane \(\pi_2 : A_2 x + B_2 y + C_2 y + D_2 = 0\). According to the assumption the plane \(\pi_2\) must be perpendicular to the previously determined plane \(\pi_1\). As in the case of plane \(\pi_1\) the main condition for determining plane \(\pi_2\) will be
\[
\sum_{i=1}^{n} |\pi_2 P_i|^2 = \sum_{i=1}^{n} (d''_i)^2 \rightarrow \min. \tag{9}
\]
where:
\[ d_i'' = \frac{|A_i x + B_i y + C_i z + D_i|}{\sqrt{A_i^2 + B_i^2 + C_i^2}}. \] (10)

Observation equation for each distance will become:
\[ dd_i'' = \frac{\partial d_i''}{\partial A_2} dA_2 + \frac{\partial d_i''}{\partial B_2} dB_2 + \frac{\partial d_i''}{\partial C_2} dC_2 + \frac{\partial d_i''}{\partial D_2} dD_2 + d_{err}'' \] (11)

where:
\[ \frac{\partial d_i''}{\partial A_2} = \pm x_i \cdot \left[ \left( A_0^0 \right)^2 + \left( B_0^0 \right)^2 + \left( C_0^0 \right)^2 \right] - A_0^0 \cdot A_2^0 x_i + B_2^0 y_i + C_2^0 z_i + D_2^0, \]
\[ \frac{\partial d_i''}{\partial B_2} = \pm y_i \cdot \left[ \left( A_0^0 \right)^2 + \left( B_0^0 \right)^2 + \left( C_0^0 \right)^2 \right] - B_0^0 \cdot A_2^0 x_i + B_2^0 y_i + C_2^0 z_i + D_2^0, \]
\[ \frac{\partial d_i''}{\partial C_2} = \pm z_i \cdot \left[ \left( A_0^0 \right)^2 + \left( B_0^0 \right)^2 + \left( C_0^0 \right)^2 \right] - C_0^0 \cdot A_2^0 x_i + B_2^0 y_i + C_2^0 z_i + D_2^0, \]
\[ \frac{\partial d_i''}{\partial D_2} = \pm 1 \cdot \left[ \left( A_0^0 \right)^2 + \left( B_0^0 \right)^2 + \left( C_0^0 \right)^2 \right]. \]

and the coefficients \((A_0^0); (B_0^0); (C_0^0); (D_0^0)\) are the approximate values of plane \(\pi_2\) which is perpendicular to plane \(\pi_1\). Realizing additional assumption about perpendicularity of these two planes \(\pi_1\) and \(\pi_2\) we can enter on the unknown \(A_2^0, B_2^0, C_2^0, D_2^0\) condition of the form:
\[ F(A_2, B_2, C_2, D_2) = A_1 A_2 + B_1 B_2 + C_1 C_2 = 0 \] (12)

From the condition (12) we obtain an additional equation that can be attached to the equations (11). The resulting system of equations, after putting appropriate weights into equals which force planes perpendicular, is solved by the least squares method. Determined equation planes constitute the equation of a straight line (Fig. 3) in form:
\[
\begin{align*}
    A_1 x + B_1 y + C_1 y + D_1 &= 0 \\
    A_2 x + B_2 y + C_2 y + D_2 &= 0
\end{align*}
\] (13)

Spatial parameters for course of this line, expressed by deviation angle from the vertical and the deflection angle in relation to the horizontal axis of structures, constitute the goal of proposed calculations.
Taking into account the location of the coordinate system, as associated with the axes of building symmetry, these angles can be determined after calculating the vector $\vec{v}$ from the following dependence

$$\vec{v} = [A_1, B_1, C_1] \times [A_2, B_2, C_2].$$  \hspace{1cm} (14)

or which the angle in relation to plane OXY has notation

$$\varphi = \arcsin \left( \frac{|\vec{n} \cdot \vec{v}|}{|\vec{n}| \cdot |\vec{v}|} \right).$$  \hspace{1cm} (15)

where:

$\vec{v}$ - directional vector of straight line $l$,

$\vec{n}$ - perpendicular vector to plane OXY.

Orthogonal projection of $l$ -line on OXY plane of assumed coordinates system allows to determine the direction of the building’s main axis inclination. For this purpose we shall calculate coordinates of two points at the determined major axis, respectively, on the first floor and on last floor of the building. This angle, indicated in figure 4 by symbol $\alpha$, is the second parameter describing the general inclination of the building.

REFERENCES


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