1. INTRODUCTION

Geometrical shaping of roads and railway tracks belongs to an interesting area of the engineering of geodesy. Other disciplines also deal with this subject matter, but the widest is considered in geodesy. In this paper there will be presented a geometrical shaping concept of the new roundabout called turbo-circuits. The history of a roundabout reaches the end of the XX century when they were called squares around monument or monumental buildings. Circular movement was held round a central two-way point. One-way movement was introduced in the beginning of the XX century. The vehicles on the roundabout were given preference to vehicles driving on the roundabout. The growth of intensity of movement caused the formation of vehicle queues. The growth of the diameter of the central island became the solution for this problem. In 1966 in Great Britain was introduced the principle of precedence of vehicles which are on the roundabout. It allowed the reduction of the diameter of the central island considerably. The practical investigation of the relating safeties of movement and traffic capacities caused, that as early as the eightieth of the XX century the small and middle-sized roundabouts became popular not only in England but also in many European countries.

A roundabout is defined as a crossing of roads with a central island which forces the movement round this island on a one-way road. The movement is held in the opposite direction to the movement of the hands of a clock. In comparison to traditional crossings roundabouts have the advantage of mainly safety, it is caused by the smaller quantity of possible points of conflict. Moreover, significant influence on the growth of safety of movement has the limitation of speed on the roundabout, thanks to what drivers have more time to undertake proper decisions. In professional literature it was underlined, that usually larger capacity on roundabouts stands out in comparison to canalized crossings what considerably influences the pollution of the environment. The central island most often has the shape of a wheel. In recent years appeared an idea of two circles shifted with the width of a belt - called turbo-circuits (turbo roundabout). The description of these roundabouts and idea of their formation is introduced mainly in foreign works [1-4], and also Polish ones linked to source works, i.e. [5-7]. It this study is shown the modification of turbo-circuits with an ellipse in the central island. It is indicated by a possibilities of applying a Pascal’s limacon and for shaping of the turbo-circuits. The preliminary considerations the relating uses of turbo-circuits for help the contained ellipses are in work [Grabowski 2008].

2. THE ROUNDABOUTS

The roundabouts shaped like a circle for the sake of size of the central island and outer diameter divide according to, by design instruction of the intersection of roads on four groups: mini, small, medium and large size. In the table 1 is contained the radius of circle requirements by Guidelines of planning roads (Tracz and other, 2001).
### Table 1. Design requirements concerning roundabout

<table>
<thead>
<tr>
<th>Type</th>
<th>Building-up area</th>
<th>Outside building-up area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameter [m]</td>
<td>Applying roundabout</td>
</tr>
<tr>
<td></td>
<td>island</td>
<td>outer</td>
</tr>
<tr>
<td>Roundabouts</td>
<td>Amount of belts</td>
<td>Diameter [m]</td>
</tr>
<tr>
<td>Mini</td>
<td>-</td>
<td>4-10</td>
</tr>
<tr>
<td>Large</td>
<td>-</td>
<td>&gt;37</td>
</tr>
</tbody>
</table>

3. **THE IDEA OF WORKING OF TURBO-CIRCUIT**

The idea of turbo-circuits was created in Holland 15 years ago. The main principle depends on the maneuver of interlacing despite the occurrence of two lanes of movement on the roundabout which do not intertwine. A vehicle coming to the roundabout is forced to choose one of the lanes which is determined by the driver.

![Fig. 1. View of shield of typical turbo - circuit [3].](image)
The driver already before the crossing has to decide in what direction to drive because the special separators between the lanes of movement separate the cars, which is an obstacle making it impossible to cross and there is no possibility to change a lane on the roundabout (Fig. 1).

The turbo-circuit is determined by two or more concentric circles shifted by the axis of the roundabout with the width of a lane. This shift causes, that entering on the internal lane the vehicle, after crossing the axis is on the extreme belt automatically. This way the maneuver of interlacing by two lanes on the roundabout is eliminated in the area of shield of the roundabout. The shield of turbo-circuit can be shaped both from semicircles, and also from quarters of circles.

The turbo-circuit shaped from two semicircles, with shifted centers along settled straight lines about the same interval (the width of belt) in relation to the centre of the shield. It is easy to notice, that such a way of formation possesses the special reason, when the intensity of movement is in one direction. If it is hard to distinguish one direction which is predominant then the turbo-circuit can be shaped from four quarters of a circle shifted by the axes so, that their centers create a square.

The form of a turbo-circuit can significantly differ. In case of a three-lane-entering of turbine roundabout the shape of the central island can have an asymmetric shape. Of course the final shape of the turbo-circuit depends on the existing conditions of movement, as well as the designer's imagination.

In professional literature there are mentioned the following characteristic features of turbo-circuits:

- the spiral horizontal road signs which should be joined with a spiral form of the shield of roundabout;
- the possibility of choice of direction of driving only on the inlet, without the possibility of changing a lane;
- two-belt inlets - at least two, and at least one two-belt outlet;
- no more than two belts of movement on the shield of roundabout in the zone of inlet (what increases the safety);
- the lack of possibility of turning round on one of the directions of movement (in the case of lack of extension of ring of the roundabout);
- the separating shield of lanes of movement.

We should underline clearly, that turbo-circuit is a comparatively young solution, therefore there is a lack of normative regulations defining the basic elements of organization of movement, rules for marking signs for this type of roundabouts, which has a special meaning in this type of solution. Well designed horizontal marking on entrance to the lane, as well as on the road sign before the roundabout will allow the driver to recognize clearly the way of driving in the chosen direction. The necessary marking should come from the proper group of signs in the branch p-8, that is, arrows illustrating from which side we can ride not only onto the entrance but inside the roundabout. If the given direction of movement turning left requires a detour of the roundabout, it then should be clearly marked on the lane of the road. A separator makes it impossible to change a lane, inside and on entering the roundabout. The separator should be a concrete element 30 cm wide and 7 cm high. The separators should be painted bilaterally by a continuous line about 10 cm thick, 15 cm from the edge of the concrete and additionally there should be a line created from punctual reflective elements laid 20 cm from the centre of the continuous line. To make possible an outflow of water the separators should have a non-continuous shape.
4. SHAPING OF TURBO-CIRCUIT BY MEANS OF ELLIPSE

An ellipse is defined as a gathering of points of an area, for which the sum of distances from two data of points \( F_1 = (-c,0) \) and \( F_2 = (c,0) \) is constant and larger than from distances of these points. The points \( F_1 \) and \( F_2 \) are called focuses of ellipse. The equation of ellipse which focuses lie symmetrically on axis \( 0x \) in relation to the beginning of the coordinate system (Fig. 2) has the form

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

where: \( a \) - great semi-axe, \( b \) - small semi-axe.

For all points on ellipse we have, e.g.

\[ |F_1P_1| + |F_2P_1| = |F_1P_2| + |F_2P_2| = |F_1P_3| + |F_2P_3| = 2a \]

Shaping the shield of turbo-circuit we can conduct similarly to the forming of roundabout by the circle utilization. Semi-ellipses about centres the shifted along axis about width of belt of shield roundabout \( s \) will be the base of creation of external edge of central island. In peculiarity the length of semi-axe both ellipses need not be equal.

It in aim of form of spiral belts of shield, should be outline next semi-ellipses about axises \( (a + s) \) and \( (b + s) \), that is enlarged about value of width of belt. He in this way be becomes created the internal belt of the movement after he will crossing axis it will stand up as external belt. Each the time the centre all semi-ellipses after one side of axis is common and shifted in relation to centre semi-ellipses the outlined after opposite side of axis about even value of width of belt \( s \). Obvious it is, that the value of semi-axe of applied ellipse can not any, the relation of length of large semi-axe to length of small semi-axe can not cross sure exacting the settlement of value. When \( a = b \) the bottom value of quotient \( a/b = 1 \), then the ellipse is the circle. We will establish on basis of range of size central island the size of relation of semi-axe ellipse which is the base of constructing the turbo-circuit valid for classic roundabouts. Founding, that creating ellipse the internal belt of movement of turbo-circuit can not be shaped by smaller radius than minimum admissible in case of circular roundabout, and also that none of semi-axe can be longer than maximum radius for the same group of roundabouts.

The radius of curvature of ellipse in any point \( (x_0, y_0) \) is expressed by formula

\[
r = a^2b^2 \frac{\sqrt{\frac{x_0^2}{a^4} + \frac{y_0^2}{b^4}}}{(\frac{x_0^2}{a^4} + \frac{y_0^2}{b^4})^3}
\]
The maximum values of curvature the ellipse assumes in points \( A_1 = (-a, 0) \) and \( A_2 = (a, 0) \). Substituting the co-ordinates of point \( A_2 = (a, 0) \) to formula (2) we will receive on minimum value of radius of ellipse curvature about data the semi - axe \( a \) and \( b \).

\[
r = \frac{b^2}{a}
\]

(3)

Acceptance of length one semi - axe and the minimum value of radius of curvature, it permits with above mentioned formula to mark admissible value of length of second semi - axe of ellipse.

5. SETTLEMENT OF RANGE OF CHANGEABILITY RELATION SEMI – AXES ELLIPSE

Let's trace establishing the relation of semi - axe of ellipse for small the and average turbo-circuit in built-up area.

In compliance with the Instruction [Tracz and other 2001] two - belt small circular roundabout in built-up area, it should have possessed since 17m the diameter of island to 25m. Accepting the maximum value of large semi - axe \( a = 12,5\text{m} \) as well as the minimum radius of curvature \( r = 8,5\text{m} \), from formula (3) we will receive the minimum value of length of semi - axe \( b \)

\[
b = \sqrt{a r} = \sqrt{12,5 \cdot 8,5} = 10,3\text{m}
\]

Value \( b = 10,3\text{m} \) is the minimum possible length of semi – axe \( b \) by maximum length of semi - axe \( a = 12,5\text{m} \), for which in no place of drive shaped by this ellipse track will not be larger curvature than recommended by Instruction. The upper border of quotient of semi - axe will carry out \( \frac{a}{b} = \frac{12,5}{10,3} = 1,21 \). Therefore basic project condition by shaping two-belts small rondo-circuit for help of ellipse is described by inequality

\[
1 \leq \frac{a}{b} \leq 1,21
\]

For \( a / b = 1 \) the curve is circle, meanwhile for \( a / b = 1,21 \) ellipse possesses maximum admissible flattening. The fulfilment of this condition has obviously to set by semi - axe which of length belongs to interval \(< 10,3\text{m};12,5\text{m} \>). If large semi-axe has maximum length \( a=12,5\text{m} \) then minimum length of a second semi-axe is \( b=10,3\text{m}, \) of cause it can assume value \( b \in <10,3; 12,5> \). For another fixed value \( a \) by designer suitable minimum value the second semi-axe is can read from table 2.

The borders of size of central island the two belts average roundabout in built-up area should be contained in interval for 25 m to 37 m. Accepting the maximum length of semi - axe \( a = 18,5\text{m} \) as well as the minimum radius of curvature \( r = 12,5\text{m} \), with formula (3) we will receive the minimum value of semi - axe \( b \)

\[
b = \sqrt{a r} = \sqrt{18,5 \cdot 12,5} = 15,2\text{m}
\]

<table>
<thead>
<tr>
<th>Size of round</th>
<th>Parameters of ellipse</th>
<th>The range of changeability of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>a [m] 8,5 9 10 11 12 12,5 - -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b [m] 8,5 8,7 9,2 9,7 10,1 10,3 - -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a/b 1,00 1,03 1,09 1,13 1,19 1,21 - -</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>a [m] 12,5 13 14 15 16 17 18 18,5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b [m] 12,5 12,7 13,2 13,7 14,1 14,6 15,0 15,2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a/b 1,00 1,02 1,06 1,09 1,13 1,16 1,20 1,22</td>
<td></td>
</tr>
</tbody>
</table>
The value of length of semi-axe \( b = 15.2 \text{ m} \) is the minimum value, if \( a = 18.5 \text{ m} \). The upper border of relation \( a/b \) will carry out, therefore the range of changeability of relation semi-axe \( a \) and \( b \) for average turbo-circuits in built-up area carries out. For \( a/b = 1 \) curve is circle, meanwhile for and \( a/b = 1.22 \) ellipse possesses maximum admissible flattening. They to fulfill passed project condition both semi-axe of ellipse have to be to comprise in interval of length 15.2 m to 18.5 m.

Carrying out analogous arguments for 2 - belt small and we will receive on built-up area average turbine roundabout the upper admissible values of quotient and / b which was has taken down in Table 3.

<table>
<thead>
<tr>
<th>Size of round</th>
<th>Parameters of ellipse</th>
<th>The range of changeability of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>( a ) [m]</td>
<td>10 11 12 12.5 - - - - - - - - - - - -</td>
</tr>
<tr>
<td></td>
<td>( b ) [m]</td>
<td>10 10.5 11.0 11.2 - - - - - - - - - - - -</td>
</tr>
<tr>
<td></td>
<td>( a/b )</td>
<td>1.00 1.05 1.09 1.12 - - - - - - - - - - - -</td>
</tr>
<tr>
<td>medium</td>
<td>( a ) [m]</td>
<td>12.5 13 14 16 18 20 22 23 23.5</td>
</tr>
<tr>
<td></td>
<td>( b ) [m]</td>
<td>12.5 12.7 13.2 14.1 15.0 15.8 16.6 17.0 17.1</td>
</tr>
<tr>
<td></td>
<td>( a/b )</td>
<td>1.00 1.02 1.06 1.13 1.20 1.27 1.33 1.35 1.37</td>
</tr>
</tbody>
</table>

The of turbo-circuits shaped from utilization the ellipse shield gives the designer on her geometry the larger possibilities of influence. Let's notice that circle used to in projecting circular roundabouts it is special ellipse by chance, so formulation this contains classic approach to projecting roundabout. The growth of value of quotient \( a/b \) causes the growth of flattening the ellipse. Using this fact, in dependence from direction structure and generic reigning on data crossing designer, it can more adjust to motive reigning on such crossing needs. Obvious it is, that all dimensions of semi-axe as well as their relations should fulfil introduced in tables 2 and 3. requirements. If it on data crossing steps out one predominant under in relation to intensity the direction of movement, the use of ellipse be indicated in formation the turbo-circuit then particularly (Fig.3).

![Fig. 3. Example of turbo-circuit from utilization the ellipses by predominant one direction of movement.](image)

They the larger difference in strength of intensity two directions of movement, this the ellipse about larger flattening should be applied. The exact relationships is pass on
present stage of investigations among these sizes hard, e.g., the difference of intensity and the value of flattening. This requires mainly observation and empirical investigations.

6. **SHAPING OF TURBO-CIRCUIT BY MEANS OF PASCAL’S LIMACON**

Pascal’s limacon is the geometrical place of points fulfilling condition $OM = OP \pm l$ (Fig. 4), by pole lies on circle about centre in point $(a/2, 0)$ and radius $a/2$. Pascal’s limacon equation in rectangular coordinates system possesses equation

$$(x^2 + y^2 - ax)^2 = l^2(x^2 + y^2), \quad a > 0, \ l > 0$$

in parametrical form:

\[
\begin{align*}
  x &= a \cos \varphi + l \sin \varphi \\
  y &= a \cos \varphi \sin \varphi + l \sin \varphi
\end{align*}
\]

in polar form:

$$\rho = a \cos \varphi \pm l$$

where $a$ is diameter received circle, meanwhile $l$ accepted length.

The curve shape is dependent from relation $a$ to $l$ (variant I, $\frac{a}{l} \leq \frac{1}{2}$; variant II, $\frac{1}{2} < \frac{a}{l} < 1$; variant III, $\frac{a}{l} > 1$). For aims of formation of roundabout can be useful only variant I for which $l \geq 2a$, (Fig. 4). Pascal’s limacon vertex possess co-ordinates $A = (a + l, 0)$ and $B = (a - l, 0)$. In case when $a = l$ Pascal’s limacon carries name cardioid. In variant I, if $a$ aims to zero by constant $l$ then Pascal’s limacon shape aims to circle about radius $l$, similarly if $l$ grows up to infinity by constant $a$, then shape larger and larger Pascal’s limacon approach to circle about radius $l$.

Taking under attention the shape of Pascal’s limacon seems, that he can be used near asymmetrical shield of roundabout, when the intensity on one main direction is considerably larger than on remaining (Fig. 5). It requires this the additional investigation of shape in dependence from quotient $\frac{a}{l} \leq \frac{1}{2}$.
7. CONCLUSIONS AND FINAL ATTENTIONS

Turbine roundabouts are a new solution, which possess in relation many advantages to circular roundabouts functioning universally. These roundabouts can be a supplement for the existing offer of circular roundabouts. The basic advantage of turbine roundabouts is the considerably larger efficiency of movement. The lack of interlacing on the shield of this roundabout causes, that the capacity of these turbo-circuits is larger by about 12% to 20% in comparison to classic circular roundabouts. The turbine shaped roundabout with an ellipse broaden in peculiarity an offer for crossings with an unequal amount of movement and diverse direction structure. The opinion of conditions of movement and the test of defining the mathematical description for ellipse shaped roundabout requires also the execution of many empirical investigations. In the present stage of considerations it can be supposed, that the capacity of ellipse sized roundabout will be larger than the circular turbine roundabouts. It will mainly depend on the size of the disproportion of intensity of movement on two main directions of movement.

The turbo-circuits are the formation of a crossing permitting a larger capacity of vehicles in a safe way similar to a one lane roundabout. This is a comparatively new form of crossing and its introduction, particularly in the initial period, requires the users' education and training of drivers with the rules of moving on such a crossing. Obviously preceded this should be with a suitable instruction on how to construct such a roundabout.

REFERENCES

Głębocka M. M. 2007: Koncepcja geometrii skrzyżowań z wykorzystaniem rond. (Conception of geometry of crossing from utilization the roundabouts). The Graduate's work. Technical University in Białystok.
Sawyer M., Słowińska M., Sakłak In. 2005: Bezpieczeństwo ruchu na rondach dwupasowych. (The safety of movement on roundabouts the two belts). Municipal transportation and Regional No. 2 Warsaw.