The branch-and-bound method and genetic algorithm in avoidance of ships collisions in fuzzy environment

Mostefa Mohamed-Seghir, Ph. D.
Gdynia Maritime University

ABSTRACT

Marine navigation consists in continuous observation of the situation at sea, determination the anti-collision manoeuvre. So it necessary to determine ship safe trajectory as a sequence of ship course changing manoeuvres. Each manoeuvre is undertaken on the basis of information obtained from the anti-collision system ARPA. This paper describes a method of safe ship control in the collision situation in a fuzzy environment based on a branch and bound method and a genetic algorithm. The optimal safe ship trajectory in a collision situation is presented as multistage decision-making process.

Keywords: safe ship control; optimal control; safe trajectories; a branch and bound method; genetic algorithm; ship control; fuzzy set theory

INTRODUCTION

The research for effective methods to avoidance ship collisions has become important with the increasing size, speed and number of ships participating in sea transport. Since when were debut the application of the ARPA the safety of maritime navigation has increased.

A new tendency within the contemporary domain of ship control involves an automation process of selecting an optimal manoeuvre or optimal safety trajectory, based on the information obtained from the anti-collision system [3, 4, 5, 8, 9]. This paper discusses in detail the ship optimal position determining process, involving stages of ship trajectory, based on the kinematic model. It is assumed that the motion of targets is uniform and occurs as a straight line. Due to the fuzziness of the process, individual approach of a particular officer.navigator, the decision-making process is also, to some extend, an ambiguous evaluation of the safe approach distance and safe time to avoid collision maneuver. Moreover, it is assumed that an optimal safe trajectory in a collision situation is a multistage decision-making process in a fuzzy environment. The maneuverability parameters of the ship as well as the navigator’s subjective assessment are taken into consideration in the process model figure 1.

PROCESSES OF SAFE SHIP CONTROL IN FUZZY ENVIRONMENT

In order to describe the safe ship trajectory, the movement of a ship returning by her rudder in deep water was described. Still, the safe ship trajectory may prove little inefficient to evaluate the ship properties. Therefore in order to precisely calculate a ship dynamic properties we used parameters of the transfer function or the advance time $t_a$ and maximum angular speed $\omega$ [2, 5, 6].

The model of safe ship trajectory can be represented by the state equation:

$$f (X, S) \rightarrow X \times S$$

(1)
The branch-and-bound method and genetic algorithm in avoidance of ships collisions in fuzzy environment

\[ X_{k+1} = f(X_k, S_k), t=1,2,\ldots, n \]  (2)

where:
\[ X_t, X_t \in X = \{a_0, a_1, \ldots, a_{p-1}, a_p, a_{p+1}, \ldots, a_n\} \] - set of real ship position co-ordinates
\[ S_t \in U = \{c_0, c_1, \ldots, c_m\} \] - control set

The process comes to an end when a ship attains back points (final points) called the final states \( W \subset X \[ [1, 2 \right]. \)

\[ W = \{a_{p+1}, a_{p+2}, a_n\} \]  (3)

The set of final states must satisfy this condition:
\[ c_{opt}, \mu_R \leq \mu_{Rsafe} \]  (4)

where:
\[ c_{opt} = (\psi_{opt}, V_{opt}) \] - optimal control,
\[ \mu_R \] - membership function of fuzzy set collision risk.

This membership function of fuzzy set collision risk can be presented in the form Figure 2, [2, 3]:

\[ Z \subseteq X \times X \]  (5)

\[ \mu_R: X \times X \rightarrow [0,1] \in R \]  (6)

\[ \mu_R(k,j) = \frac{1}{\exp(\lambda_{RD}(k,j)CPA^j + \lambda_{RT}(k,j)TCPA^j)} \]  (7)

**Fig. 2. Graphical interpretation membership function of fuzzy set collision risk**

Similarly, the membership function of fuzzy set of goal can be written in the form:

\[ G \subseteq X \times S \]  (8)

\[ \mu_G: X \times U \rightarrow [0,1] \in R \]  (9)

\[ \mu_G(k,j) = 1 - \frac{1}{\exp(\lambda_g(k,j)CPA^j)} \]  (10)

Now, it the membership function of fuzzy set constraints must be defined as constraints of maneuver at each step:

\[ C \subseteq X \times S \]  (11)

\[ \mu_C: X \times U \rightarrow [0,1] \in R \]  (12)

\[ \mu_C(k) = \frac{1}{\exp(\lambda_c(k)|V\cos(\psi) - V\cos(\psi-1)|CPA^j)} \]  (13)

The fuzzy set decision is determined as the fuzzy set \( D \subseteq X \times S \). It is a result of an operation "∗" of the fuzzy set of a goal and fuzzy set of constraints:

\[ D = G \ast C \]  (14)

\[ \mu_D(\ldots) = \mu_G(\ldots) \ast \mu_C(\ldots) \]  (15)

where:
\[ \lambda_{RD}, \lambda_{RT}, \lambda_c \] - navigator’s subjective parameters,
CPA - the Closest Point of Approach,
TCPA - the Time to Closest Point of Approach,
\[ V \] - ship speed,
\[ \Psi \] - ship course.

In order to find a solution to this the author proposed the following methods.

**BRANCH-AND-BOUND METHOD**

A fuzzy decision is a result of a certain compromise between these sets \( G \) (fuzzy set of goals) and \( C \) (fuzzy set of constraints), if the trajectory is called sequence states attained, then membership function of fuzzy set decision define as [3, 4].

\[ \eta_0 = \mu_C(S_0) \wedge \mu_G(X_1) \]

\[ \eta_1 = \mu_C(S_0) \ast \mu_G(X_0) \ast \mu_C(S_1) \wedge \mu_G(X_2) = \eta_0 \ast \mu_C(S_1) \wedge \mu_G(X_2) \]  (17)

\[ \eta_k = [\mu_C(S_0) \ast \mu_G(X_k)] \wedge [\mu_C(S_{k+1}) \ast \mu_G(X_{k+2})] \wedge [\mu_C(S_{k-1}) \ast \mu_G(X_{k-2})] = \eta_{k-1} \wedge [\mu_C(S_{k+1}) \ast \mu_G(X_{k+2})] \]  (18)

\[ \eta_{n-1} = [\mu_C(S_{n-1}) \ast \mu_G(X_{n-1})] \wedge [\mu_C(S_{n-1}) \ast \mu_G(X_{n})] \wedge [\mu_C(S_{n-1}) \ast \mu_G(X_{n})] = \eta_{n-2} \wedge [\mu_C(S_{n-1}) \ast \mu_G(X_{n})] \]  (19)

If range controls a mount \( S_0, S_1, \ldots, S_k, k < n-1 \), it for each \( L > k, L < n-1 \) it gets

\[ \eta_k \geq \eta_L \]  (19)
From this inequality emerge, that each value $\eta_L$ cannot be greater than value $\eta_k$ in other case at use of operation minimum $\land$. This way, it is possible to ascertain progressing, that inequality gets:

$$\eta_k \geq \eta_n = \mu_D(S_0, S_1, ..., S_{n-1}|X_0)$$  \hspace{1cm} (20)

To suppose, that it obtain k control stage and certain state of process state, now it must be choose optimal state from states achieved earlier.

To continue this procedure until we obtain final state, the process ends and we get optimal safe ship’s trajectory in collision situation.

**GENETIC ALGORITHM METHOD (AG)**

In the context of the multistage fuzzy control, an individual approach is understood as a sequence of particular control stages $S_0, ..., S_{n-1}$. According to this approach of an individual, the fuzzy environment is assessed using a membership function of fuzzy decision. On the basis of this decision several potential solutions can be selected. The set of these solutions is termed as the population. It is assumed that the algorithm will operate on a population of a certain size, which is initially randomly generated. Some members of the population, which play the role of “parents”, are reproduced by crossover and mutation,
whereas new solutions are „children”, The best and strongest of them all “survives”, this means that it participate further in this process. At the end of the process one can expect to find a very good, or perhaps even the optimal solution.

Determining a safe trajectory of a ship, as formulated above, and finding the optimal sequence can be done through [1, 2, 6, 7]:

$$\mu_D(S_1, S_2, ..., S_n | X_0) = \max_{S_0, ..., S_n} \mu_D(S_0, S_1, ..., S_n | X_0) =$$

$$= \max_{S_0, ..., S_n} \left[ \mu_C(S_0) \land \mu_C(X_1) \land \mu_C(S_1) \land \mu_C(X_2) \land \ldots \right]$$

Where at each stage (t-1), control $S_{t-1} \in U$, the fuzzy constraints $\mu_C(S_{t-1})$ are imposed, and at stage t on state $X_t$ the fuzzy goals $\mu_C(X_t)$ are imposed.

Before a genetic algorithm can be used some assumptions must be made as follows:

- the problem is a series of controls $S_0, ..., S_{N-1}$, due to the relief of genetic algorithm and simplification of the results analysis.
- the problem is a series of controls $S_0, ..., S_{N-1}$, due to the relief of genetic algorithm and simplification of the results analysis.
- we assumed that a ship speed is constant and the control $S_{t+1}$ at stage $t + 1$ is defined as the angle of the course $\Psi_{t+1}$, relative to the previous angle of the course $\Psi_t$ at stage $t$,
- the actual encoding does not change. We assumed that the real representation of each gene $S_t \in [0.360]$ is evenly distributed, which is natural in this case,
- purpose function of each individual is a membership function of the fuzzy decision type, of minimum ($\land$) given by the formula,

$$\mu_D(S_0, S_1, ..., S_n | X_0) =$$

$$= \mu_C(S_0) \land \mu_C(X_1) \land \mu_C(S_1) \land \mu_C(X_2) \land \ldots$$

- individuals with the highest value of the purpose function have the largest share in the next parental population, and weaker individuals are rejected in the selection process,
- selection process is reproduction - proportional or ranked,
- cross-sectional averaging is used as the most suitable for encoding floating point,
- there occur perturbations of gene mutations, acting in accordance with the Cauchy distribution with a certain probability of occurrence in successive generations,
- creating the next generation does not reject all parental individuals of the population, but the best of them attach to the descendants of individuals,
- stopping conditions should involve at least the assumption that the algorithm terminates the calculation when the improvement is smaller than a certain threshold.

**SIMULATION RESEARCH**

The exemplary results of the performed computer simulations are as follows.

The first, the computer simulation involved 20 targets. Through the adjustments we can determine whether the designated safe trajectories are indeed optimal (Fig.4).

<table>
<thead>
<tr>
<th>Target</th>
<th>$N_j$</th>
<th>$D_j$</th>
<th>$V_j$</th>
<th>$\Psi_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>220.00</td>
<td>3.90</td>
<td>15.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>287.00</td>
<td>6.80</td>
<td>14.00</td>
<td>27.00</td>
</tr>
<tr>
<td>3</td>
<td>315.00</td>
<td>4.20</td>
<td>5.00</td>
<td>180.00</td>
</tr>
<tr>
<td>4</td>
<td>330.00</td>
<td>7.00</td>
<td>10.00</td>
<td>150.00</td>
</tr>
<tr>
<td>5</td>
<td>335.00</td>
<td>3.30</td>
<td>11.00</td>
<td>297.00</td>
</tr>
<tr>
<td>6</td>
<td>339.00</td>
<td>5.30</td>
<td>10.00</td>
<td>90.00</td>
</tr>
<tr>
<td>7</td>
<td>341.00</td>
<td>7.40</td>
<td>12.50</td>
<td>90.00</td>
</tr>
<tr>
<td>8</td>
<td>352.00</td>
<td>13.10</td>
<td>16.00</td>
<td>162.00</td>
</tr>
<tr>
<td>9</td>
<td>0.00</td>
<td>10.50</td>
<td>2.00</td>
<td>180.00</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
<td>7.00</td>
<td>10.00</td>
<td>117.00</td>
</tr>
<tr>
<td>11</td>
<td>7.00</td>
<td>8.10</td>
<td>14.40</td>
<td>225.00</td>
</tr>
<tr>
<td>12</td>
<td>20.00</td>
<td>8.80</td>
<td>14.00</td>
<td>225.00</td>
</tr>
<tr>
<td>13</td>
<td>21.00</td>
<td>7.00</td>
<td>10.50</td>
<td>135.00</td>
</tr>
<tr>
<td>14</td>
<td>37.00</td>
<td>5.00</td>
<td>7.00</td>
<td>135.00</td>
</tr>
<tr>
<td>15</td>
<td>26.00</td>
<td>2.20</td>
<td>14.00</td>
<td>45.00</td>
</tr>
<tr>
<td>16</td>
<td>45.00</td>
<td>5.90</td>
<td>14.50</td>
<td>285.00</td>
</tr>
<tr>
<td>17</td>
<td>142.00</td>
<td>6.40</td>
<td>15.00</td>
<td>0.00</td>
</tr>
<tr>
<td>18</td>
<td>161.00</td>
<td>3.10</td>
<td>10.50</td>
<td>27.00</td>
</tr>
<tr>
<td>19</td>
<td>309.00</td>
<td>3.20</td>
<td>11.00</td>
<td>153.00</td>
</tr>
<tr>
<td>20</td>
<td>50.00</td>
<td>3.90</td>
<td>2.00</td>
<td>210.00</td>
</tr>
<tr>
<td>Owen</td>
<td>11.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 4.** The result of simulation of the collision situation in passing with 20 moving targets. a) Branch-and-Bound Method, b) Genetic Algorithm
The branch-and-bound method and genetic algorithm in avoidance of ships collisions in fuzzy environment

In the case of the method of branch-and-bound. The ship holds his course given only after 24 minutes made the manoeuvre rate (ψ = 345°) to the left, after this stage, returns to the set course. After 6 minutes, the ship performs again a few manoeuvres to the left avoiding the last conflict of the object (9) before returning to a given course.

In the case of the method of branch-and-bound.

In this situation, the manoeuvre was performed on the right course (ψ₀ = 12.5°), the own ship returns to a given course by making some quick manoeuvres to the left passing few targets.

CONCLUSIONS

- In this paper it presented tow methods based on fuzzy set theory to solve task of optimal safe ship trajectory in collision situation, according to the international Rules of the Road at Sea.
- This work showed a possibility of using a branch-and-bound method and genetic algorithms in a fuzzy environment as a method to solve the problem related to determining an optimal safe ship trajectory in collision situations with the use of computer software.
- It is possible to effectively solve tasks of determining a safe trajectory of a ship as a multistage decision-making process in a fuzzy environment.
- A navigator controls the ship with according to his individual assessment of the risk collision. This is an individual decision, which is not only subject to a condition of passing a greater distance from its set point.
- This present paper showed that the suggested idea of a fuzzy set theory application is a promising way to solve the considered task and design novel anti-collision systems in the future. The fuzzy set theory can be applied in many domains.

BIBLIOGRAPHY


CONTACT WITH THE AUTHOR

Mostefa Mohamed-Seghir, Ph. D.
Faculty of Marine Electrical Engineering,
Gdynia Maritime University,
Morska 81-87
81-225 Gdynia, POLAND
e-mail: mosem@am.gdynia.pl