Floatability and stability of floating dock-docked ship system

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ABSTRACT

An analysis of floatability and stability of the floating dock-docked ship system is the subject of this paper. These properties are considered for a cubicoid box dock whose dimensions are close (almost identical) to those of SINE 212 CD dock designed by SINUS design office. The analysis is first of all aimed at determination of the above mentioned hydromechanical properties in the light of the relevant requirements of PRS dealing with minimum freeboard of the dock and its initial metacentric height. It has been concluded that the analyzed dock reveals the designed docking capability which is not constrained by the PRS requirements though the dock in question is fitted with the roofing unfavourable for dock stability. It has been shown that assessing the transverse stability of the dock as well as of docked ship (floating object) by means of a stability factor instead of metacentric height, is reasonable. The work was performed within the frame of EUREKA – E! 2968 ECOLOGICAL DOCK research project.

Key words: floating dock, docked object, floatability, stability, stability factor.

1. INTRODUCTION

Essential conditions of usefulness (functionality) of floating dock are, among other, possibilities of maintaining the required values of its floatability and stability.

Correctly designed dock should have such floatability (immersability and load-carrying ability) which can ensure, in the range of its allowable draughts, docking (bringing-in and lifting) the floating objects of demanded values of such parameters as weight and main dimensions. Hence the ensuring of the ability amounts to determining an appropriate design arrangement of the dock, i.e. its main dimensions and correct subdivision determining its ballasting capability.

Correctly designed and used dock must be transversely stable, i.e. both in the case of the whole floating dock/docked object system and the docked object itself.

Therefore the dock must have such features due to which, in every phase of its functioning, it can float in the upright position or at most with heel angle of a given allowable value. Also, it should be fitted with such technical devices and service manuals which make it possible to keep docked objects in the upright position effectively and permanently.

This paper presents results of identification and analysis of floatability and stability of a box-shaped cubicoid dock having its main particulars and performance parameters close to those of SINE 212-CD dock described in [1].

The research task in question was realized as the part of EUREKA – E! 2968 ECOLOGICAL DOCK project, titled: „Ecological floating dock – special hydromechanical problems”.

2. GEOMETRICAL AND HYDROSTATIC CHARACTERISTICS OF FLOATING DOCK

The main hydrostatical characteristics of cubicoidal floating dock, depending on its length $L$, draught $T$ and geometry of its transverse cross-section as in Fig.1, are expressed as follows:

\[ V(T) = \begin{cases} LBT & \text{for } T \in (0; h) \\ LBh + 2Lb(T - h) & \text{for } T \geq h \end{cases} \] (1)

Volumetric displacement of the dock:

\[ z_B(T) = \begin{cases} 0.5T & \text{for } T \in (0; h) \\ \frac{h^2(B - 2b) + 2bT^2}{2(B - 2b)h + 2bT} & \text{for } T \geq h \end{cases} \] (2)

Ordinate of the centre of the volume $V(T)$:

\[ r_0(T) = \begin{cases} \frac{B^2}{12T} & \text{for } T \in (0; h) \\ \frac{b(3B^2 - 6Bb + 4b^2)}{6[ Bh + 2b(T - h)]} & \text{for } T \geq h \end{cases} \] (3)

Initial transverse metacentric radius:

The hydrostatical characteristics $V(T), z_B(T)$ and $z_M(T)$ of the dock whose dimensions $L, B, b$ and $h$ are given in Fig.1 and Tab.1, obtain numerical values given in Tab.2 and shown in Fig.2.

\[ z_M(T) = z_B(T) + r_0(T) \]

Ordinate of the centre of the volume $V(T)$:

\[ z_B(T) = \begin{cases} 0.5T & \text{for } T \in (0; h) \\ \frac{h^2(B - 2b) + 2bT^2}{2(B - 2b)h + 2bT} & \text{for } T \geq h \end{cases} \] (2)

Ordinate of the centre of the volume $V(T)$:

\[ r_0(T) = \begin{cases} \frac{B^2}{12T} & \text{for } T \in (0; h) \\ \frac{b(3B^2 - 6Bb + 4b^2)}{6[ Bh + 2b(T - h)]} & \text{for } T \geq h \end{cases} \] (3)

Initial transverse metacentric radius: $r_0(T)$

The hydrostatical characteristics $V(T), z_B(T)$ and $z_M(T)$ of the dock whose dimensions $L, B, b$ and $h$ are given in Fig.1 and Tab.1, obtain numerical values given in Tab.2 and shown in Fig.2.

Tab. 1. Main particulars of the dock.

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Length of pontoon and side walls</td>
<td>$L$</td>
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<tr>
<td>Total breadth</td>
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<td>42.00 m</td>
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<tr>
<td>Side wall breadth</td>
<td>$b$</td>
<td>4.00 m</td>
</tr>
<tr>
<td>Pontoon depth</td>
<td>$h$</td>
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</tr>
<tr>
<td>Dock mass</td>
<td>$P_d$</td>
<td>8200 t</td>
</tr>
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<td>Height of dock mass centre</td>
<td>$z_{G1}$</td>
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</tr>
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<td>Height of keelblocks</td>
<td>$s$</td>
<td>1.80 m</td>
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Tab. 2. Values of hydrostatic characteristics of the dock of the dimensions as in Tab.1.

<table>
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<tr>
<th>T [m]</th>
<th>V [m³]</th>
<th>zₙ [m]</th>
<th>rₙ [m]</th>
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<td>37188</td>
<td>6.777</td>
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<td>17.45</td>
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</table>

at the draught T = 3.375 m the functions rₙ(T) and zₘ(T) are discontinuous. Their corresponding values are given in the shadowed line.

4. FLOATABILITY OF THE DOCK

For the intended use of the dock first of all two its floatability states determined by its characteristic draughts, are of importance (see Fig. 1), i.e.:

* by the draught Tₘ, i.e. the draught at which effective bringing - in – the dock operation of the ship (object) at its draught Tₛ, is possible.

Fig.2 Hydrostatic characteristics of the dock.
Floatability and stability of floating dock-docked ship system

4.1. Nominal (design) draught $T_M$

The design draught of the dock, $T_M$, is directly determined as a result of simultaneous fulfillment of two requirements:

- the requirement, resulting from the classification society rules, that the minimum freeboard value $F_D$ defined as $F_D = H - T_M$ (see Fig. 1), is to be maintained, which means that:

$$H - T_M = F_D \geq F_D^*$$  \hspace{1cm} (6)

- the fundamental design requirement which determines the maximum draught $T_S$ of the ships (objects) intended for docking in the designed dock. The draughts $T_M$ and $T_S$ are mutually connected by the obvious relation:

$$T_M \geq T_S + h + s + w$$  \hspace{1cm} (7)

in which the remaining quantities have the following meanings:

- $h$ and $s$ – pontoon depth and height of keelblocks, respectively (see Fig.1)
- $w$ – distance between keel of the ship brought into the dock and the bed of keelblocks, in which some margin for ship trim is also included.

Assuming that the dock of the dimensions given in Tab.1 has to satisfy the freeboard regulations determined in PRS rules [2], i.e. that its design freeboard value $F_D$ cannot be smaller than $F_D^* = 1 \text{ m}$, one can determine, using the relationships (6) and (7) and under assumption that the distance $w \approx 0.30 \text{ m}$ is sufficient [3], that:

- the maximum draught of the dock, $T_M$, can be $T_M = 12 \text{ m}$ and that
- at the draught $T_M$ it is able to receive ships (objects) of the maximum draught $T_S = 6.525 \text{ m}$.

The above determined immersability features of the dock are merely potential ones as they result only from the linear dimensions of the dock and they at most determine extreme (inexceedable) values of the draughts $T_M$ and $T_S$. Ballasting capability of the dock, i.e. the maximum mass of water ballast which can be intentionally and effectively put into the dock, decides whether the draught values may be really reached.

Hence the water ballast mass, $M_R$ (or its volume $V_R$), necessary to immerse the dock to the draught $T = T_M = 12 \text{ m}$ should be determined and compared with the dock’s ballasting capacity, i.e. the maximum ballast mass $M_{M}$ (or its volume $V_{M}$) which can be taken in and distributed in its designed ballast tanks.

The dock of the draught $T$ will remain in equilibrium of flotation if for that draught the following inequality is satisfied:

$$P_D + P_W + M_R = \rho V(T)$$  \hspace{1cm} (8)

where:

- $P_D + P_W$ – total mass of the dock and its required working stores
- $M_R = \rho V_R(T)$ – the required mass of water ballast of the volume $V_R(T)$
- $V(T)$ – volumetric displacement of the dock
- $\rho$ – density of ballast and overboard water.

Hence the water ballast volume $V_R$, necessary to reach the draught $T$, is as follows:

$$V_R(T) = V(T) - \frac{P_D + P_W}{\rho}$$  \hspace{1cm} (9)

Assuming that:

- $P_D = 8200 \text{ t}$ (see Tab.1)
- $P_W = 150 \text{ t}$ (acc. [1])
- $\rho = 1.005 \text{ t/m}^3$ (with accounting for hull plating thickness)

one can state on the basis of the expression (9) that:

- the dock can be effectively immersed to the draught $T = T_M = 12 \text{ m}$ and at the draught
- some ballast volume margin amounting to $V_M - V_R = 2814 \text{ m}^3$, equivalent to about 9.3% of the dock ballasting capacity $V_M = 30333 \text{ m}^3$ (see Tab.3), still remains.

4.2. Dock’s operational draught $T_P$

The range of possible values of the dock’s operational draught $T_P$ (see Fig.1) is in particular determined by the condition of maintaining the minimum freeboard value $F_D^*$ = 0.20 m compliance with the PRS rules [2]:

$$F_D = h - T_P \geq F_D^*$$  \hspace{1cm} (10)

The resulting maximum value of the draught $T_P$ = 12 m determines the maximum lifting capacity of the dock, $U$, or/and its maximum load-carrying ability $N$ equal to the maximum mass of the ship (object) which can be docked in it.

And, in accordance with the obvious equation:

$$U + P_D = N + P_W + M_{RE} + P_D = \rho V(T_P')$$  \hspace{1cm} (11)

where:

- $M_{RE} = \rho V_{RE}$ – residual ballast mass

the following can be stated:

the maximum buoyancy of the dock is:

$$\rho V(T_P') = 22783 \text{ t}$$

its maximum lifting capacity is:

$$U = N + M_R + P_W = 14583 \text{ t}$$

5. TRANSVERSE STABILITY OF THE FLOATING DOCK - DOCKED SHIP SYSTEM

The problem of transverse stability of floating dock is here considered in two practically distinct aspects. Two cases of the stability are analyzed and assessed, namely:

- of the entire dock-ship system when the docked ship rests with its whole length on keelblocks of the floating dock
- of the ship itself in every phase of its docking in/out process.
In both the cases only the so called initial stability, i.e that considered only within the range of small heel angles, is investigated, that, in real dock working conditions, fully covers practical problems of its stability.

An initial stability measure is assumed the stability factor \( w \) defined in Appendix I and identified as \( w_{SD} \) factor for the dock-ship system and \( w_{SS} \) factor for the ship itself.

Positive values of the factor \( (w > 0) \) mean that the considered object is unconditionally stable; whereas negative ones \((w < 0)\) show its initial absolute unstability.

The unconditional stability of docked object (ship) is here assumed the sufficient and unique criterion of its stability. Whereas the stability of the dock-ship system is assessed from the point of view of formal (legal) requirements, i.e. the criteria \( w_{SD} \) whose values for the considered system are determined on the basis of the relevant PRS rule requirements [2]. It means that in this case, is of importance the relative stability of the object, which takes place and is acceptable only when the following inequality is satisfied:

\[
 w_{SS} > w_{SD}.
\]

The so defined stability is here determined and assessed for the dock-ship system in which:

- the dock characterized in Ch.2 and 3 operates within the range of draughts \( T_D \) \( < 3.00 \text{m} \); \( 12 \text{m} \) >
- the ship of the dimensions and hydrostatic features described in Appendix II, is the docked object.

### 5.1. Stability requirements

Stability of the dock-ship system is here assessed in the light of the relevant rule requirements (criteria) of PRS [2]. They mainly amount to the following regulations:

1) During ship lifting/launching operation the initial metacentric height of the dock, \( G_m \), cannot be smaller than 1.4 m. However for the docks of the load-carrying capacity \( N \geq 8000 \text{t} \) is recommended the height \( G_m \) to be not smaller than 3.0 m, that can be expressed as follows:

\[
 G_m(T) \geq G_m^{(1)}
\]

where:

\[
 G_m^{(1)} = (1.4 \text{m}; 3.0 \text{m})
\]

2) The static heel angle \( \phi \) of the dock-ship system, caused by the heeling moment, \( M_w \), resulting from the wind pressure \( p = 490 \text{ Pa} \), should not be greater than 1.5°, that can be written as follows:

\[
 \tan \phi = \frac{M_w}{9.81G_m(T)pV(T)} \leq 0.0262
\]

where:

\[
 \rho V(T_D) \text{ [t]} = \text{dock mass (buoyancy)}
\]

pertinent to its draught \( T_D \)

\[
 M_w(T_D) = 0.001p A_w(T_D) h_w(T_D) \text{ [kNm]}
\]

\[
 A_w(T_D) \text{ [m}^2\text{]} = \text{area of windage}
\]

\[
 h_w(T_D) \text{ [m]} = \text{height of centre of the area A}_w \text{ over dock waterline}.
\]

For use of the requirements (12) and (13) in this work they have been transformed into the form of an equivalent criterion of stability, i.e. the factors \( w_{SD} \) (see comments in App.I).

Hence:

the requirement (12) obtains the form:

\[
 w_{SD}(T_D) = \rho V(T_D) G_m(T_D) \geq
\]

\[
 w_{SD}^{(1)} = G_m^{(1)} \rho V(T_D)
\]

the requirement (13) obtains the form:

\[
 w_{SD}(T_D) \geq w_{SS}^{(2)} = 1.906 A_w(T_D) h_w(T_D)
\]

The limiting values of the dock stability factors \( w_{SD}^{(1)} \) and \( w_{SD}^{(2)} \) are presented in Tab.4, and the values \( w_{SS} \) are given for three variants of the dock design:

- for the dock without any roofing (a hypothetical one only)
- for the dock half covered by a movable roofing whose side windage area, a part of the entire windage area \( A_w(T_D) \), is equal to \( A_{w1} = 3230 \text{ m}^2 \) and spread over the length \( l \) equal to the dock’s half length \( L = 170 \text{ m} \)
- for the dock fully covered by the roofing whose side windage area is \( A_{w2} = (3230 + 2975) = 6205 \text{ m}^2 \) and spread over the full length of the dock.

<table>
<thead>
<tr>
<th>Dock draught ( T_D ) [m]</th>
<th>Dock volumetric displacement ( V ) [m³]</th>
<th>Required values ( G_m^{(1)} = 1.40 \text{m} )</th>
<th>Recommended values ( G_m^{(2)} = 3.00 \text{m} )</th>
<th>Dock covered over half length</th>
<th>Dock covered over full length</th>
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It can be observed (see Tab.4) that for maintaining the required relative stability of the dock the following criteria are decisive:

- the criterion (15) of the limiting values \( w_{SD}^{(2)} \), in the case of either fully or partly covered dock
- the criterion (14) of the limiting values \( w_{SD}^{(1)} \), in the case of not covered dock (hypothetical only).

### 5.2. Assessment of stability of the floating dock-docked ship system

In Tab.5 are contained values of the stability factors \( w_{SD}(T_D) \) and \( w_{SS}(T_D) \) together with their limiting values \( w_{SD} \) and \( w_{SS} \geq 0 \), calculated and presented in App. I (Tab.1.1).
Floatability and stability of floating dock-docked ship system

### APPENDIX I

#### Models for calculation of initial stability of the dock-ship system and docked ship itself

The necessary condition of transverse stability (appropriate stable equilibrium) of every free-floating object is its capability to generate „automatically” such moment $M_R(\phi)$ which, in the case of inclining the object by the angle $\phi$, will so act as to restore its initial position, back from the heel angle $\phi$. It means that the free-floating object will be stable then and only then if all its immanent (internal) forces and moments generate such resultant moment $M_R(\phi)$ whose derivative is:

$$\frac{\partial M_R(\phi)}{\partial \phi} < 0 \quad (1.1)$$

The moment $M_R(\phi)$ which satisfies the condition (1.1), is righting moment.

**1. Stability of the dock-ship system**

Internal forces and moments which act on a free-floating dock loaded with a ship, are the following (Comp. Fig.1.1a):

- the total weight of the dock (together with stores and ballast):
  $$F_{GD}(T_D)g$$
- buoyancy of the dock:
  $$F_{BD}(T_D)g = \rho g V(T_D)$$
- pressure resultant–load exerted on the dock by the ship:
  $$R_D(T_D)g = \left[ F_{GS} - F_{BS}(T_S^*) \right] g \quad (1.2)$$

$w_{SD}(T_D)$ and $w_{SS}$ values, and the values $w_{SD}(T_S^*)$ and $w_{SS}^*$

| Dock’s draught $T_D$ | $3.000$ | $3.175$ | $3.375$ | $4.000$ | $5.175$ | $6.000$ | $7.000$ | $8.000$ | $9.000$ | $10.000$ | $11.000$ | $12.000$
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|
| Stability factor $10^4 w_{SD}(T_D)$ | $79.55$ | $79.82$ | $24.14$ | $24.29$ | $24.69$ | $27.86$ | $31.70$ | $34.87$ | $38.10$ | $4.68$ | $48.15$ | $49.73$
| Limiting values $10^4 w_{SS}$ | | | | | | | | | | | | |
| Fully covered dock | $35.08$ | $34.81$ | $34.51$ | $33.58$ | $31.87$ | $30.70$ | $29.31$ | $27.95$ | $26.62$ | $25.32$ | $24.06$ | $22.82$
| Ship’s draught $T_S^*$ | - | - | - | - | - | $0.825$ | $1.825$ | $2.825$ | $3.825$ | $4.825$ | $5.800$ | $5.800$
| Stability factor $10^4 w_{SD}(T_S^*)$ | - | - | - | - | - | $-6.80$ | $-5.05$ | $-3.78$ | $-2.66$ | $-1.48$ | $0.89$ | $0.89$
| Stability criterion | $w_{SS}(T_S^*) > w_{SS}^* > 0$

(*) the shadowed values show a shortage of the relative stability – in the case of dock, and the unconditional stability – in the case of ship.

From the above presented data the following results:

1) the considered dock, when lifting the example ship of the draught $T_S = 5.8 \text{ m}$ and total mass $F_{GS} = 10032 \text{ t}$, i.e. that close to the largest ship (see p.4.1 and 4.2) permitted to be docked in it,

- does not satisfy the PRS stability requirements, if fully covered and the docking phase corresponds to the dock’s draught values from the interval : $7.000 \text{ m} \geq T_D \geq 3.375 \text{ m}$
- always satisfies (for every allowable dock draught $T_D \leq T_{P_{,TM}}$) the requirements if only covered over its half length.

Taking into account that in the course of bringing the ship into the dock and lifting it, the dock is not entirely covered

- the moment $M_{RS}$ resulting from the forces $F_{GS}$ and $F_{BS}$ acting on the ship:

$$M_{RS}(T_S^*) = \left[ F_{BS}(T_S^*) z_{MS}(T_S^*) - F_{GS} z_{GS}^* \right] g$$

- the moment:

$$Q_{RD}(T_D) = \rho g \sum i_D(T_D)$$
generated by free surfaces of water in ballast tanks.

The quantities appearing in the expressions (1.2) have the following meaning:

- $F_{GS} \land F_{BS}$ — ship mass and buoyancy, respectively
- $z_{GS}^*$ — ordinates of ship mass centre and its initial metacentric point, defined in the ship – fixed reference frame (with respect to the point $K$ – see. Fig.1.1).
- $T_D$ and $T_S^*$ — draughts of the dock and ship, respectively, for which the following relation is valid: $T_D = T_S^* + a$, where $a = h + s = 5.175 \text{ m}$ (see Fig.1.1 and Tab.1).

If the dock is inclined by a small positive angle $\phi$ the moment $M_{RD}(\phi,T_D)$ due to the above specified internal forces and moments, acting on it and defined with respect to the point $K_D$ (see Fig.1.1a), is expressed as follows:

$$M_{RD}(\phi,T_D) = -w_{SD}(T_D) g \phi \quad (1.3)$$

where:

- $g > 0$ — gravity acceleration
- $w_{SD}(T_D)$ — moment factor of $M_{RD}(\phi,T_D)$ equal to:

$$w_{SD}(T_D) = F_{BD}(T_D) z_{MD}(T_D) - F_{GD}(T_D) z_{GD}(T_D) + F_{GS}(z_{GS} + a) + F_{BS}(T_S^*) [z_{MS}(T_S^*) + a] - \rho \sum i_D(T_D) \quad (1.4)$$
Floatability and stability of floating dock-docked ship system

The so determined values of the factor $w_{SD}(T_D)$ for the investigated dock loaded by the ship described in App. II, are presented in Tab. 1.1.

From the condition (1.1) it results that the moment $M_{RD}(\phi, T_D)$ determined by the expressions (1.3) and (1.4) will be really the righting moment of the dock if and only if the following inequality is satisfied:

$$M_{RD}(\phi, T_D) = - w_{SD}(T_D) < 0 \quad (1.5)$$

It means that the dock will be really (unconditionally) stable if the factor $w_{SD}(T_D)$, called here the mass factor of stability, is positive:

$$w_{SD}(T_D) > 0 \quad (1.6)$$

In the case of unconditional assessment of stability (with respect to formal legal criteria), its condition determined in the domain of the factor $w_{SD}$ amounts to the inequality:

$$w_{SD}(T_D) > w'_{SD}(T_D) \quad (1.7)$$

in which $w'_{SD}(T_D)$ is the appropriate limiting value (see. Ch.5.1).

Commonly used measure of initial transverse stability of surface floating objects is the initial metacentric height.

**Tab. 1.1.** Stability characteristics of the floating dock-docked ship system and the docked ship itself.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Symbol</th>
<th>Unit</th>
<th>Values at the system draught $T_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buoyancy of the system</td>
<td>$F_{BD}$</td>
<td>[t]</td>
<td>$21527$ $22783$ $24218$ $25073$ $26678$ $27806$ $29173$ $30540$ $31907$ $34640$ $36007$</td>
</tr>
<tr>
<td>Ordinate of the system metacentric point</td>
<td>$z_{MD}$</td>
<td>[m]</td>
<td>$50.50$ $47.89$ $22.14$ $21.51$ $20.49$ $19.89$ $17.93$ $17.64$ $17.41$ $17.23$ $17.14$ $17.12$ $17.11$</td>
</tr>
<tr>
<td>Ship weight loading the dock</td>
<td>$R_D$</td>
<td>[t]</td>
<td>$10032$ $10032$ $10032$ $10032$ $10032$ $8993$ $7417$ $5686$ $3849$ $1933$ $0$ $0$</td>
</tr>
<tr>
<td>Ballast mass</td>
<td>$M_b$</td>
<td>[t]</td>
<td>$3145$ $4401$ $5836$ $6691$ $8296$ $10463$ $13406$ $16504$ $19708$ $22991$ $26290$ $27657$</td>
</tr>
<tr>
<td>Ordinate of the ballast mass centre</td>
<td>$z_b$</td>
<td>[m]</td>
<td>$0.44$ $0.61$ $0.81$ $0.93$ $1.16$ $1.46$ $1.53$ $1.24$ $1.04$ $0.89$ $0.78$ $0.74$</td>
</tr>
<tr>
<td>Mass of the dock together with ballast</td>
<td>$F_{GD}$</td>
<td>[t]</td>
<td>$832942$ $835596$ $558380$ $280297$ $284297$ $315815$ $345052$ $376822$ $409094$ $447001$ $483228$ $499057$</td>
</tr>
<tr>
<td>Ordinate of the centre of the mass $F_{GD}$</td>
<td>$z_{GD}$</td>
<td>[m]</td>
<td>$9.47$ $8.64$ $7.91$ $7.56$ $7.03$ $6.52$ $5.88$ $5.15$ $4.56$ $4.08$ $3.69$ $3.55$</td>
</tr>
<tr>
<td>Stability factor of the system</td>
<td>$w_{SD}$</td>
<td>[tm]</td>
<td>$795485$ $798232$ $241366$ $242938$ $246948$ $278555$ $317046$ $348725$ $381016$ $416751$ $481496$ $497257$</td>
</tr>
<tr>
<td>Stability factor $w_{SD}$ and $\Delta w_{SD}$</td>
<td>$w_{SD}$</td>
<td>[tm]</td>
<td>$832942$ $835596$ $280297$ $284297$ $315815$ $345052$ $376822$ $409094$ $447001$ $483228$ $499057$</td>
</tr>
</tbody>
</table>

For the docked ship

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Symbol</th>
<th>Unit</th>
<th>Values at the docked ship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship draught</td>
<td>$T_S$</td>
<td>[m]</td>
<td>$0.825$ $1.825$ $2.825$ $3.825$ $4.825$ $5.800$ $5.800$</td>
</tr>
<tr>
<td>Ship buoyancy</td>
<td>$F_{BS}$</td>
<td>[t]</td>
<td>$10032$ $2615$ $4346$ $6183$ $8099$ $10032$ $10032$</td>
</tr>
<tr>
<td>Ordinate of the ship metacentric point</td>
<td>$z_{MS}$</td>
<td>[m]</td>
<td>$24.40$ $16.40$ $12.80$ $10.80$ $9.80$ $10.20$ $10.20$</td>
</tr>
<tr>
<td>Ship stability factor</td>
<td>$w_{SS}$</td>
<td>[tm]</td>
<td>$- 68046$ $- 50512$ $- 37769$ $- 26622$ $- 14838$ $8928$ $8928$</td>
</tr>
</tbody>
</table>
Floatability and stability of floating dock-docked system

\[ h_0 = G M_0 = Z_{MB} - Z_G \]

where: \( M_0 \) - initial metacentric point, and \( G \) - centre of mass of the object. The relevant stability factor is then expressed as follows:

\[ w_{SD} = F_B h_0 = F_G h_0 \]

where \( F_B \) and \( F_G \) are values of buoyancy and weight of the floating object, respectively, and, as defined, positive and equal to each other. The absolute stability condition \( w_{SD} > 0 \) amounts then to the following:

\[ h_0 > 0 \]

However, the interchangeable and equivalent application of the conditions: \( w_{SD} > 0 \) and \( h_0 > 0 \) is limited to the cases in which values of \( F_B \) and \( F_G \) can be easily and unambiguously determined, and first of all it concerns the metacentric height \( h_0 \). This is always possible when the quantities \( F_B \) and \( F_G \) are homogeneous and location of the point \( G \) of the considered object is unquestionable. Otherwise if at least one of the quantities \( F_B \) and \( F_G \) is not homogeneous, the metacentric height \( h_0 \) usually is a conventional quantity and its value – relative and ambiguous. However the factor \( w_{SD} \) remains objective and unambiguous. Therefore this factor should be used to obtain an objective and right assessment of stability.

From the relations described by the expressions (1.2) ÷ (1.4) it results that the righting moment \( M_{RS}(\phi, T_S^*) \) for the dock loaded by a part of weight of the docked ship, is sum of several very different components. First of all for this reason it was decided to measure stability of the dock, but also that of several very different components. In the instant when the ship loses its stability, acts the righting moment:

\[ T \phi \]

Therefore the following statement should be accepted:

In all the cases in which floatation equilibrium of objects is not determined only by the equality of mass and buoyancy of the objects, their stability should be measured by means of the stability factor.

\[ \text{APPENDIX II} \]

Characteristics of docked ship

In this work a general cargo ship of the main dimensions given in the table below, was assumed the docked object.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length b.p. ( L_{pp} )</td>
<td>[m]</td>
<td>150.00</td>
</tr>
<tr>
<td>Total length ( L_C )</td>
<td>[m]</td>
<td>161.00</td>
</tr>
<tr>
<td>Breadth ( B )</td>
<td>[m]</td>
<td>22.92</td>
</tr>
<tr>
<td>Hull depth ( H )</td>
<td>[m]</td>
<td>13.30</td>
</tr>
<tr>
<td>Design draught ( T_K )</td>
<td>[m]</td>
<td>8.75</td>
</tr>
<tr>
<td>Docking draught ( T_d )</td>
<td>[m]</td>
<td>5.80</td>
</tr>
<tr>
<td>Docked ship mass ( F_{GS} )</td>
<td>[t]</td>
<td>10032</td>
</tr>
<tr>
<td>Ordinate of ship mass centre ( Z_{GS} )</td>
<td>[m]</td>
<td>9.31</td>
</tr>
</tbody>
</table>

In Fig.2.1 are presented the following hydrostatic characteristics of the ship: the ship hull volumetric displacement \( V_S(T_S^*) \) and the ordinate of initial metacentric point \( Z_{MS}(T_S^*) \).

BIBLIOGRAPHY