DETERMINING CO-OPERATION CHARACTERISTICS OF THE NAVAL GAS TURBINE WITH POWER RECEIVER USING THE TECHNIQUE OF PLANNING EXPERIMENT

Bogdan Pojawa
Polish Naval Academy
Department of Mechanical and Electrical Engineering
Śmidowicza Street 69, 81-103 Gdynia, Poland
tel.: +48 58 6262757, fax: +48 58 6262648
e-mail: b.pojawa@amw.gdynia.pl

Kamil Borsuk
Polish Naval Academy
Department of Mechanical and Electrical Engineering
Śmidowicza Street 69, 81-103 Gdynia, Poland

Abstract
The process of exploitations naval gas turbines bases on their characteristics, in this on characteristic of co-operation of engine with power receiver. In particular, if a controllable pitch propeller is it. The ship’s documentation does not often contain that kind of characteristic. There is need the study of method her determine therefore. It undertaking the test of determination the characteristic of co-operation of chosen naval gas turbines with power receiver, it was decided to execute in analytic way, using the technique of planning experiment which also allows for efficient and effective testing. Taking preliminary character of examinations into account they decided to carry them out on the laboratory position with the gas turbine engine GTD-350 and single-stage reduction gearing H-564 co-operating with Froude HWZ-3 water brake.

In the article theoretical bases of planning experience, a manner of preliminary implementations of studies as well as their results were presented. It the investigation of adequacy for received results of investigations was conducted, in this statistical and technical analysis. The conclusions on basis of received results of investigations of adequacy were expressed.

Keywords: a characteristics of the naval gas turbine, design of experiments, experiment design, a naval gas turbine

1. Introduction

During the operation of multi-drive systems, a very important issue with respect to its strength, is the evenness of load of the propulsion motors. This is particularly important when two or more engines work on a single line of shafts. There are many ways to implement uniform load running engines. However, the best way is to control their work according to the torque. Exploiter manages the load of the engines according to the characteristics contained in the operating instructions or in the case of automatic control systems, it does not control it directly, and an automatic control system takes over. For example, automatic load control system of marine gas turbine engine LM 2500.

This system carries out a process to maintain a constant speed of a sailing ship choosing both: the right pitch of the adjustable propeller and stream of the fuel. Completion of the uniformity of loads is not carried out on the basis of the torque but determined on the basis of measurement and the gas-thermodynamic parameters. The so-called torque computer carries it out, which sets (online) the torque of free power turbine based on the measurement of selected physical quantities characterizing the mechanical work of the engine. With the mathematical model, by which torque is appointed, missing, there is a problem in its periodic review, e.g. during the diagnostic tests.
The above became the inspiration for the preliminary tests aimed at checking if it is possible to develop a reliable mathematical model allowing designating the characteristics of various conditions for cooperation between the marine gas turbine engine and the receiver in the form of a pitch of the adjustable propeller. To determine the mathematical model the experiment planning theory was used, which also allows for efficient and effective testing. Given the preliminary nature of research, it was decided to carry them out on a gas turbine engine GTD-350.

2. Preliminary examinations

In order to determine the characteristics of the mentioned cooperation, we should consider, not only one propeller characteristics, but also their family. Family of these characteristics creates a field of cooperation between free power turbine engine and a pitch of adjustable propeller. From the operator's perspective and on the basis of characteristics of marine gas turbine engines, characteristics should show the dependence of effective energy given to the receiver and the energy expressed by the useful torque, depending on:

- parameters characterizing the energy state of the gas generator exhaust:
  - rotation speed of the gas generator \( n_{GG} \),
  - temperature of the exhaust gas behind a combustion chamber \( T_3 \) or behind gas generator \( T_{04} \),
  - pressure of the air behind compressor \( P_2 \).

The most common parameter is the rotation speed of the gas generator \( n_{GG} \).

- parameters characterizing the cooperation of a free power turbine with a propeller, with its pitch/increase set. Usually this is the speed of a free power turbine \( n_{PT} \).

Finding the characteristics of this mathematical model is made in accordance with the theory of experimental design \([1, 5]\). Therefore, first step is to characterize and determine the rules of the test object. It was decided to carry out tests on a gas turbine engine GTD-350. The laboratory stand represents a miniature ship's propulsion system with a turbine engine. Its main elements are two-rotor gas turbine engine GTD-350, single-stage reduction gearing H-564 and the energy receiver in the form of a water brake Froude HWZ-3. Due to its principle of operation, the brake charges the engine, in a similar fashion to the propeller with controllable pitch \([3, 4]\). Laboratory is shown in Fig. 1.

![Fig. 1. The laboratory stand with the gas turbine engine GTD – 350](image)

Next, the set of values characterizing the object of research were determined. The input quantities are the rotation speed of the gas generator \( n_{GG} \) and the rotation speed of the power turbine \( n_{PT} \). A useful torque \( M \) is the output. The set of fixed size \((S)\), whose values do not change during the study, we include for example parameters of the test object in question. Constants are often overlooked because of the established and inordinate influence on the size of the output.
during the study. However, distorting values ($Z$) are the other parameters whose values may change during the experiment and which may affect the output. These parameters are the temperature, pressure and relative humidity of the environment and the condition of various engine components. Therefore, in further part of the research, results of the measurements will be brought to normal atmospheric conditions. Perfect weather conditions shall be barometric pressure $p_{0\,WZ} = 101325\,\text{Pa}$, absolute temperature $T_{0\,WZ} = 288.15\,\text{K}$ and absolute humidity $\varphi = 0$. Fig 2 is showing the elements characterizing the considered object of examinations.

![Fig. 2. Elements characterizing the considered object of examinations](image)

On the basis of the values characterizing the object, its quality mathematical model was determined:

$$M(n_{GG}, n_{PT}) = 0.$$  

(1)

An approximation function was assumed to be in the form of a polynomial. The most commonly used polynomials of approximation are [2,5]:

- linear polynomials,
- linear polynomials with interactions (interactions taken into account),
- square polynomials,
- square polynomials of the first order with interactions.

During further investigation, it was decided to make a simultaneous analysis of several polynomials of approximation in order to select the most suited for the measurement results. For the approximation, we used the method of least squares [6]. Given the above, we decided to take the functions of research object in the form of the following polynomials:

- linear function without interaction, expressed with formula:
  $$M = b_0 + b_1 \cdot n_{PT} + b_2 \cdot n_{GG},$$  
  (2)

- linear function with interaction, expressed with formula:
  $$M = b_0 + b_1 \cdot n_{PT} + b_2 \cdot n_{GG} + b_{12} \cdot n_{PT} \cdot n_{GG},$$  
  (3)

- square function without interaction, expressed with formula:
  $$M = b_0 + b_1 \cdot n_{PT} + b_2 \cdot n_{GG} + b_{11} \cdot n_{PT}^2 + b_{22} \cdot n_{GG}^2,$$  
  (4)

- square function with interaction of the first order, expressed with formula:
  $$M = b_0 + b_1 \cdot n_{PT} + b_2 \cdot n_{GG} + b_{11} \cdot n_{PT}^2 + b_{12} \cdot n_{PT} \cdot n_{GG} + b_{22} \cdot n_{GG}^2.$$  
  (5)

Another very important step was the adoption of relevant research program, from which the measurements have been taken and allowing for a determination of polynomials of approximation. On the end, for further study an intent, poly-selective, rotatable design static program was chosen. This program allows determining linear and quadratic functions of the research object, as well as
Pojawa, K. Borsuk

allows the designation of approximated value of the output with the same accuracy in all directions of the delineated response surface \([1, 5]\). Research on the basis of the poly selective program requires a compromise between informativeness and efficiency. The already mentioned ability to complete the program \([5]\) is also necessary. This condition, in case of the considered object, comes down to testing available associations of the values \(n_{GG}\) and \(n_{PT}\) in individual measuring points. For the minimum approximation error, magnitude of the individual values should correspond to the square roots of Chebyshev polynomials. Elements are determined from the dependence \([1, 5]\):

\[
\hat{x}(r) = -\cos \frac{\pi \cdot (2 \cdot u - 1)}{2 \cdot n}, \quad (6)
\]

where:

- \(u\) – next agreement (measuring point) of program,
- \(n\) – number of arrangements of the program.

In rotation program input quantities take five values, denoted symbolically: \(-\alpha, -1, 0, 1, +\alpha\). Measurement points, consisting of a combination of \(\pm 1\) are called the kernel of the program, \(\pm \alpha\) points are so-called star points, and zero points constitute the centre of the program. Using the rotation program increasing the number of measurements in the centre of the program is recommended, particularly in case of objective impossibility of implementing the program at some, particular stage \([1, 5]\). Not one, but five measurements in the centre of program were accepted.

Intrinsic values of individual standardized input values are being calculated from the pattern:

\[
x = \bar{x} + \hat{x} \cdot \Delta x, \quad (7)
\]

where:

\[
\Delta x = \frac{x_{\text{max}} - x_{\text{min}}}{2 \cdot \alpha}, \quad (8)
\]

\[
\bar{x} = \frac{x_{\text{max}} - x_{\text{min}}}{2}, \quad (9)
\]

where \(\hat{x}\) - standardized value \((-\alpha, -1, 0, 1, +\alpha)\).

Standardized value \(\alpha\) is called the size of the stellar arm. In the experiment planned for the two input quantities, star arm is \(\alpha = 1.414\) \([5]\). Rotational program of two-factor, no repetition experiment are shown in Tab. 1. Ranges of values of input elements are the following: for the rotation speed of the gas generator \(n_{GG} \in (57;80)\) and for the rotation speed of the power turbine \(n_{PT} \in (60;100)\).

On the basis of this program, measurements were made for set operating conditions at each load. Measurements were made with the help of the measuring-registering system \([3]\). For each load, physical values at the time of 10 s, with a sampling rate of 10 Hz were determined and then averaged. At the next stage, the results were brought to the SI unit system, ambient pressure was included during determining the absolute pressure, and then the results were brought to normal (perfect) atmospheric conditions. Based on that the following were calculates: the power, torque and specific fuel consumption for each of the registered loads. The rotation speeds of the gas generator \(n_{GG}\) and the power turbine \(n_{PT}\) were described in percentage values due to the existence of this unit on control and measuring instruments by which the engine load was steered. Taking measurement was preceded by calibration of the individual measuring circuits together with determining their uncertainties.

3. Statistical analysis and content-related of findings

A statistical analysis was conducted on the basis of preliminary study results. Its aim was to obtain the dependence approximating the useful torque of free power turbine on the parameters characterizing the energy state of the engine’s exhaust, with the highest possible accuracy. To
Determining Co-Operation Characteristics of the Naval Gas Turbine with Power Receiver Using the Technique of...

Tab. 1. Program of measurements according to the rotation program for $n_{GG} \in (57;80)$ and $n_{PT} \in (60;100)$

<table>
<thead>
<tr>
<th>No. of the measuring point $u$</th>
<th>Place of the measurement</th>
<th>Standardized value input elements</th>
<th>Input values real as the function of the elements standardized</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{n}_{PT}$</td>
<td>$\hat{n}_{GG}$</td>
<td>$n_{PT} [%]$</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>65.86</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>1</td>
<td>65.86</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>94.14</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>94.14</td>
</tr>
<tr>
<td>5</td>
<td>1.414</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>-1.414</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1.414</td>
<td>80</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>-1.414</td>
<td>80</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
<td>70</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>70</td>
</tr>
</tbody>
</table>

determine the association of approximation, adopted polynomials were used (2), (3), (4), (5). These approximation polynomials were evaluated statistically, using the measures of the accuracy of approximations, which include [1, 5, 6]:

- average error of the dividend of the rate of the regression, $S_{bi}$,
- quotient of the value of rates of the regression and the average error of the dividend of rates of the regression $t = \frac{b}{S_i}$, which apart from being used for the evaluation the accuracy of the estimation, would also be regarded as the test of the significance of parameters in the process of the statistical verification of the model,

- sum of remainders $MS$,
- rate of the determination $R^2$.

Best-fitting approximation polynomial in terms of statistical is one for which the sum of the residual MS is the smallest and the coefficient of determination is closest to unity. In addition, one should note the value of the average estimation error [5, 6]. Tab. 2 shows the values of characteristic quantities and measurements of the assessment of matching approximation functions of the research object.

Assuming the above and the data contained in Tab. 2, it was found that the best function of the test object is a second degree polynomial (square) with the interaction of the first order, due to the smallest sum of remainders, and the closest to unity coefficient of determination. In addition, the polynomial is characterized by small values of the average estimation error, as evidenced by the value of $t$. Finally, the ratio sought approximate dependence took the form:

$$M = 3109.007 + 3.009 \cdot n_{PT} - 109.507 \cdot n_{GG} - 0.012 \cdot n_{PT}^2 + 1.169 \cdot n_{GG}^2 - 0.152 \cdot n_{PT} \cdot n_{GG}. \quad (10)$$

The resulting polynomial allows for removal of the dependence of the useful torque for any rotational speed of the gas generator $n_{GG}$ (characterized by its energy state) and any rotational speed of the free power turbine $n_{PT}$ (characterized by its load, in cooperation with a propeller on a given jump). Dependence of useful torque in the rotational speed of the generator exhaust and a free power turbine is shown in Fig. 3.
Tab. 2. Putting together characteristic elements and measures of the evaluation of fitting the approximation function of object of examinations

<table>
<thead>
<tr>
<th>No.</th>
<th>Element</th>
<th>Analysed function of the object, ( M = f(n_{PT}, n_{GG}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Linear (without interaction)</td>
</tr>
<tr>
<td>1</td>
<td>Rates of the regression ( b_i )</td>
<td>Const.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} \cdot n_{PT} ) (L)</td>
</tr>
<tr>
<td>2</td>
<td>Average mistake of the respect ( S_{bi} )</td>
<td>Const.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} \cdot n_{PT} ) (L)</td>
</tr>
<tr>
<td>3</td>
<td>Quotient ( t = \frac{b_i}{S_{bi}} )</td>
<td>Const.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (L) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{PT} (Q) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( n_{GG} \cdot n_{PT} ) (L)</td>
</tr>
<tr>
<td>4</td>
<td>Sum of rests</td>
<td>MS</td>
</tr>
<tr>
<td>5</td>
<td>Rate of the determination</td>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

Fig. 3. Relation of the useful torque \( M \) in the function of the rotation speed of the power turbine \( n_{PT} \) and the gas generator \( n_{GG} \) for the scope of the scheme of experience.
Accuracy of resultant approximating function was found by determining the standard deviation [6]:
\[
\sigma[M(n_{PT}, n_{GG})] = 3.5 \text{ N·m},
\]
as well as on the basis of the relative average square error [6]:
\[
\sigma_{M(n_{PT}, n_{GG})} = 0.32 \%.
\]

In the next stage, chart of evaluation was drawn. In the evaluation, we have included the following elements: effects of matching and the influence of individual input variables and their interactions on the torque, at a reliability level \( \alpha = 0.05 \), which is presented in the Fig. 4.

![Fig. 4. Graph of effects standardized for the useful torque M](image1)

From the Fig. 4, one can easily read which effects of input variables have the greatest impact on the output rate to be determined. Individual factors are ranked according to their absolute value, which also is illustrated in column. Based on the coefficients presented in Fig. 4, it is concluded that both the rotational speed of the gas generator and free turbine speed drive have a major impact on the useful torque. Given the taken role of the object (10) the most significant effects are linear, and the least important factor is the square effect of the rotational speed of free power turbine.

Performed next was the approximated graph of the measured values of useful torque, as shown in Fig. 5. In addition, we placed an amount of remainders, which, rounded up, equals to \( MS = 21.4 \text{ N·m} \).

![Fig. 5. Relation of measured values and approximate of useful torque M](image2)
The above chart can be used to test the adequacy of pre-approved functions as a comparison of the measured value of torque during the measurements with calculated values obtained by approximation polynomial. Adopted function of the object of research is well reflected in the actual object of study, because more points are arranged along a straight line, and the sum of remainders is small.

The final stage of the study was the analysis of the substantive results of the research, consisting of the execution of logical assessment obtained according to the actual phenomena occurring in the test object. One of the most common forms of substantive examination is to study the adequacy of the resulting approximate dependencies. The study was based on a comparison of the adequacy of the characteristics of co-operation of free power turbine with adjustable pitch propeller, for selected values of adjustment H and rotational speed of the gas generator $n_{GG}$, obtained from measurements and calculations. Graphical representation of the results of adequacy is shown on Fig. 6.

Based on the above figure and the results of the calculations contained in Tab. 2 it can be concluded that the resulting polynomial allows setting a very accurate area of cooperation for GTD-350 engine with an adjustable pitch propeller. Higher inaccuracies only exist in the areas that were not covered by the experimental design. This applies in particular to the area including the gas generator speeds close to nominal. The area covered by the experimental errors of approximation do not exceed 0.5%.

4. Conclusion

On the basis of findings of conducted preliminary examinations the following conclusions can be drawn:

- There is a possibility of drawing up the credible mathematical model letting the energy appoint characteristics for different conditions for cooperation of the ship's gas turbine engine with the receiver in the form of the controllable pitch propeller.
The relation of approximation drawn up allows, in the indirect way, to appoint the useful torque of the power turbine from parameters characterizing an energy state of the gas generator of the considered gas turbine engine, for any parameters of the state surroundings.

The relation drawn up allows for approximating the useful torque, but with the accuracy sufficing its engineering applications.

Earned abilities and computational experience can be used in the usage of ship's gas turbine engines, in particular for construction of steering systems with straining the engine according to the torque and a security system of the engine before overloading.

References