Toward higher spot detectability of Bayesian image reconstruction algorithms using the L-filter penalty

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Abstract

Iterative Bayesian image reconstruction algorithms using the L-filter penalty allow effective noise reduction and stabilization of the iteration process. They create more visually pleasing images compared to those obtained with Bayesian algorithms using median root prior. The L-filter, which is a generalization of median root prior, is very flexible, i.e. its weights can be modified. That gives possibility of compromising between median and averaging while designing the prior. A way of optimizing such weights is presented in this article. The stochastic search was utilized in order to find the L-filter weights for which the contrast recovery of small spots can be increased. This is important because such image quality is critical when detecting tumors in medical images or material defects in industrial tomography. The methodology presented here allowed finding parameters resulting in higher contrast recovery than with the weights originally proposed without deteriorating the other image properties.

Streszczenie

Iteracyjne Bayesowskie metody rekonstrukcji obrazów z użyciem L-filtu pozwalają na efektywną redukcję szumu, stabilizację procesu iteracyjnego oraz tworzenie obrazów wizualnie atrakcyjniejszych w porównaniu do tych otrzymanych ze zwykłymi priorytetem medianowym. L-filt, który jest uogólnioną formą priorytetu medianowego jest bardzo elastyczny. To pozwała na kompromis pomiędzy medianą, a średnianiem. W artykule przedstawiony został sposób optymalizacji omawianych wag filtrowych. Wykorzystano przeszukiwanie stochastyczne w celu znalezienia wag L-filtu, dla których odzyskanie kontrastu niewielkich plamek jest zwiększone. Jest to ważne ponieważ taka własność obrazu jest krytyczna przy wykrywaniu tkanek nowotworowych w obrazowaniu medycznym lub defektów materiałowych w tomografii przemysłowej. Zaprezentowana metodologia pozwoliła na znalezienie takich wag dla których odzyskanie kontrastu jest większe aniżeli dla wag zaprezentowanych początkowo.

I Introduction

Image reconstruction from projections (IRP) is, in principle, an attempt to recover the spatial distribution of a certain entity within the evaluated object from a set of measurements (projections) taken at different orientations. It is an emerging interdisciplinatory field of research of which resulting algorithms are applicable to many engineering problems, mainly medical imaging. IRP in medical imaging concerns X-ray tomography (CT), emission tomography (ECT), ultrasonography (USG) and magnetic resonance imaging (MRI) [1].

The scope of this article is on ECT but the methodology presented here may be useful elsewhere. In ECT the projection data suffer from high noise occurrence. This and additionally a finite number of projections make the reconstruction problem ill posed. Therefore, the algorithms used here should be designed in such a way as to take into account only possible information about the imaging system as well as the object under study.

Therefore, statistical nature of the radioactive decay should be of prime importance in the models upon which the algorithms in ECT are built. Among the models used [2], i.e. continuous-continuous (C-C), continuous-discrete (C-D), discrete-discrete (D-D) the latter fulfills that requirement the most. Well-known method based on D-D model is Maximum Likelihood - Expectation Maximization (ML-EM) of Shepp and Vardi [3]. The algorithm proposed by the authors seeks the solution that fits the data but the resulting reconstruction is overly noisy.

One of the possible ways of coping with that problem is Bayesian approach. In this case two probability models are involved [4]. The first model describes the contribution of each image element to the overall number of counts in a particular detector element whereas other physical circumstances from the true isotope concentration. Such approach allows noise reduction and stabilization of the iteration process [5]. However, setting the appropriate Bayesian parameters is a difficult task. We presented the methodology of optimization of such parameters in [6] based on stochastic search. The proposed framework was utilized with the Bayesian algorithms using median root prior, the L-filter, Finite Impulse Response (FIR) filter and median hybrid, and the Huber penalty. All the methods mentioned are presently state of the art in the field and ensure effective noise reduction without significant deteriorating the edges. But, the penalty may cause decreasing of contrast recovery of small objects in the image, i.e. tumors in medical images or material defects in industrial tomography.

This article presents attempts to increase the detectability of small spots through optimizing the weights of the L-filter. This work is to some extent complementary to our previous study [6] where the prior weight and the basis function bandwidth for each penalized algorithm were optimized as to maximize the accuracy.
II Implementation of the reconstruction algorithm

The iterative Bayesian reconstruction was implemented in the form of one-step late formula proposed by Green [4]

\[
C_j^{(k+1)} = \frac{\sum_i p_j a_{ij} C_j^{(k)}}{\sum_i a_{ij} + \beta M_j - M_j}
\]

where \(C_j\) represents means counts at image element \(j\) (basis function coefficient, see below), and \(p\) denotes the acquired number of counts in the detector element \(d\). The coefficients \(a_{ij}\) in the system matrix represent the probability of emission in the image element \(j\) being detected in detector unit \(i\). \(M\) is a hyperparameter which will be described in the next section, \(\beta\) is a prior weight.

One iteration consists of the following operations. Firstly, the measured projection value is compared to the corresponding computed projection value by division. The resulting correction image is scaled and modified depending on the penalty term. Then it is backprojected to the image, which is an input for the next iteration.

The projection/backprojection operators were implemented in a way presented in [7]. The algorithm is called splatting. It makes use of rotationally symmetric image elements called blobs. To make the process more efficient the projection of such image element is stored in a look-up table (footprint). Using blobs provides increased accuracy of the reconstruction. Additionally when appropriately tuned, blobs may act as regularization though limiting the bandwidth of the object function [6]. Thus image representation is a convenient way of slightly applying smoothing whereas nonlinear L-filter may be used to enhance the edges.

The reconstruction was accelerated using subsets of projections [8]. The number of subsets was set relatively high, i.e. 30 due to demands of many reconstructions to be performed during the optimization process. The projections were ordered according to Weighted Distance Scheme (WDS) projection access scheme. Benefits of using WDS were presented in previous issue of this journal [9].

III The L-filter penalty

The hyperparameter \(M\) in the correction formula presented in the previous section is a product of the L-filter. This is a linear combination of the ordered values. Because of that it is less likely that L-filter outputs of adjacent window locations are the same [10] as it happens for basic median root prior [11]. The penalty reference using the L-filter is computed as

\[
M_j = L(c_n | n \in N_i; w) = \sum_{n \in N_i} w_n c_n
\]

\(N_i\) indicates the neighboring image coefficients of \(c\) within a mask. Those image coefficients are ordered (subscript in parentheses) according to their quantitative values and the weight \(w\) in the sum is taken according to their rank. The spatial location of the pixels is ignored.

The weights are normalized so that the sum of \(w\) is equal to 1.

IV Optimization of the L-filter weights

Global optimization was performed using Controlled Random Search of Price [12]. The author’s version, known as CRS2, was chosen because it is simpler and requires less computer storage than the original version. Yet, both versions have similar performance. More detailed description of the implementation utilized here can be found in [6].

The optimization criterion chosen in this study was normalized root mean square (NRMS) error of the image coefficients within the digitized and the reconstructed phantom. The projections of the mathematically defined phantom (see table 1) were generated using software simulator. Such obtained projection data were contaminated by Poisson distributed noise, assuming the detection of 3-10⁶ events. Sixty parallel projections were generated within 180° degree arc. The detector size as well as the image size was equal to 129.

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<th>Y₀</th>
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V Results

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<td>The weights optimized here</td>
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<td>0.01</td>
<td>-0.03</td>
<td>0.02</td>
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</tr>
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</table>

The simulation study was performed using 90 parallel projections of the Shepp-Logan head phantom uniformly distributed within 180°-degree arc. The projections ordered using WDS were grouped into 18 subsets. The discrete footprint table had 300 subdivisions. Ray driven splatting was chosen because of higher accuracy and smaller computation cost of over pixel driven splatting. Additionally, the former seems to be more suitable for OS-EM [8] that is pro-
jection oriented. The caching scheme on a ray level (see [7]) was used in the ray-driven splatting for the purpose of additional gain in speed. The \( \alpha \) and \( \beta \) parameters were kept constant. Their values \( \alpha = 12.76, \beta = 0.6 \) were based on optimization for the L-filter with the weights originally proposed (see [6]).

Figure 1 presents the profiles within two proximate hot spots in the phantom. Decreasing of contrast recovery for the L-filter penalty with the weights proposed in [10] compared to basic median root prior [11] can be seen. On the other hand the weights found using the methodology presented above provide image reconstruction with higher contrast recover than those in [10].

![Figure 1: Profiles within two proximate hot spots in the phantom. Symbols denote the following settings of the L-filter: ORG – the weights originally proposed in [10], MRP - basic median root prior [11], OPT – the optimized values of weights found here.](image)

Visual assessment of the images (figure 2) reveals no significant difference between the reconstructions obtained with basic median root prior and the L-filter with the optimized weights. Deterioration of the edges can be seen in the case of the reconstruction with the weights presented in [9].

## VI Conclusions and future work

Initial experiments with the optimization of the L-filter coefficients were presented in this work. Global optimization was utilized using the CRS procedure that was proven to be efficacious in performing the specified task. The initial experiments show flexibility and the potential of the L-filter penalty to be used in IRP.

It can be concluded that the L-filter penalty may be used not only as a combination of median and smoothing as originally proposed but actually as a way to enhance the edges providing additional regularization using blobs. Five dimensional optimization performed here should rather be extended to seven dimensions and include \( \alpha \) and \( \beta \) because modifying the weights of the L-filter makes the penalty properties different. Therefore the prior strength as well as the blob bandwidth should be appropriately changed.

Future investigations will be focused on incorporating numerical observers and practical studies using PET phantom (see [5]). The final goal is to establish methodology of task specific optimization of the free parameters in real imaging systems.

## References:


### Tytuł: Zwiększanie detekcji plamek w Bayesowych algorytmach rekonstrukcji obrazów z użyciem L-filtra

*Artykuł recenzowany*