Laboratory investigations of dynamic properties of accelerometers with fractional orders for application in telematic equipment

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ABSTRACT
The paper presents laboratory studies on measuring accelerometers, which were modelled in the classical differential equations, as well as the fractional calculus. Measurement errors were examined and the classical and fractional models in terms of dynamic properties were compared. The advantages of fractional calculus in modelling dynamic elements were also indicated.

KEYWORDS: fractional calculus, measuring transducer, measurement errors

1. Introduction
The recent dynamic development of the research into the use of fractional calculus for the analysis of dynamic systems encouraged the author to attempt its use for the analysis and modelling of transducers and measurement systems. The differential equation describing an absolute movement of the transducer’s seismic mass [4], [6], [7], [8] takes the form:

\[
\frac{d^2}{dt^2} y(t) + 2\zeta\omega_0 \frac{d}{dt} y(t) + \omega_0^2 y(t) = 2\zeta\omega_0 \frac{d}{dt} x(t) + \omega_0^2 x(t)
\] (1)

A relative shift of the seismic mass is introduced in equation (1):

\[
w(t) = y(t) - x(t)
\] (2)

changes it into:

\[
\frac{d^2}{dt^2} w(t) + 2\zeta\omega_0 \frac{d}{dt} w(t) + \omega_0^2 w(t) = -\frac{d^2}{dt^2} x(t)
\] (3)

Taking into consideration the assumption that the dynamic behavior of the element responsible for damping is better described by the fractional derivative, equation (3) is written down as:

\[
\frac{d^2}{dt^2} w(t) + 2\zeta\omega_0 \frac{d^{(\nu)}}{dt^{(\nu)}} w(t) + \omega_0^2 w(t) = -\frac{d^2}{dt^2} x(t)
\] (4)

Generalizing equation (4) in view of the fact that integer order derivatives in the integral equation derivative are a special case of non-integer derivatives, we can write down:

\[
A_1 \frac{d^{(\nu)}}{dt^{(\nu)}} y(t) + A_2 \frac{d^{(\nu)}}{dt^{(\nu)}} y(t) + A_3 \frac{d^{(\nu)}}{dt^{(\nu)}} y(t) =
\]

\[
= B_1 \frac{d^{(\nu)}}{dt^{(\nu)}} x(t) + B_2 \frac{d^{(\nu)}}{dt^{(\nu)}} x(t) + B_3 \frac{d^{(\nu)}}{dt^{(\nu)}} x(t)
\] (5)

2. Identification of transducer dynamics
In order to identify sensor dynamics, a measurement system was constructed (Figure 1). The DelaTron accelerometer, Type4507, manufactured by the Bruel&Kjaer company, characterized by sensitivity of 10.18 mV/ ms⁻² was examined. The sensor was placed on the electrodynamic inductor.
A model accelerometer produced by VEB Metra, type KB 12, sensitivity of 317 mV/ms² was aligned in one axis with the examined sensor. The input signal was the vibrations of the inductor plate actuated by a sinusoidal signal from the generator. The model signal was the one from the KB 12 sensor, whereas the signal examined was the signal from the 4507 sensor.

The main objective of the study was identification of the mathematical model of the 4507 sensor on the basis of signals received from the sensors. The identification method applied here was ARX [1], [2], [9] – the examined signal was compared with the model signal and on the basis of the comparison discreet transmittance of the examined sensor was determined.

Signals were collected at a sampling frequency of 10 000 Hz each with the use of the measurement card. The sampling time used in the ARX method was 0.0001 s. The voltage-source signals were examined. Then they were converted/translated into acceleration. Identification was accomplished with the use of the MATLAB&Simulink package [10] (Fig. 2).

As a result of the ARX identification method, the examined sensor transmittance looks as follows:

\[
G(z) = \frac{0.79196z^2 + 0.51435z}{z^2 - 0.6439z + 0.048034}
\]  (6)

Figure 3 depicts frequency characteristics of the sensor's amplitude and phase.

As a result of the ARX identification method, the identified signal and the signal characteristics in the model have the same amplitude and there is no phase shift between these signals (Figure 5.)

In order to compare characteristics from the model sensor, examined sensor and the obtained model of the examined sensor, the system presented in figure 6 was built.

Figure 4 shows the signals entering the system identification block. The model signal amplitude differs from the amplitude of the identified signal.
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The comparison of the characteristics indicates that signals from the examined sensor and the examined sensor model have identical amplitudes.

3. Measurement error analysis

As a result of conducted measurements (section 1) high values of error peaks were observed. They are a consequence of the determination of error for the characteristics of variables over time, which change their values from positive to negative. Sensitivity of the examined sensor is much lower than that of the model sensor – thus we deal with the cases when the model sensor displays the acceleration value close to zero, whereas the examined sensor, due to its low sensitivity, indicates zero. Hence peaks in the characteristics of errors. The lowest error value is reached at values close to the amplitude, the highest – at those close to 0.

The bigger the difference between frequencies of the examined characteristics and the characteristics at which identification was accomplished, then the bigger the difference between the median relative error of the sensor and of the model is.

4. Comparison of the integer and fractional order models of the accelerometer

It can be concluded that the sensor's model reproduces the model signal with the relative error larger by 0.3619% than the sensor's error. This value occurs at examining the characteristics of the same frequency as in the case of the examined sensor identification. When the frequency of the examined characteristics is different from that at identification, then the error values will be higher. The relative error values for the sensor and its model for different frequencies are shown in Table 1.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Sensor's relative error [%]</th>
<th>Model's relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>45.2213</td>
<td>30.8089</td>
</tr>
<tr>
<td>200</td>
<td>22.9227</td>
<td>30.2997</td>
</tr>
<tr>
<td>300</td>
<td>29.1945</td>
<td>29.5564</td>
</tr>
<tr>
<td>400</td>
<td>70.60.78</td>
<td>28.3097</td>
</tr>
<tr>
<td>500</td>
<td>90.5626</td>
<td>26.0184</td>
</tr>
</tbody>
</table>

In order to check whether the model based on the fractional order equation describing the dynamic behavior of the object reproduces the model signal better than the “classical” model, on the basis of the sensor transmittance model (6) determined by the ARX method, a group of models was determined by means of fractional order equations. Our investigations started from one \( V_2 \) fractional order responsible for damping. The order of the \( V_2 \) derivative changes the range of values from 0.94 to 2.08 by a 0.02 step.

Frequency characteristics of the models' amplitude and phase are depicted in figures 10 and 11.

![Fig.10. Amplitude and phase characteristics for different \( V_2 \)](image-url)
In summary, we can conclude that out of the group of characteristics of the \( V_2 \) fractional order the closest to the ideal one with reinforcement and phase shift equal 0 is the characteristic for the order equal 1. Due to the way of transmittance determination of fractional coefficients describing the sensor’s dynamic behavior, the amplitude and phase characteristics differ from the same characteristics determined for the “classical” notation of dynamic behavior (transmittance is different).

On the basis of amplitude and phase characteristics of the sensor’s model obtained by the ARX method and the sensor’s model determined by the “fractional order method” it can be concluded that the fractional order model reproduces the sensor’s dynamic behavior far more accurately:

- amplitude and phase characteristics are closer to the linear characteristic in a larger scope of signal processing;
- in the case of frequency above 1 Hz, amplitude and phase frequencies are almost linear: magnitude is within the boundaries from -2.02 to -2.03 dB, and the phase shift is within the range from 0.04\(^\circ\) to 10\(^\circ\). In the case of the “classical”
model of the transducer model, one cannot think about such
great linearity.

Determination of the median relative error for the examined
characteristics, let us claim that:
• median relative error in the case the fractional order model’s
response is examined is constant up to the third decimal place
(this is confirmed by linearity of earlier obtained Bode’s plot
of frequency characteristics);
• in each examined case there is an advantage when the sensor’s
fractional model is used, the more so, the higher the difference
between the frequency at which the “classical” model was
determined (300 Hz) and the frequency of the examined
characteristics. For the cases examined, the percentage decrease
in the median error ranges from 5.2144 to 10.0049 %.

5. Examination of the
accelerometer models of ν₁ and ν₂ fractional orders

Bode’s plot of frequency characteristics for the V₁ and V₂ order
combinations was examined (Table 4).

Table 4. Order combinations of orders in equation (5)

<table>
<thead>
<tr>
<th>ν₁</th>
<th>0.94</th>
<th>0.96</th>
<th>0.98</th>
<th>1</th>
<th>1.02</th>
<th>1.04</th>
<th>1.06</th>
<th>1.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν₂</td>
<td>1.94</td>
<td>1.96</td>
<td>1.98</td>
<td>2</td>
<td>2.02</td>
<td>2.04</td>
<td>2.06</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Fig.16. Bode’s characteristics for ν₁ = 1 and ν₂ = 2 determined by
the method for fractional orders

Linearity starts from ca. 0.5 Hz at magnitude equal -2.02 dB
and the maximum phase shift equal 0.05° (the peak in the next
figure). The phase shift for the frequency of 1 Hz equals 0.005°.

It is worth noticing that Bode’s characteristics in the case of
fractional V₁ only and V₁ and V₂ are of a different shape when
it comes to low frequencies. Above 1 Hz it is practically of no
importance whether it is only V₁ which is non-linear, or V₁ and
V₂. Frequency characteristics are practically identical. Thus, the
use of non-linear V₁ only has the same effect as using fractional
(non-integer) V₁ and V₂. The very method of determining the
sensor model’s dynamic behavior affects accuracy of such a model
processing. In the case of classical and fractional models for
identical (integer) orders the observed processing accuracy is
higher in the case of the “fractional” type model.

6. Conclusion

The use of the fractional calculus for describing characteristics
of dynamic systems seems justified for the following reasons:
• Global research into numerous physical phenomena (description
of properties of viscoelastic materials, liquid permeation through
porous substances, electric load transfer through an actual
insulator, heat transfer through a heat barrier, or descriptions
of friction, [3], [11], [12], [14]), showed that fractional calculus
describes this type of phenomena more accurately than classical
mathematical analysis.
• Continuous physical phenomena of the real world should be
described “intuitively” by means of differential equations of
orders taken from the set of real numbers and not only, integer
numbers, i.e. discrete. Classical integrals and integer order
derivatives are only specific cases of the fractional calculus.
• The fact that in previous decades researchers from different
areas of science did not use the fractional calculus is accounted
for by the author by the lack of IT tools having great computing
potential which in our times are widely accessible.

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