GRAVIMETER WITH TWO-DIMENSION DIGITAL PROCESSING OF MEASURING INFORMATION

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Abstract: Gravimeter with two-dimensional digital data processing about acceleration of gravity is considered in a paper. The result of measurements in gyroscopic gravimeter contains errors. An influence on gravimeter inertial absolute acceleration and other disturbing influences cause the errors of measurements. These of acceleration arise by work gravimeter onboard the plane in structure of air gravimetric system. The structural scheme of gyroscopic gravimeter is proposed in a paper for multiple precision. This construction is providing immediate measurements of acceleration of gravity. The possibility of digital data processing is considered in view of a two-dimensional character of an array of measuring information.

1. INTRODUCTION

The most perspective type of gravimeter based on dynamically customized gyroscope. The examples of realisations such gravimeter are reduced in (Odintsov, 1982). However many errors is inherent in it gravimeter. These errors can essentially distort result of measurements.

These errors are stipulated by that gravimeter measures a collection of acceleration of gravity (useful component of result of a measurement) and inertial absolute acceleration (parasite signal calling an errors of result of measurements). The inertial absolute acceleration is called by vertical acceleration of the plane, on which is installed gravimeter. The magnitude of such parasite signal can exceed magnitude useful component in result of measurements. The forward and angular vibrations of the plane also can reduce in essential errors of result of measurements.

Development of methods of compensating of these errors is actual task. The good result can be received on the based of applications of the digital computing device. This computing device fulfils algorithmic handling of result of measurements. Such handling includes a filtration of a measuring information from a feeler of gravimeter. Compression of a measuring information also applied for effective saving of this information to memory of computing device.

For effective application of methods of algorithmic handling it is desirable, that gravimeter measured immediately value of acceleration of gravity. For this purpose it is necessary to bring in some modifications to a construction of gravimeter. Centre of mass of a rotor of a two-degree dynamically customised gyroscope is displaced along it of axes of rotation. Also into the scheme of gravimeter are entered a transmitter of an angle of a turn, low-pass filter and device of an evaluation of an output signal gravimeter (digital computing device).

2. STRUCTURE AND MATHEMATICAL MODEL OF GRAVIMETER

Gravimeter (Fig. 1) contains a two-degree dynamically customized gyroscope (1) with a rotor (2). To a gyroscope are connected a transmitter of an angle of a turn (3), transmitter of a moment (4), low-pass filter (5) and device (6) for an evaluation of an output signal of gravimeter. The centre of mass of a rotor is displaced concerning it of an axes of rotation on magnitude \( l \).

The multiple precision of measurements of acceleration of gravity is ensured because the centre of masses of a rotor is displaced concerning it of an axes of rotation. Therefore this gravimeter fulfils an immediate measurement of acceleration of gravity. These measurements are fulfilled with the help of transmitter of an angle of a turn.

This property is a possibility to execute a filtration of a measuring signal. The most part of a power of useful component of a measuring signal is concentrated on frequencies below 0.1 radian/sec (Line 2, Fig. 2). The most part of a power of errors in a measuring signal is concentrated on frequencies above 0.1 radian/sec (Line 1, Fig. 2). Therefore low-pass filter can execute a filtration of errors in a measuring signal. The device of an evaluation of an output signal will transform a result of a filtration to an output signal of gravimeter.

Also at use of the modern digital computing device there is a possibility to fulfil a filtration of a measuring signal in view of its two-dimensional character. Thus it is necessary to accumulate result of measurements and to consider them in fixation to coordinates of points on a surface of the Earth.

Thus, the essential errors of measurements are eliminated from an output signal of gravimeter. If these errors
to not eliminate, their magnitude can be commensurable with a value of a useful signal. Therefore offered method of handling of a measuring information allows essentially to increase of measurement accuracy of acceleration of gravity.

\[ M_g = mgl \cdot \cos \alpha , \]  

(1)

where \( m \) – mass of a rotor, \( l \) – displacement of a centre of masses of a rotor concerning it of an axes of rotation, \( \alpha \) – angle of a deviation of a rotor.

The moment \( M_h \) from vertical acceleration \( \dot{h} \) along an axes of sensitivity of gravimeter is equalled:

\[ M_h = mhl \cdot \cos \alpha . \]  

(2)

The moment \( M_T \) elastic forces of the elements of a suspension bracket of a rotor is equalled:

\[ M_T = C_x \alpha , \]  

(3)

where \( C_x \) – rigidity of the elastic elements of a suspension bracket at their torsion.

The centrifugal moment \( M_u \) is equalled:

\[ M_u = I \gamma^2 \sin \alpha , \]  

(4)

where \( I \) – moment of inertia of a rotor, \( \gamma \) – angular velocity of rotation of a rotor.

If to accept, that \( \alpha \ll 1 \) radian, it is possible to note:

\[ C_x \alpha + I \gamma^2 \alpha = -mlg + m\dot{h} . \]  

(5)

From here, by designating \( k = \dot{N} + I \gamma^2 \), we obtain:

\[ \alpha = -\frac{ml}{C + I \gamma^2} (g + \dot{h}) = -\frac{ml}{k} (g + \dot{h}) . \]  

(6)

By designating \( S = ml/k \), we obtain an entering signal of a low-pass filter:

\[ T = \frac{1}{S} \alpha = -g + \dot{h} . \]  

(7)

From here

\[ \alpha = \left(-g + \dot{h}\right)S. \]  

(8)

Thus, the angle \( \alpha \) deviation of a rotor is proportional to acceleration of gravity \( g \) and vertical acceleration \( \dot{h} \) of the plane.

The output signal of a transmitter of an angle of a turn passes through the amplifier (in a Fig. 1 is not shown) with an amplification factor \( 1/S \) and come on an input of a low-pass filter. At an output signal of a transmitter of an angle of a turn also there are errors. These errors stipulated by forward and angular vibrations of the plane, on which gravimeter is installed. In view of these errors the input signal of a low-pass filter is defined of the formula:

\[ T = \frac{1}{S} \alpha = -g + \dot{h} - \left(R_x \alpha - R_y \right) \beta - \frac{B}{ml} \left(\dot{\omega}_x + \dot{\omega}_y \right) + \frac{M_{ij}}{ml} \beta , \]  

(9)

where: \( R_x, R_y \) – projection on axes \( 0x, 0y \) accelerations of forward vibrations of the plane, \( \beta \) – constant of proportionality, \( B \) – moment of inertia of a rotor of a gyroscope, \( \dot{\omega}_x, \dot{\omega}_y \) – projection on axes \( 0x, 0y \) accelerations of angular vibrations of the plane, \( M_{ij} \) – moment of tool errors of a two-degree dynamically customized gyroscope.
3. FILTRATION OF MEASURING INFORMATION IN GRAVIMETER

Frequency spectrum of acceleration of gravity and errors is different (Fig. 2, Lines 2 and 1). The dominant frequency of the first signal is equalled 0.00175 radian/sec. Dominant frequency of the second signal is equalled 0.269 radian/sec.

The low-pass filter has cutoff frequency 0.1 radian/sec and fulfills a filtration of a signal $T'$ with the purpose of an elimination of errors. In an outcome on an exit of a filter the output signal $T''$ is obtained.

For a low-pass filter it is possible to note a relation (Goldenberg et al., 1990):

$$T'(\tau) = \int_{-\infty}^{\infty} w(t-\tau)T(\tau)d\tau,$$

where $w(\cdot)$ – weight function of a filter.

The weight function of a filter is equalled:

$$w(t) = \int_{-\infty}^{\infty} W(j\omega)e^{j\omega t}d\omega = 2\omega_0\left[\frac{\sin\omega_0t}{\omega_0t}\right],$$

where $W(j\omega)$ – transfer function of this filter.

The output signal of a low-pass filter acts in a device of an evaluation of an output signal of gravimeter. This device calculates an output signal under the formula:

$$T''(\tau) = \int_{-\infty}^{\infty} 2\omega_0\left[\frac{\sin\omega_0(t-\tau)}{\omega_0(t-\tau)}\right]T'(\tau)d\tau.$$  

In this case $\tau=2\pi n$ – slice of time of an evaluation of an output signal, $n=1, 2, \ldots$ – amount of full turnovers of an exterior framework of a gyroscope.

In an outcome the output signal $T''$ is obtained which contains a measuring information about acceleration of gravity $g$. In this signal there are no all errors which dominant frequency exceeds 0.1 radian/sec. These errors are stipulated: accelerations of forward vibrations with dominant frequency 3140 radian/sec; accelerations of angular vibrations with dominant frequency 6.7, 10, 20, 40 and 60 radian/sec.

During evaluations $\tau=2\pi n$ errors of a gyroscope completely will be eliminated. For a full turnover of an exterior framework these errors in turn equalled positive and negative values. On the average this error will be equalled to zero.

Thus, in an output signal of gravimeter a series of errors of measurements completely is compensated. It allows essentially increasing measurements accuracy of acceleration of gravity.

4. METHODS OF TWO-DIMENSIONAL DIGITAL PROCESSING OF MEASURING INFORMATION IN GRAVIMETER

The filtration of the gravity measuring information can be executed by the one-dimensional filter for the array of data. These data are received along one line of flight of aircraft gravity system. Such filtration is carried out during flight in rate of receipt of the data or at processing gravity measurements after flight.

In the given research other variant is offered also. It consists in formation two-dimensional array of the gravity measuring information on anomalies of acceleration of gravity. The array is formed in fixation to coordinates of points of a surface of the Earth, in which these data were received. This array corresponded to series lines of flight. After that carry out a filtration of the generated array with the help of two-dimensional digital low-pass filter. Two-dimensional correlation in a useful signal about anomalies of acceleration of gravity in addition take into account. Such filtration can be executed at processing results of gravity measurements after flight.

Interrelation of such approach with the basic requirements to gravimetric survey on some district are considered.

Known regional and detailed gravimetric survey (Malovichko and Kostitsyn, 1992). The regional gravimetric survey is displayed on cards of scale 1:200 000 with section of isolines through 2 mGal. The gravimetric survey is continuous. The cards, made by results of survey, give representations about general structure of an abnormal field, his basic features and regularity. The cards of anomalies are used at the decision of astronomical and geodetic tasks.

The anomalies at regional shooting also provide the decision of tectonic tasks.

The detailed gravimetric survey will be carried out in conditions, when the regional survey is already carried out and basic regularity and the properties of an abnormal field are known. The detailed survey differs from regional survey to structure of a network and flights, scales of cards and accuracy of definition of anomalies, ways of their processing and interpretation (Kostitsyn, 1989; Malovichko et al., 1989). In Table 1 (Malovichko and Kostitsyn, 1992; Instruction on Gravimetric Investigation, 1980) the basic characteristics of detailed gravimetric survey for flat country are given.

**Tab. 1. The basic characteristics detailed gravimetric survey for conditions of flat country**

<table>
<thead>
<tr>
<th>Scale of gravimetric cards and diagrams</th>
<th>Section of isolines [mGal]</th>
<th>Root-mean-square errors of definition [m]</th>
<th>Step h of measurements along profiles [m]</th>
<th>Distance between profiles [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:50000</td>
<td>0.50</td>
<td>0.70</td>
<td>40</td>
<td>100…500</td>
</tr>
<tr>
<td>1:50000</td>
<td>0.25</td>
<td>0.35</td>
<td>40</td>
<td>50…250</td>
</tr>
<tr>
<td>1:25000</td>
<td>0.26</td>
<td>0.35</td>
<td>20</td>
<td>50…250</td>
</tr>
<tr>
<td>1:25000</td>
<td>0.20</td>
<td>0.25</td>
<td>20</td>
<td>20…100</td>
</tr>
<tr>
<td>1:10000</td>
<td>0.20</td>
<td>0.20</td>
<td>4</td>
<td>20…100</td>
</tr>
<tr>
<td>1:10000</td>
<td>0.10</td>
<td>0.10</td>
<td>4</td>
<td>10…50</td>
</tr>
</tbody>
</table>

At detailed survey the network consists of parallel profiles, distance between which in (3..5) times more step on a profile. In this connection the accuracy of study of an abnormal field on profile is higher, than on interprofiles section.

Thus, by results of gravimetric survey two-dimensional
array of the gravity measuring information can be generated. This array is constructed in fixation to coordinates of points of a surface of the Earth, in which these data were received. In this case two-dimensional correlation of anomalies of acceleration of gravity on a surface of the Earth also take account. The crimp have casual not correlated character (different noise and casual indignation) or character of periodic vibrations. These vibrations have one-dimensional correlation along a line of moving gravimeter and mobile basis. Two-dimensional filtration is possible of effectively to separate of useful signal from crimp handicapes in generated two-dimensional array of the gravity measuring information.

Methods, known from digital image filtration, can be used for formation of amplitude-frequency response of the two-dimensional filter (Gonsales, Woods, 2005; Adaptive Filters, 1988). The amplitude-frequency response so that maximum to keep the useful information on fine details of an arrangement of anomalies of acceleration of gravity on a surface of the Earth. Simultaneously periodic crimp effectively deleted. The frequency of this crimp corresponded to certain local sites of amplitude-frequency response of two-dimensional digital filter. As the consequence, accuracy of the gravity measuring information is raises.

For researches the results of gravity measurements on range by the size 650x650 km with a step 2.5...5.0 angular minutes on a longitude and latitude are used (Bezvesilnaya, 2001). The given method of a filtration has excluded many errors from the gravity measuring information. These errors are caused: forward vibrating accelerations with prevailing frequency 3140 radian/sec; angular vibrating accelerations with prevailing frequency 20 radian/sec, which is equal to frequency of own fluctuations of gravimeter (most dangerous case of the main resonance); angular vibrating accelerations with prevailing frequency 40 radian/sec and 60 radian/sec; angular vibrating acceleration with frequency 6.7 and 10 radian/sec (harmonic fluctuation). Also level of casual noises and disturbances which run on gravimeter has decreased. The accuracy of the gravity measuring information on anomalies of acceleration of gravitation has increased on (20...25) % in comparison with an one-dimensional way of a filtration.

The compression of measuring information can be applied in this gravimeter. It ensures more effective storage of such information in memory of the digital computing device. The compression is fulfilled by the following method:

1. The two-dimensional array of digital references of gravimetric measuring information is formed. Thus the digital references of a gravimetric measuring information note in this array in fixation to coordinates of points on a surface of the Earth.
2. Fulfil a low-frequency filtration of gravimetric measuring information. Thus take into account singularities of frequency spectra of a useful signal and errors (Fig. 2). In an outcome of a filtration from measuring information the errors are eliminated.
3. Fulfil compression of a two-dimensional array of digital references of a gravimetric measuring information with the help of one of methods of compression of two-dimensional digital arrays (videoimages). The methods based on wavelets and fractals (Walstead, 2003) are most effective in this case. Thus the parameters of a method of compression select by such, what the distortions of a measuring information did not exceed a error of measurements. This error included error of a measurement of acceleration of gravity, errors of a measurement of a height and navigational parameters.

The gravimetric measuring information is submitted as sequence of digital references. These references characterize size of acceleration of gravity in certain points of a surface of the Earth. A latitude and longitude of these points will derivate two-dimensional space. Therefore from references the two-dimensional array of the gravimetric measuring information can be generated. Volume of this array can be very large. Therefore actual there is a compression of such information. It also is influenced by limited volume of storage devices for accumulation and storage of the gravimetric measuring information.

The compression of the gravimetric measuring information provides compact storage of this information. Thus the conditions are observed:

- maintenance of specific accuracy of representation of this information at a minimum allowed degree of compression;
- or maintenance of a specific degree of compression of this information at minimum allowed( permissible) accuracy of representation.

In a method of compression the low-frequency filtration of the gravimetric measuring information is executed. The basis of such filtration is difference in spectral density of a useful signal and handicapes (Fig. 2). In an outcome of a filtration initial errors of the measuring information essentially decrease. The transformed constituent of an error also decreases. She is stipulated by presence of a set of computing operations in a method of compression. In an outcome the tolerance by the methodical constituent of an error stipulated by application of the algorithm of compression with an exclusion of a part of the information can be increased.

The increase of a degree of compression of the gravimetric measuring information is provided because of applications of effective methods of compression of two-dimensional arrays of the information because of fractals. Such approach for the information on objects of a natural origin especially is effective. It concerns also gravimetric measuring information. Thus the parameters of a method of compression are selected so that the errors of the gravimetric measuring information after compression have not exceeded specific values.

5. CONCLUSIONS

In gravimeter the algorithmic handling of a measuring information allows to separate a useful signal of acceleration of gravity from errors. These errors stipulated vertical acceleration, forward and angular vibrations of the plane, on which is installed gravimeter. Thus, in an output signal
of gravimeter a series of errors of measurements completely is compensated. The exactitude of measurements of acceleration of gravity makes 1 mGal.

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