FRICIONAL HEAT GENERATION DURING COLD ROLLING OF METALS

Olena YEVTUSHENKO*

*Department of Automatics and Robotics, Faculty of Mechanical Engineering, Bialystok Technical University, ul. Wiejska 45c, 15-351 Bialystok, Poland
lenay@pb.edu.pl

Abstract: The problem of transient frictional heating in cold rolling of metals is considered. For this purpose the equations of heat conduction for the rolls and the rolled strip are solved by the Laplace integral transform method. The evolution of the contact temperature and its dependence from sliding speed is discussed.

1. INTRODUCTION

In conventional cold rolling, a cold metal strip is passed through the cold rolls. The heat due to friction is generated in a contact zone and it is induced by rolls and a strip. For cooling of the rolls water can be used.

It was established that temperature is one of the factors which significantly affects the cold rolling efficiency (Kovalev S.I. et al, 1982). The theoretical model for the temperature of rolls and a strip due to its frictional heating has been presented by Wilson W.R.D. et al, (1989) and Yamamoto H. et al, (1993) using the finite element approach. This model assumes that the material of the rolls is homogeneous and isotropic. The temperature regimes of the rolls under the assumption that the rolls are micro periodically stratified concentric cylinders composed of two different materials have been investigated by Matysiaik et al, (1998).

In present paper the solution of the boundary-value problem of heat conduction for homogeneous rolls and strip by the Laplace integral transform method is obtained.

2. STATEMENT OF THE PROBLEM

The contact geometry of the strip and rolls is shown in Fig. 1.

It is assumed that:

a. friction heat is generated on the interfaces and completely distributed between the strip and rolls;

b. plastic work of deformation occurs with heat generation in the interior of the strip;

c. length of the arc of the contact is very much smaller than the strip thickness and, hence, the temperature field in the nip region is one-dimensional with respect to the spatial coordinate;

d. rotational velocity of rolls suffices to neglect the heat conduction in the circumferential direction. The heat conduction in the axial direction is also ignored;

e. outside the contact region the rolls are cooled with water.

On this assumptions, the boundary-values heat conduction problem for the rolls may be written in the form (Luikov, 1968)

\[
\frac{\partial^2 T(r,t)}{\partial t^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(r,t)}{\partial r} \right) = \frac{1}{k_R} \frac{\partial^2 T(r,t)}{\partial t} , \quad 0 < r < R , \quad t > 0, \quad (1)
\]

\[
T(r,0) = T^0_R, \quad 0 \leq r \leq R, \quad (2)
\]

\[
K_R \left. \frac{\partial T(r,t)}{\partial r} \right|_{r=R} = \left\{ \begin{array}{l}
q_R, \quad 0 \leq \varphi \leq \varphi_0 , \quad t > 0, \\
n h[T_W - T(R,t)], \quad \varphi_0 < \varphi \leq 2\pi , \quad t > 0, 
\end{array} \right. \quad (3)
\]

\[
T(0,t) < \infty, \quad (4)
\]

where \( T \) is the temperature, \( r \) is the radial coordinate, \( \varphi \) is the angular coordinate, \( \varphi_0 \) is the contact angle, \( t \) is the time, \( R \) is the roll radius, \( q_R \) is the intensity of the heat flow, \( K_R \) is the thermal conductivity, \( k_R \) is the thermal diffusivity, \( h \) is the heat exchange coefficient, \( T_W^0 \) is the initial temperature of the rolls, \( T_W \) is the temperature of cooling water.

The intensity of the heat flow \( q_R \) generated at the interface and directed into the roll is determined by Johnson K.L., (1987):

\[
q_R = (1 - \lambda) q_f, \quad q_f = f \left| V_R - V_S \right| \rho_m , \quad (5)
\]
where $\lambda$ is the heat partition factor, $V_R$ is the sliding speed of the roll, $V_S$ is the speed of the strip, $p_m$ is the average contact pressure, $f$ is the coefficient of friction.

The heating time $t_R$ and cooling time $t_C$ for one rotation of the rolls are given by

$$t_R = \frac{R\phi_0}{V_R}, \quad t_C = \frac{2\pi R}{V_R} - t_R.$$  

(6)

The average thickness $H_m$ of the rolled strip in the contact region is given by Matysiak et al, (1998):

$$H_m = \frac{H_1 + 2H_2}{3}$$  

(7)

and the contact angle

$$\phi_0 = \sqrt{\frac{H_1 - H_2}{R}},$$  

(8)

where $H_1$ is initial, $H_2$ is final thickness of the rolled strip.

According to the assumptions a) – e), the boundary value problem of heat conduction for the rolled strip has the form:

$$\frac{\partial^2 T(z,t)}{\partial z^2} + q_p = \frac{1}{k_S} \frac{\partial T(z,t)}{\partial t}, \quad \left| z \right| < \frac{1}{2} H_m, \quad t > 0$$  

(9)

$$T(z,0) = T_S^0, \quad \left| z \right| < \frac{1}{2} H_m,$$  

(10)

$$K_S \frac{\partial T(z,t)}{\partial z} = \pm q_s, \quad z = \pm \frac{H_m}{2},$$  

(11)

where $z$ is the coordinate, $K_S$ is the thermal conductivity, $k_S$ is the thermal diffusivity, $q_p = \lambda q_f$ is the intensity of the frictional heat flow directed into the strip.

Specific power of the heat source caused by the plastic deformation of the strip is determined as

$$q_p = \frac{k}{K_S} \frac{d\varepsilon}{dt}.$$  

(12)

The plastic strain rate is found from the rough formula (Yamamoto H. et al, 1993):

$$\frac{d\varepsilon}{dt} = \frac{1}{t_S} \ln \left( \frac{H_1}{H_2} \right),$$  

(13)

where $t_S = R\phi_0 / V_S$ is the contact time for the rolled strip.

Let us consider the case of cold rolling where the strip slips relative to the rolls at all the points in the contact region. Then, according to Orowan's theory of cold rolling (Orowan E., 1943), two zones of local speed of the rolled material can be defined. At entry the strip is moving slower than the roll surfaces ($V_S < V_R$); at exit the strip is moving faster ($V_S > V_R$). The vertical plane where $V_S = V_R$ is termed neutral. Location of the neutral angle $\phi_n$ is found from the relations (Johnson K.L., 1987):

$$\phi_n = \phi_0 \phi_n^* = -\frac{1}{2} \frac{\gamma_2}{\gamma_1} \left[ 1 - \exp \left( -\frac{\gamma_1}{2} \right) \right].$$  

(14)

$$\gamma_1 = 2f\phi_0 \frac{R}{H_m}, \quad \gamma_2 = \frac{\phi_0}{f}.$$  

(15)

The strip speed at arbitrary vertical plane is assumed to be constant and equal to

$$V_S = V_R \frac{H_n}{H_m},$$  

(16)

where $H_n = H(\phi_n)$.

The average pressure over the arc of the contact is calculated by the equation (Johnson K.L.,1987):

$$p_m = k \left[ 4 \frac{(\gamma_1 + \gamma_2)}{\gamma_1^2} (\exp(-\gamma_1 \phi_n^*) - 1) + \gamma_2 + 4 \gamma_2 \phi_n^* - 2 \gamma_2 \phi_n^2 \right],$$  

(17)

where $k$ is the yield stress of the strip material in simple shear.

3. SOLUTION OF THE PROBLEM

The boundary-value heat conductivity problems (1) - (4) and (9) - (11) are solved by the method of Laplace integral transform. For the region $0 < \phi < 2\pi$ where the roll is heated, we obtain

$$T(r,t) = T_R^0 + \frac{q_R R}{K_R} \left[ \frac{2k_R r}{R^2 t} + 1 \left( \frac{r}{R} \right)^2 \right] - \frac{1}{4} - 2 \sum_{n=1}^{\infty} \frac{J_0(rn^*)R}{\alpha_n^2 J_0(\alpha_n)} \exp \left[ -k_R \left( \frac{\alpha_n^2}{R} \right)^2 t \right]$$  

(18)

where $J_n (\cdot)$ is the Bessel function of the first kind of the order $n$, $\alpha_n > 0$, $n=1, 2, \ldots$, are zeros of the Bessel function $J_n(\cdot)$, i.e., $J_n(\alpha_n)=0$.

For the region $\phi_0 < \phi < 2\pi$ where the roll is cooled, we have

$$T(r,t) = T_W - 2Bi(T_W - T_R^0) \times$$

$$\times \sum_{n=1}^{\infty} \frac{J_0(\beta_n r/R)}{\beta_n J_0(\beta_n)} \exp \left[ -k_R \left( \frac{\beta_n^2}{R} \right)^2 t \right]$$  

(19)

where $\beta_n > 0$, $n=1, 2, \ldots$ are the positive roots of equation $\beta_n J_1(\beta_n) - Bi J_0(\beta_n)$,

(20)

$Bi = hR / K_R$ is the Biot number.

The solution of the boundary value problem of heat conduction (9) – (11) is

$$T(z,t) = T_S^0 + q_p k_S t + \frac{q_S H_m}{K_S} \left[ \frac{2k_S}{H_m^2} t + \frac{z^2}{H_m^2} - \frac{1}{12} \right]$$  

(19)
where \( |z| \leq H_m/2, \ t \geq 0 \).

4. HEAT PARTITION RATIO

The unknown heat partition ratio \( \lambda = q_S/(q_f + q_S) \) of frictional heat between the roll and the strip can be determined from the condition that the temperature of the roll in the contact region at time \( t = t_R \) is equal to that of the strip surface \( z = H_w/2 \) at \( t = t_S \):

\[
\lambda = \frac{K_S}{q_f H_m \Psi_S} \left[ T_R^0 - T_S^0 - q_p k_S l_S + \frac{q_R R}{K_R \Psi_R} \right],
\]

(22)

where

\[
\Psi_R = 2 \frac{k_R l_R}{R^2} + \frac{1}{4} - \frac{2}{\alpha_n} \sum_{n=1}^{\infty} \frac{1}{(2 \pi n)^2} \exp \left( -\frac{\alpha_n^2 k_R l_R}{R^2} \right),
\]

(23)

\[
\Psi_S = 2 \frac{k_S l_S}{H_m^2} + \frac{1}{6} - \frac{4}{\alpha_n} \sum_{n=1}^{\infty} \frac{1}{(2 \pi n)^2} \exp \left( -\frac{(2 \pi n)^2 k_S l_S}{H_m^2} \right).
\]

(24)

The total heat flow generated at the interface will direct into the roll at \( \lambda = 0 \) and into the strip at \( \lambda = 1 \).

5. NUMERICAL ANALYSIS AND DISCUSSION

All calculations by formulas (18)–(24) were performed with experimental data borrowed from article of Yamamoto H. et al. (1993). The numerical examinations of contact temperature and heat partition ratio for the system consisting of stainless steel SUS 430 rolls \( (K_e = 50.2 \text{ W/(m}^2\text{C}) \), \( k_e = 1.33 \times 10^3 \text{m}^2/\text{s}) \) and high-conductivity copper strip \( (K_s = 381 \text{ W/(m}^2\text{C}) \), \( k_s = 1.02 \times 10^{-4} \text{m}^2/\text{s}) \) and high-conductivity copper strip \( (K_s = 381 \text{ W/(m}^2\text{C}) \), \( k_s = 1.02 \times 10^{-4} \text{m}^2/\text{s}) \).

Radius of rolls is \( R = 38 \text{ mm} \), the strip thickness at the entry into the contact zone is \( H_t = 1.78 \text{ mm} \). Reduction in the strip thickness \( d = (H_1 - H_2)/H_1 \) in the cold rolling made up to 20%. The cold rolling conditions were as follows: \( f = 0.1 \), \( T_w = 45^\circ\text{C} \), \( h = 5500 \text{ W/(m}^2\text{C}) \).

For the first rotation of the roll, \( T_R^0 = T_{R0} = 45^\circ\text{C} \). For the \( m \)-th rotation \( T_R^{(m)} = (T_R^{(m-1)} + T_R^{(2m)})/2 \), where the average temperature in the contact region \( (0 \leq r \leq R, 0 \leq \varphi \leq \varphi_0) \) is found from solution (18) by the formula:

\[
T_R^{(m)} = T_R^{(m-1)} + \frac{q_R R}{K_R \Psi_R}, \quad m = 2, 3, ...
\]

(25)

In the region \( (0 \leq r \leq R, \varphi_0 \leq \varphi \leq 2\pi) \), where the roll is cooled, the average temperature has, in view of solution (19), the following form:

\[
T_R^{2(m)} = T_W - 2Bi(T_W - T_R^{(0(m-1))}) \times \sum_{n=1}^{\infty} \frac{1}{\alpha_n^2} \exp \left( -\frac{\alpha_n^2 k_S l_S}{H_m^2} \right) \left[ \frac{\beta_n}{R} \int J_0(\beta_n) \left( \beta_n J_0(\beta_n) + Bi J_1(\beta_n) \right) \right]
\]

(26)

where \( m = 2, 3, ... \).

The time-dependent heat partition factor \( \lambda \) at \( d = 20\% \) for different sliding speeds of the roll: \( 1-V_R = 1.6 \text{m/s}, 2-2.8 \text{m/s}, 3-4.5 \text{m/s}, 4-6.7 \text{m/s} \).

The heat partition factor \( \lambda \) (22) - (24) and the contact temperature \( T \) increase most rapidly when the rolls make the first rotation of the duration \( t = t_R + t'_R \), and with time they reach steady state (Fig. 2, 3). It is seen that the rate of friction heat directed into the strip and contact temperature increases with increasing of the roll speed. The amount of heat entering the roll decreases with increasing \( V_R \), i.e., with increasing the power of frictional heat release (Fig. 2).

Fig. 2. The time-dependent heat partition factor \( \lambda \) at \( d = 20\% \) for different sliding speeds of the roll: \( 1-V_R = 1.6 \text{m/s}, 2-2.8 \text{m/s}, 3-4.5 \text{m/s}, 4-6.7 \text{m/s} \).

Fig. 3. The evolution of the contact temperature \( T \) at \( d = 20\% \) for different sliding speeds of the roll: \( 1-V_R = 1.6 \text{m/s}, 2-2.8 \text{m/s}, 3-4.5 \text{m/s}, 4-6.7 \text{m/s} \).

With increasing the parameter \( d \) defining the relative reduction in the strip thickness, the amount of friction heat directed into the roll increases (Fig. 4), while the contact
temperature is elevated (Fig. 5). At low sliding speed (<3 m/s), nearly linear dependence of the contact temperature on the parameter $d$ should be noted.

![Diagram](image1)

**Fig. 4.** Dependence of the heat partition ratio $\lambda$ on the sliding speed of the roll $V_R$ in its first rotation for different values of the strip thickness reduction $d$: 1–$d$=20%, 2–25%, 3–30%, 4–35%

Thus, it was shown that in cold rolling of high-conductivity copper the temperature in the strip-roll contact region calculated by the proposed model is greater than 100°C. This is in agreement with results of Yamamoto H. et al (1993).

![Diagram](image2)

**Fig. 5.** Contact temperature $T$ versus sliding speed of the roll $V_R$ in its first rotation for different values of the parameter $d$: 1–$d$=20%, 2–25%, 3–30%, 4–35%

It should be recognized that for study of the frictional heating effect we dropped deliberately some features of cold rolling which are important from a processing standpoint.

REFERENCES


GENERACJA CIEPŁA NA SKUTEK TARCIA PODCZAS ZIMNEGO WALCOWANIA METALI

Streszczenie: Rozpatruje się zagadnienie nieustalonego nagrzewania tarczowego podczas zimnego walcowania metali. W celu tym za pomocą transformacji całkowej Laplace’a otrzymano rozwiązanie równań przewodnictwa ciepła dla wałków i warstwy. Zbadano ewolucję temperatury oraz jej zależność od prędkości poślizgu.