_forecast model for real time reliability of storage system based on periodic inspection and maintenance data_.

**FORECAST MODEL FOR REAL TIME RELIABILITY OF STORAGE SYSTEM BASED ON PERIODIC INSPECTION AND MAINTENANCE DATA**

**MODEL DO PROGNOZOWANIA NIEZAWODNOŚCI SYSTEMU MAGAZYNOWANIA W CZASIE RZECZYWISTYM W OPARCIU O DANE Z PRZEGŁĄDÓW OKRESOWYCH ORAZ DANE EKSPLOATACYJNE**

In recent years, storage reliability has attracted much attention for increasing reliability requirement. In this paper, forecast models for real-time reliability of storage system under periodic inspection and maintenance are presented, which is based on the theories of reliability physics and exponential distribution. The models are developed under two newly-defined imperfect repair modes, i.e., Improved As Bad As Old (I-ABAO) and Improved As Good As New (I-AGAN). A completion method for censored life data is also proposed by averaging the residual lifetime. According to the complete and censored lifetime data, parameters in the models are estimated by applying maximum likelihood estimation method and iterative method respectively. A numerical example of a storage system is given to verify the feasibility of the proposed completion method and the effectiveness of the two models.

**Keywords:** storage reliability, real-time reliability, periodic inspection and maintenance, censored data.

1. **Introduction**

In engineering practice, there exist such kinds of systems. They are kept in storage state for most of their lifetime, but must keep high mission reliability and be ready for operation whenever needed. Strategic missile, spare parts for nuclear power plant, and fire protection system are typical examples of such systems. Periodic testing, replacement and other maintenance measures are necessary to avoid or reduce the occurrence of failure [6, 9, 12].

In the past decades, storage reliability has attracted much attention from both academia and engineering field. Merren studied the periodic test problem of electronic equipment in storage, and an algorithm for computing the reliability was developed based on test data [8]. Aneziris presented a method for calculating the dynamic reliability of safety systems, which was based on the theory of Markov chain [2]. Ref. [4] proposed an availability model for storage products under periodic inspection, but they ignored the initial failure. Zhao and Xie studied the problem of potential storage reliability estimation with initial storage failure [13]. Zhang and Zhao established a storage reliability model under periodic inspection [14]. Ref. [15] studied the storage reliability of missile-engine with Bayesian approach. Yang used stochastic process method to describe the degradation process of the components with multi-performance parameters, and proposed a storage reliability evaluation model [11]. Ref. [13] introduced an approach to evaluate the storage reliability of engine control circuit module.

Up to now, most existing research mainly concentrated on the data analysis methods for the components of a system. Real-time reliability estimation of products in storage on the system level has not received sufficient attention. Most studies assume the system is in perfect initial state and there is not much effort placed on empirical study based on actual storage reliability data. As for repair efficiency, two basic assumptions are known As Bad As Old (ABAO) and As Good As
New (AGAN). However, these two extreme cases seldom happen in most practical systems [7]. More reasonable repair models need to be developed to get a closer description of the real situation.

In this paper, real-time reliability evaluation models are established based on periodic inspection data, and two newly-defined imperfect repair modes are considered, i.e., Improved-ABAO and Improved-AGAN. A new completion method is also proposed to convert censored life data into complete data. Based on complete and censored lifetime data, the parameters in the models are estimated with maximum likelihood estimation method and iterative method, respectively. A numerical example is given to illustrate the models and method.

2. Reliability model for storage system

Due to different reasons, not all the storage systems are shipped out of the factory with reliability testing. Thus the initial use reliability is

\[ R(t) = P(T > t), t ≥ 0 \]

where \( R \) is the inherent reliability of a storage system, \( R_o = P(A); R(t) = P(T > t | A) \) is the conditional reliability of the system, which changes with storage time and \( R(0) = 1 \). Therefore, the system reliability can be represented as

\[ R(t; \theta) = R_o R_s(t; \theta), \theta \in \Theta \]

where \( \Theta \) is a measurable parameter set.

In order to keep high mission reliability, periodic inspection and maintenance are needed. Based on the difference of maintenance, two types of forecast models for real-time reliability are given as follows.

2.1. Reliability forecast model under I-AGAN

Assume that the system reliability is put into use at \( t = 0 \) and it is inspected at intervals of time \( t \geq 0 \). If partial failure or aging occurs, the maintenance is carried out. After \( x(0 < x < t) \) time maintenance, the system is restored and keeps on storing. When the maintenance is perfect and leaves the system as if it were new, we call it repair under AGAN[7]. In engineering practice, however, maintenance can increase system reliability but the failure rate usually shows an increase tendency owing to the degradation of material strength. Taking account of this, we propose a new repair mode called Improved-AGAN (I-AGAN) (See Fig. 1).

Under I-AGAN, three assumptions are made as below: (1) The system is stored under natural conditions, and the lifetime \( t \) obeys exponential distribution. (2) In the initial time, the system reliability is \( R(0) = R_o \leq 1 \) and lifetime \( t \) obeys exponential distribution with parameter \( \theta_o \). After the \( k \)th maintenance, lifetime obeys exponential distribution with parameter \( \theta_k \) and \( \theta_0 \geq \theta_1 \geq \theta_2 \geq \ldots \geq \theta_{k-1} \geq \theta_k \) (3) The inspection interval is \( \tau \). The maintenance time \( x_i \) \( (k = 1, 2, \ldots) \) is a stochastic variable that varies with failure.

From Fig. 1, the reliability before the \( k \)th maintenance \( R(t + \sum_{i=1}^{k-1} x_i) \) is \( R_o (k + \sum_{i=1}^{k-1} x_i) \). According to the definition under I-AGAN, after each maintenance the system is restored to its initial state, so the reliability at time \( k + \sum_{i=1}^{k-1} x_i \) is \( R_o (k + \sum_{i=1}^{k-1} x_i) \leq R_o (k + \sum_{i=1}^{k} x_i) = R_0 = R(0) \), where \( k = 1, 2, 3, \ldots, n \); \( n \) is times of maintenance.

Denote \( F(t) \) as the failure distribution function of the system at \( t \) and \( F(t) = 1 - R(t) \). Hence, \( F(t) \) obeys the following distribution:

\[ F(t) = 1 - R_o \exp \left( -\frac{t - \tau - \sum_{i=1}^{r} x_i}{\theta} \right), r = \left[ \frac{t - \tau - \sum_{i=1}^{r} x_i}{\tau} \right] \]

(4)

Suppose \( \lambda \), \( (i = 1, 2, \ldots, k) \) is the system failure rate after the \( i \)th maintenance. As \( \lambda_i = 1/\theta_i \), obviously, \( \lambda_k \leq \lambda_{k-1} \leq \ldots \leq \lambda_1 \leq \lambda_0 \). According to the reliability physics and engineering practice, after the \( k \)th maintenance the failure rate satisfies the following equation:

\[ \lambda_k = \lambda_0 \exp(-k\beta), k = 1, 2, \ldots, n \]

(5)

where \( \beta(\beta > 0) \) is the degradation factor. It can also be rewritten as \( \theta_k = \theta_0 \exp(-k\beta), k = 1, 2, \ldots, n \).

From Eqs. (4), (5), the real-time reliability at time \( t \) is:

\[ R(t) = R_o \exp \left( -\lambda_0 \exp(-k\beta)(t - \tau - \sum_{i=1}^{r} x_i) \right), r = \left[ \frac{t - \tau - \sum_{i=1}^{r} x_i}{\tau} \right] \]

(6)

Based on the maintenance strategy, it can be obtained that:

\[ \lim_{t \to +kT + \sum_{i=1}^{k} x_i} \left[ \frac{t - \tau - \sum_{i=1}^{r} x_i}{\tau} \right] = k - 1 \]

\[ \lim_{t \to +kT + \sum_{i=1}^{k} x_i} \left[ \frac{t - \tau - \sum_{i=1}^{r} x_i}{\tau} \right] = k, k = 1, 2\ldots, n \]

(7)

From Eq. (6), it can be concluded that the system real-time reliability before the \( k \)th maintenance time \( t = k\tau + \sum_{i=1}^{k-1} x_i \) is:

\[ R_k(t) = R_o \exp(-\lambda_0 \exp(-k\beta \tau)), k = 1, 2, \ldots, n \]

2.2. Reliability forecast model under I-ABAO

When the maintenance leaves the system in the same state as it was before failure, we call it repair under ABAO[7]. In practice, however, maintenance may leave the system reliability higher than it was before failure but lower than its initial state \( R_o \). Additionally, material strength degradation causes failure rate to increase. Taking account of this, we
propose another imperfect repair mode called Improved-ABAO (I-ABAO), with the same assumptions in Section 2.1 (See Fig. 2).

According to reliability characteristics, we can divide a storage system into two subsystems. Subsystem 1 requires regular inspection and maintenance with an invariable or increasing failure rate. Subsystem 2 has a high reliability and does not require regular inspection and maintenance, but aging phenomenon may occur.

Under I-ABAO, after each maintenance the system reliability \( R(t) \) cannot be restored to the initial reliability, that is:
\[
R(t_k + \sum_{i=1}^{k} x_i) < R_0 = R(0) .
\]
During the storage, the real-time reliability of Subsystem 1 can be obtained by:
\[
R_{1k}(t) = \exp\left[-\lambda_0 \sum_{i=1}^{k} x_i (t - \tau - \sum_{j=1}^{i} x_j) / \tau\right],
\]
where \( \lambda_0 \) is the degradation coefficient. From Eq. (2), it can be concluded that the system reliability at time \( t \) is:
\[
R(t) = R_0 R_{1k}(t) = R_0 R_{1k} R_{2k}(t) = R_0 \exp\left[-\lambda_0 \sum_{i=1}^{k} x_i / \tau - \sum_{i=1}^{k} x_i (t - \tau - \sum_{j=1}^{i} x_j) / \tau - \delta t\right],
\]
where \( \delta \) is the degradation coefficient. Insert it into Eq. (17) and the first iterative parameter set \( \hat{\Theta}_1 \) is:
\[
(17)
\]

Before the \( k \)th inspection, which is at time \( t = t_k + \sum_{i=1}^{k-1} x_i \), the system reliability \( R_k(t) \) is:
\[
R_k(t) = R_0 \exp\left[-\lambda_0 \sum_{i=1}^{k} x_i + \delta (t_k + \sum_{i=1}^{k-1} x_i) / \tau\right], \quad k = 1, 2, \ldots, n
\]

### 3. Completion of censored data

Regular inspection is necessary for the storage system, but the failures are seldom found exactly at the inspection time. So it is difficult to obtain complete data, and the field lifetime data are mostly censored. If the censored data are simply regarded as complete data or treated with interpolation method, large errors will be brought out. The completion method of censored life data is discussed below.

### 3.1. Completion method of censored data

Let \( T_1, T_2, \ldots, T_n \) be the lifetimes of storage system; \( t_1, t_2, \ldots, t_n \) be the start times of inspection and \( a(T) \) be the residual lifetime of the \( i \)th system. If \( t_i \) is right censored or completely censored, \( I = 0 \); Otherwise, if \( t_i \) is left censored, \( I = 1 \). It can be obtained that
\[
a(T_i) = (T_i - t_i)(1 - I_i), \quad I_i = I(t_i \leq T_i), i = 1, 2, \ldots, n .
\]
Consequently, the complete data of \( t_i(i = 1, 2, \ldots, n) \) is:
\[
(13)
\]

Accordingly, when the failure data of the \( i \)th system is right censored, the average residual lifetime is:
\[
(14)
\]

Therefore, under periodic inspection the average residual lifetime of the system is:
\[
(15)
\]

If inspection data are discontinuous, the average residual lifetime is:
\[
(16)
\]

where the measurable parameter set \( \theta_1, \theta_2, \ldots, \theta_m \in \Theta \) and \( a(T) \) are the implicit functions of unknown parameters, which can be solved by iterative algorithm [10]. Assuming that the system lifetime \( T \) has density function \( f(\xi; \Theta) \), the likelihood function of parameter set \( \Theta \) within the complete sample data \( \xi_1, \xi_2, \ldots, \xi_m \) is \( L(\xi; \Theta) = \prod_{i=1}^{m} f(x_i; \Theta) \), so the likelihood equation is:
\[
(17)
\]

Suppose the initial value of parameter estimation is \( \Theta_0 = \left\{ (\theta_1)_0, (\theta_2)_0, \ldots, (\theta_m)_0 \right\} \). Insert it into Eq. (17) and the first iterative parameter set \( \Theta_1 = \left\{ (\theta_1)_1, (\theta_2)_1, \ldots, (\theta_m)_1 \right\} \) can be obtained. Circulating the iterative procedure, the \( k \)th iterative parameter set \( \Theta_k = \left\{ (\theta_1)_k, (\theta_2)_k, \ldots, (\theta_m)_k \right\} \) can also be obtained. Thereby, the maximum likelihood estimator set of \( \Theta \) is the convergent parameter set \( \hat{\Theta} = \lim_{k \to \infty} \Theta_k \).

### 3.2. Censored data completion of Weibull & exponential type system

For most storage system, the lifetime obeys Weibull distribution or memoryless exponential distribution. Based on cumulative damage effect, the method to convert censored data of Weibull storage system into complete ones is given below. Assume that the lifetime \( T \) has the density function:
\[
f(t, \alpha, \lambda) = \lambda \alpha (\lambda t)^{\alpha - 1} \exp\left[-(\lambda t)^{\alpha}\right], \quad t \geq 0
\]

where \( \lambda, \alpha \) are parameters to be estimated. The likelihood function under the case of the complete sample is:
and the maintenance time is negligible compared to the storage time, supposing that the periodic inspection interval is the Bayes estimation of use reliability, the estimator and use data set collected from periodic inspection, where reliability function is:

\[
\hat{\lambda}_0 = \exp(-\frac{\hat{\lambda}_0}{\alpha})
\]

\[
\hat{\beta}_0 = \exp(-\frac{\hat{\beta}_0}{\tau})
\]

\[
\hat{\delta}_0 = \exp(-\frac{\hat{\delta}_0}{k})
\]

Therefore, the logarithmic likelihood equations are:

\[
\sum_{i=1}^{n} T_i^a \ln T_i \left( \frac{1}{\alpha} - \frac{1}{n} \sum_{i=1}^{n} T_i^a \right) = 0
\]

\[
\sum_{i=1}^{n} T_i^a \beta T_i \left( \frac{1}{\beta} - \frac{1}{n} \sum_{i=1}^{n} T_i^a \right) = 0
\]

\[
\gamma - \frac{1}{n} \sum_{i=1}^{n} T_i^a = 0
\]

If \( x_i \) is left censored, it can be obtained from Eqs. (13), (15) that:

\[
T_i = \frac{\lambda_i^a \mu^a \exp(-\tau dt)}{\exp(-\mu_i \tau dt)}
\]

\[
\tau_i = \left[ (i-1)\tau + \sum_{j=1}^{i-1} x_j \right]^{\alpha}
\]

\[
\delta_i = \exp(-\frac{\hat{\delta}_0}{k})
\]

\[
\beta_i = \exp(-\frac{\hat{\beta}_0}{\tau})
\]

\[
\alpha_i = \exp(-\frac{\hat{\alpha}_0}{\tau})
\]

\[
\lambda_i = \exp(-\frac{\hat{\lambda}_0}{\tau})
\]

\[
T_i = x_i + \frac{\tau}{\lambda}
\]

For the exponential storage system, take \( a = 1 \), then the completion of left censored data and right censored data are given as follows, respectively.

\[
T_i = x_i + \frac{\tau}{\lambda}
\]

If \( x_i \) is right censored, then:

\[
T_i = \frac{\lambda_i^a \mu^a \exp(-\tau dt)}{\exp(-\mu_i \tau dt)}
\]

\[
\tau_i = \left[ (i-1)\tau + \sum_{j=1}^{i} x_j \right]^{\alpha}
\]

\[
\delta_i = \exp(-\frac{\hat{\delta}_0}{k})
\]

\[
\beta_i = \exp(-\frac{\hat{\beta}_0}{\tau})
\]

\[
\alpha_i = \exp(-\frac{\hat{\alpha}_0}{\tau})
\]

\[
\lambda_i = \exp(-\frac{\hat{\lambda}_0}{\tau})
\]

\[
T_i = x_i + \frac{\tau}{\lambda}
\]

\[
\sum_{i=1}^{n} \tau_i^a \ln \tau_i \left( \frac{1}{\alpha} - \frac{1}{n} \sum_{i=1}^{n} \tau_i^a \right) = 0
\]

\[
\sum_{i=1}^{n} \tau_i^a \beta \tau_i \left( \frac{1}{\beta} - \frac{1}{n} \sum_{i=1}^{n} \tau_i^a \right) = 0
\]

\[
\gamma - \frac{1}{n} \sum_{i=1}^{n} \tau_i^a = 0
\]

4. Parameter estimation of reliability model

When the censored data have been converted into complete data, the estimator \( \hat{\Theta} \) of parameter set can be obtained by iteration of Eq. (17). Given the distribution function \( F(t) = F(t; \Theta) \), the real-time reliability function of storage system can be obtained as:

\[
R(t) = \hat{R}_0 \cdot \exp\left[ -\lambda_0 t^\alpha \right]
\]

For the original censored data, \( (N_i, S_i, t_i), i=1, 2, \ldots, n \) is the failure data set collected from periodic inspection, where \( S_i \) is the number of non-failed products within \( N_i \) of total stored products at time \( t_i \). Supposing that the periodic inspection interval \( \tau \) is a fixed time unit, and the maintenance time is negligible compared to the storage time, only parameters in Eqs. (8), (12) need to be estimated.

\[
R_k(t) = \hat{R}_0 \cdot \exp\left[ -\lambda_0 t^\alpha \right], \quad k = 1, 2, \ldots, n
\]
5. Case study

In this section, a storage system composed of 18 subsystems is considered. It is maintained every other year to keep its availability. Compared with the storage time, the time consumed by maintenance is negligible. Those subsystems which have serious failure or have no value for maintenance will be withdrawn from storage, while others will be restored to storage state. Maintenance will continue until all the subsystems are withdrawn from storage. The detailed data are shown in Table 1.

5.1. Reliability function of Weibull & exponential type system

At the beginning of storage, all the 18 subsystems satisfy mission reliability. According to Eq. (25), the initial storage reliability $R_0$ is:

$$R_0 = \frac{1}{2} \left(1 + \frac{S_0}{N_0 + 2}\right) = 0.975$$

Rearrange the data in Table 1 on a monthly basis by left-censored data (denoted as $x^-$), right-censored data (denoted as $x^+$), and complete data (denoted as $x$): \{36+, 36+, 84+, 84+, 84+, 120+, 120+, 144+, 144+, 156+, 156+, 156+, 156+, 156+, 180+, 204+, 204+, 228+\}. If the storage lifetime obeys Weibull distribution, the complete lifetime data of the storage system $T_i$ (month) are derived from Eqs. (21), (22), (23), as shown in the sequence: (248, 248, 262, 262, 262, 267, 278, 126, 291, 298, 298, 298, 298, 156, 312, 327, 327, 343). Inserting it into Eq. (20), parameter estimators with a precision of 10-5 are obtained: $\hat{\beta} = 0.00363$. The real-time reliability function with complete lifetime data is:

$$R(t) = 0.975 \exp \left[-0.000363 \cdot t \right]$$

(35)

If the storage lifetime obeys exponential distribution, the complete lifetime data $T_i$ (month) are derived from Eq. (24), as shown in the sequence: \{1239, 1239, 1287, 1287, 1287, 156, 156, 156, 156, 156, 156, 156, 156, 156, 156, 156, 156, 156, 156\}. The estimator of parameter $\lambda$ is:

$$\hat{\lambda} = \frac{1}{n} \left[ \sum_{i=1}^{n} T_i \right]^{-1} = 8.3126 \times 10^{-4}.$$  

So the real-time reliability function with the complete lifetime data is:

$$R(t) = 0.975 \cdot \exp \left(-0.00083126 t \right)$$

(36)

Fig. 3 illustrates the reliability change of Weibull type system and exponential type system.

5.2. Reliability analysis under I-AGAN

Based on the original exponential censored data (Table 1), when I-AGAN is adopted, the estimator of parameter $\lambda_0$ and $\beta$ in Eq. (30) can be obtained using numerical analysis and iterative method: $\hat{\lambda}_0 = 0.003865$, $\hat{\beta} = 0.1102$. Therefore, the real-time reliability under I-AGAN can be expressed as:

$$R(t) = 0.975 \exp \left[-0.0003865 \cdot \left\lfloor t \right\rfloor\cdot \left\lfloor \frac{t}{12} \right\rfloor \right] \left(t - 12 \cdot \left\lfloor \frac{t}{12} \right\rfloor \right)$$

(37)

where $\lfloor \cdot \rfloor$ represents rounding down.

To validate the effectiveness of the estimation model, reliability curves drawn according to Eqs. (36), (37) are shown in Fig. 4.
It can be concluded from Fig. 4 that during storage under I-AGAN, the reliability of exponential system decreases progressively faster over time, which accords with engineering practice. Within the first 10 years, the system keeps a reliability higher than 0.85, and within 19 years, the reliability still remains higher than 0.7.

5.3. Reliability analysis under I-ABAO

When I-ABAO is adopted, based on the data in Table 1 and Eq. (33), estimator of parameter $\lambda_0$, $\delta$, $\beta$ in Eq. (34) can be obtained using iterative method: $\lambda_0=0.001098$, $\beta=0.2015$, $\delta=0.000384$. Therefore, the real-time reliability under I-ABAO is expressed as:

$$R(t) = 0.975\exp\left[-0.001098\exp\left\{0.2015\left(\frac{t}{12}\right)\right\}\left(-12\left(\frac{t}{12}\right)\right) - 0.000384\right]$$

where $\lfloor \cdot \rfloor$ represents rounding down.

Reliability curves drawn according to Eqs. (36), (38) are shown in Fig. 5.

In Fig. 5, the reliability of exponential system under I-ABAO has similar downtrend with that under I-AGAN. Within the first 10 years of storage, the system keeps a reliability higher than 0.80. However, after 10 years, the reliability enters into a low reliability area or even fails. Compared with Fig. 4, I-AGAN can keep higher reliability.

Fig. 4 and Fig. 5 show the comparison between the reliability curves obtained from complete data and censored data under I-AGAN, I-ABAO respectively. The results show they share the similar downtrend and they are in accordance with engineering practice, which verifies the effectiveness of the proposed completion method and models.

6. Conclusion

In this paper, we develop two forecast models for real-time reliability of storage system under two repair modes. A completion method is also proposed to convert censored data into complete data by averaging the residual lifetimes. The case study shows that the system reliability estimated using the complete data shares similar changing trend with that using directly recorded censored data. The result verifies the feasibility of the proposed completion method and the effectiveness of the two models. For systems in long-time storage state under periodic inspection, the real-time reliability can be effectively estimated by applying the forecast models proposed in this paper.

In this article, it is assumed that the repair cycle is a fixed value. It should be interesting to carry out further studies on the unfixed repair cycle. Furthermore, if it is difficult to identify a suitable theoretical distribution for an application, a nonparametric approach can be employed to estimate the probability distribution based on the periodic inspection data.

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