Based on the analysis of failure interaction, imperfect preventive warranty policy is adopted for the multi-component system. Average failure rate of each warranty interval is studied and warranty cost model and availability model are built as viewed from interactive failure rate. Then Warranty period project is brought forward as an example, which can validate the feasibility of model and show the advantage of the project. The research can provide technique and methods for determining Warranty Period of multi-component system, which further enriches and perfects the warranty theory.

**Keywords:** warranty period, cost, availability, failure interaction, multi-component.

In order to prevent product failure or its serious results and keep it in a prescribed state, a series of activities performed by the manufacture alone or jointly with the user are called preventive warranty. It primarily includes trouble shooting, periodic perfect maintenance and periodic imperfect maintenance etc. The paper researches on preventive warranty which mainly contains periodic perfect maintenance policy and periodic replacement policy. Chun [2] introduced periodic preventive warranty in prior time when he studied product warranty. Jack [4] further studies the model and the product can be made to ‘repair as good as new’ after preventive warranty, which permit preventive warranty interval variable. In order to achieve the lowest warranty cost, Yeh [13] improves the model as to make the degree of the preventive warranty reach some required level. On the base of the updated warranty policy, many scholars set up preventive warranty cost model in warranty interval and study the optimal periodic preventive warranty interval, which aim at getting the lowest warranty cost [1, 4, 9]. The document [5] balances the saved and added cost by warranty products’ preventive warranty, and studies optimal preventive warranty strategy regarding product’s long term average scale of charges in minimum as goal, and determines the best preventive interval and provides the efficient algorithm. The above mentioned studies which research on warranty interval aimed at independent components and multi-component with separate failure, which affects the practice of the applications to some extent.

Along with technology development, the product with more complex and its various components with more interaction between certain parts of system, each of the failure of its own abrasion or aging, or some other units’ is the failure of the product, which make it not enough for warranty research to only pay attention to single component or multi-component system with separate failure. Therefore, based on the analysis of multi-component with failure interaction, this paper will establish warranty cost model and availability model under the imperfect preventive warranty policy, analyze cost effectiveness at unit interval, decide warranty decision-making project of multi-component system with failure interaction and validate this project.
2. Failure interaction analysis

Thomas [12] thinks that system of the maintenance interaction among the internal components can be divided into three categories: economy interaction, failure interaction, and structure interaction. The so-called economy interaction is that maintenance costs of a few parts by repairing together were lower than separately. The structure interaction is a body consisted by a number of components, which repairing some one means to repair other parts. Failure interaction is that failure of some part of the system will cause failure distribution of other parts of the system changing, so failure interaction was believed to be existed between the two parts. In early relevant documents, two conditions of failure interaction [8]: (a) a component (affecting component) fault resulted in malfunction of other components (affected components) at the rate of $p$ $(0 \leq p \leq 1$). (b) The failure of the affected components will increase the aging degree, but will not cause the immediate problems. The results of the two cases are that the failure rate of the affected components is accelerated. The components interaction will increase the failure rate, therefore the system failure rate is called relevant failure rate. Failure interaction may be stable or not[11]. When the failure interaction is stable, the affected components failure rate is higher than the independent rate, but remaining on some certain level. When it is unstable, the affected components failure rate will increase rapidly in a very short time.

According to the analysis model of the failure interaction in literature [10], for the system consisting of q components, the components’ failure interaction rate includes initial failure rate and new addition failure rate, and expressed as follows:

$$\{\lambda(t)\} = ([I]\{\lambda(t)\} + [\theta(t)]\{\lambda(t)\})B$$  

(1)

In which, $\{\lambda(t)\}$ is the vector of $q \times 1$, which shows the failure interaction rate, and $\{\lambda(t)\}B$ is the failure vector of the failure interaction $q \times 1$. $[I]$ is the unit matrix of $q \times q$, and $[\theta(t)]$ is the relevant coefficient matrix. The elements of $\theta(a)\theta(b)$ $(a = b = 1,2,\ldots,q)$ is the relevant coefficient, which shows the affected degree of component b towards component a. When $\theta(a)$ is equal to zero, there is no influence among the components; when $\theta(a)$ is equals to one, which shows that component a will cause failure of component b. The relevant coefficient can be decided by the following methods:

a) Get it by probability theory.
b) According to the experience estimation of designer, the manufacturer and maintenance personnel.
c) Based on the estimation of mechanical and kinetics.
d) Based on laboratory testing.

3. Warranty interval decision-making model

3.1. Model description and hypothesis

This paper mainly studies the two components system composed by one key component and subsystems, the system will be carried with the imperfect preventive warranty, without consideration of failure interaction in subsystems. In each imperfect preventive warranty interval, the key components will have its least warranty when it occurs failures. The failure rate remained after warranty, but will increase the subsystem failure rate $\lambda_{sb}$; on the contrary, subsystem failure will cause the failure rate of key components, and the whole system needs warranty after which the failure rate will remain.

To facilitate the research, as to multi-components we have the following hypothesis:

- Imperfect preventive warranty is adopted in warranty interval. When failure of each component occurred, warranty must be adopted. Failure rate after warranty is between as good as new and as bad as old. Failure rate of subsystem will change when warranty of key component is carried.
- The system has the characteristics of aging, and the failure rate will increase with time increases.
- The improvement in imperfect preventive warranty of the system is a constant.

3.2. Cost model

There are assumptions that imperfect preventive warranty is adopted in warranty interval, $T$ is warranty interval, each whole preventive warranty cost $Cw$ is the function of preventive warranty expected cost $Cw_0$, loss of unit time for shutdown $Cd$; and the time of each preventive warranty $T_j(Cw_0 = Cw_0^0 + Cw_0^n)$. So, warranty cost of system in interval is expressed as followed:

$$C(T, W) = nC_p + \sum_{j=1}^{n} EC_j(T) + EC(W - n(T + T_p))$$  

(2)

In which, $n$ is the number of imperfect preventive warranty in warranty interval $W$, $n = \text{int}[W/(T + T_p)]$, $EC_j(T)$ is the expected cost of $T_j(0 \leq j \leq 1)$ imperfect preventive warranty interval of the system. $EC(W - n(T + T_p))$ is the expected cost of the time between $n(T + T_p)$ to $W$ of the system.

The failure rate of key component in $j$th imperfect preventive warranty interval is as followed:

$$\lambda_{kj}(t) = \lambda_k(t - (j - 1)\alpha T)$$  

(3)

If failure happens in key component, failure rate of subsystem $\bar{\lambda}_{kj}(t)$ will increase. Based on the failure interaction, the average failure rate interaction of subsystem in $j$th imperfect preventive warranty interval is as followed:

$$\bar{\lambda}_{kj}(t) = \lambda_k(t - (j - 1)\alpha T) +$$

$$+ \theta \left[ \sum_{i=1}^{n} \eta_i \lambda_i(t - (i - 1)\alpha T) - \frac{1}{2} \eta_i^2 \lambda_i(t) \right]$$  

(4)

Failure quantities of key component in $j$th imperfect preventive warranty interval can be expressed by failure rate. It is as followed:

$$n_{kj} = \int_{(j-1)\alpha T + T_p}^{j\alpha T + T_p} \lambda_{kj}(t) dt$$  

(5)
Failure quantities of subsystem in \(j^{th}\) imperfect preventive warranty interval can be expressed by average failure rate. It is as followed:

\[
n_{j,k} = \int_{T_{j-1}+T_p}^{T_j+T_p} \lambda_{j,k} (t) dt
\]  

(6)

Each failure warranty cost \(C_{f}\) is the function of failure warranty expected cost \(C_{f}\); loss of unit time for shutdown \(C_{p}\) and the time of each failure warranty \(T(C = C_{p} + C_{f})\). According to the failure number of key component and subsystem in \(j^{th}\) imperfect preventive warranty interval, the expected warranty cost of the system in \(j^{th}\) imperfect preventive warranty interval is following:

\[
EC_j(T) = (n_{j,k} + n_{j,k})C_f
\]  

(7)

In the same way, the failure number of key component of the time between \(n(T+T_p)\) to \(W\) as follows:

\[
n_{(n+1),k} = \int_{n(T+T_p)}^{W} \lambda_{(n+1),k} (t) dt
\]  

(8)

The function of warranty cost in warranty interval can be gotten by taking formula (7) and (10) into (2):

\[
EC(W-n(T+T_p)) = (n_{(n+1),k} + n_{(n+1),k})C_f
\]  

(10)

The function of warranty cost in warranty interval can be gotten by taking formula (7) and (10) into (2):

\[
C(T,W) = nC_p + \sum_{j=1}^{n} EC_j(T) + EC(W-n(T+T_p))
\]  

\[
= nC_p + \sum_{j=1}^{n} [(n_{j,k} + n_{j,k})C_f] + (n_{(n+1),k} + n_{(n+1),k})C_f
\]  

\[
= nC_p + \sum_{j=1}^{n} \left[ \int_{T_{j-1}+T_p}^{T_j+T_p} \lambda_{j,k} (t) dt \right] C_f
\]  

\[
= \int_{n(T+T_p)}^{W} \left[ \lambda_{(n+1),k} (t) + \lambda_{(n+1),k} (t) \right] dt C_f
\]  

(11)

3.3. Availability model

Expected availability in warranty interval can be expressed as follow:

\[
A(T,W) = \frac{W - D(T,W)}{W}
\]  

(12)

\[D(T,W)\] and \[C(T,W)\] has the same expression, \[C_p\] and \[C_f\] is replaced by \[T_p\] and \[T_f\]. So expected shutdown time in warranty interval with imperfect preventive warranty interval \(T\) is as follows:

\[
ET_j(T) = (n_{j,k} + n_{j,k})T_f = \int_{T_{j-1}+T_p}^{T_j+T_p} \lambda_j(t) + \lambda_{j,k}(t) dt
\]  

(13)

In which, \(ET_j(T)\) is the expected shutdown time of \(j^{th}\) imperfect preventive warranty interval \(T\) of the system. \(ET(W-n(T+T_p))\) is the expected shutdown time of the time between \(n(T+T_p)\) to \(W\) of the system.

According to the analysis method of warranty cost of system, the expected failure warranty shutdown time of the system in \(j^{th}\) imperfect preventive warranty interval is following:

\[
ET(W-n(T+T_p)) = (n_{(n+1),k} + n_{(n+1),k})T_f = \int_{n(T+T_p)}^{W} \left[ \lambda_{(n+1),k} (t) + \lambda_{(n+1),k} (t) \right] dt T_f
\]  

(14)

The expected failure warranty shutdown time of the system between \(n(T+T_p)\) to \(W\) as follow:

\[
ET(W-n(T+T_p)) = (n_{(n+1),k} + n_{(n+1),k})T_f = \int_{n(T+T_p)}^{W} \left[ \lambda_{(n+1),k} (t) + \lambda_{(n+1),k} (t) \right] dt T_f
\]  

(15)

The function of warranty shutdown time in warranty interval can be gotten by taking formula (14) and (15) into (13):

\[
D(T,W) = nT_p + \sum_{j=1}^{n} \int_{T_{j-1}+T_p}^{T_j+T_p} \left[ \lambda_{j,k} (t) + \lambda_{j,k} (t) \right] dt T_f
\]  

\[
+ \int_{n(T+T_p)}^{W} \left[ \lambda_{(n+1),k} (t) + \lambda_{(n+1),k} (t) \right] dt T_f
\]  

(16)

The function of availability in warranty interval can be given taking formula (16) to (12).

\[
A(T,W) = 1 - \frac{1}{W} D(T,W)
\]  

\[
= 1 - \frac{1}{W} \left[ nT_p + \sum_{j=1}^{n} ET_j(T) + ET(W-n(T+T_p)) \right]
\]  

\[
= 1 - \frac{1}{W} \left[ nT_p + \sum_{j=1}^{n} \left\{ \int_{T_{j-1}+T_p}^{T_j+T_p} [\lambda_{j,k} (t) + \lambda_{j,k} (t)] dt T_f \right\} + \int_{n(T+T_p)}^{W} \left[ \lambda_{(n+1),k} (t) + \lambda_{(n+1),k} (t) \right] dt T_f \right]
\]  

(17)

3.4. Model resolution

Unit cost-effective of system is derived from cost and availability quantificationally. And scientific warranty needs to control warranty cost, at the same time to guarantee availability. So, models are analyzed by unit cost-effective, as follows:

\[
V = \frac{C(T,W)}{W - A(T,W)}
\]  

(18)

The function of Unit cost-effective can be given taking formula (5) and (11) to (18).
4. Case analysis

Diesel, as a complex equipment, is core and the key part, and the advantages and the disadvantages of whose performance take effect on the output of energy and the traction of power. Diesel mainly includes: pressure booster, oil pump, and movement components etc. The pressure booster failure caused by other components’ malfunction and failure; the movement components’ failure is fatigue-type failure, the proportion of the relevant failure is relatively small\(^1\); the oil pump failure is mainly of fatigue-type failure almost without relevant failure, which will also lead relevant failure to the pressure booster.

According to research and analysis, the diesel engines may be considered as multi-component system with failure interaction, which composed by the pressure booster and subsystems (all the rest of the components). And the discipline of the booster’ failure obeys weibull distribution:

\[
\lambda(t) = \left(\frac{m}{\eta}\right)^m \left(\frac{t}{\eta}\right)^{m-1}
\]

In which, \(m = 2\), \(\eta = 1000\). Failure rate \(\lambda_k\) of subsystem is 4.98\(\times\)10\(^{-4}\). Average time \(T^c_i\) of failure warranty is 3. Average warranty cost \(C_w\) of each failure is 300. Average warranty loss \(C_d\) of unit time is 900.

Assumptions of diesel:

Imperfect preventive warranty policy is adopted in warranty interval. Improve factor \(\alpha = 0.8\). In each imperfect preventive warranty interval, failure rate \(\lambda_k\) of supercharger will not change warranted, but failure rate \(\lambda_{sb}\) of subsystem will increase. Relevant coefficient \(\theta = 0.5\). Whereas, failure of supercharger happens immediately if failure of subsystem happen. The time of each preventive warranty \(T^c_i\) is 1. preventive warranty expected cost \(C_w\) is 300.

4.1. Calculate process

The number of imperfect preventive warranty in warranty interval \(W\) is as follows:

\[
n = \text{int}\left[ \frac{W}{T^c_i+1} \right]
\]

The whole failure warranty cost of system \(C = 300\times900\times3 = 3000\). The whole preventive warranty cost \(C^p = 100\times900\times1 = 1000\).

The failure rate of booster in the \(j\) imperfect preventive warranty interval is as follows:

\[
\lambda_{sb}(t) = \lambda_{sb}(t - (j-1)\alpha T) = \frac{2}{1000}(t - 0.8(j-1)T)
\]

Then, failure number of booster in \(j\) imperfect preventive warranty interval is as follows:

\[
n_{sb}(j) = \int_{(j-1)T^c_i+T}^{jT^c_i+T} \lambda_{sb}(t) dt = \int_{(j-1)T^c_i+T}^{jT^c_i+T} \left[ \frac{2}{1000}(t - 0.8(j-1)T) \right] dt
\]

The average failure rate interaction of subsystem in \(j\) imperfect preventive warranty interval is as follows:

\[
\bar{\lambda}_{sb}(t) = n_{sb}\lambda_{sb}(t - (j-1)\alpha T) + \theta \sum_{i=1}^{m} \frac{n_i}{2} \lambda_{sb}(t - (i-1)\alpha T) - \frac{1}{2} \frac{n_i}{2} \lambda_{sb}(t)
\]

\[
= 4.98 \times 10^{-4} + 0.5 \times \left[ \sum_{i=1}^{m} \frac{n_i}{2} \frac{2}{1000^2}(t - 0.8(i-1)T) \right]
\]

Consequently, failure number of subsystem in \(j\) imperfect preventive warranty interval is as follows:

\[
n_{sb} = \int_{(j-1)T^c_i+T}^{jT^c_i+T} \bar{\lambda}_{sb}(t) dt =
\]

\[
\int_{(j-1)T^c_i+T}^{jT^c_i+T} \left[ \frac{2}{1000^2}(t - 0.8(j-1)T) \right] dt
\]

So, the expected warranty cost of the system in \(j\) imperfect preventive warranty interval is following:

\[
EC(T) = n_{sb} + n_{sb}\bar{\lambda}_{sb}(t)
\]

\[
= 300 \times \left[ \int_{(j-1)T^c_i+T}^{jT^c_i+T} \left[ \frac{2}{1000}(t - 0.8(j-1)T) \right] dt \
+ \int_{(j-1)T^c_i+T}^{jT^c_i+T} \left[ 4.98 \times 10^{-4} + 0.5 \times \left[ \sum_{i=1}^{m} \frac{n_i}{2} \frac{2}{1000^2}(t - 0.8(i-1)T) \right] \right] dt \right]
\]

The failure number of booster of the time between \(nT^c_i\) to \(W\) as follows:

\[
n_{sb}(n) = \int_{nT^c_i+T}^{W + nT^c_i} \lambda_{sb}(t) dt = \int_{nT^c_i+T}^{W + nT^c_i} \left[ \frac{2}{1000}(t - 0.8nT) \right] dt
\]

The failure number of subsystem of the time between \(nT^c_i\) to \(W\) as follows:

\[
n_{sb}(n) = \int_{nT^c_i}^{W + nT^c_i} \bar{\lambda}_{sb}(t) dt =
\]

\[
\int_{nT^c_i}^{W + nT^c_i} \left[ 4.98 \times 10^{-4} + 0.5 \times \left[ \sum_{i=1}^{m} \frac{n_i}{2} \frac{2}{1000^2}(t - 0.8(i-1)T) \right] \right] dt
\]
The expected failure warranty cost of system of the time
between $n(T + T_p)$ to $W$ as follows:

$$
EC(W - n(T + T_p)) = \left(\sum_{i=1}^{n} r_i \frac{2}{k_i} \right) \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt
$$

$$
= 3000 \times \left[ \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt \right]
$$

(29)

Above all, warranty cost function $C(T, W)$ can be given.

In the same way, the expected failure warranty shutdown
time of the system in $j^{th}$ imperfect preventive warranty interval
is following:

$$
ET_j(T) = \left(\sum_{i=1}^{n} r_i \right) \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt
$$

$$
= 3 \times \left[ \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt \right]
$$

(30)

The expected failure warranty shutdown time of system of
the time between $n(T + T_p)$ to $W$ as follows:

$$
ET(W - n(T + T_p)) = \left(\sum_{i=1}^{n} r_i \right) \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt
$$

$$
= 3 \times \left[ \int_{\frac{2}{k_i}}^{W} \left(1 - (1 - \frac{2}{k_i}j)^{T_p} \right) dt \right]
$$

(31)

Then, availability function $A(T, W)$ in warranty interval can
be given too. Unit cost-effective function can be given as follows:

$$
V = \frac{C(T, W)}{W} \left(\frac{1}{A(T, W)}\right) = \frac{C(T, W)}{W - D(T, W)} \tag{32}
$$

Warranty cost, availability and unit cost-effective is simulated
as figure 1, figure 2 and figure 3.

According to figure 1, figure 2 and figure 3, optimal warranty
interval and imperfect warranty interval are not existed.
However, warranty interval of general system is defined to 3 year.
So, 3 projects of the diesel are as following:

1. When imperfect preventive warranty interval is the
same to warranty interval, which is 3 year, failure rate relevant
coefficient is zero, and the initialized warranty cost, availability
and unit cost-effective are as follows:

$$
T=1080 \text{ (years)}, C_{min} = 46683, A = 0.8657, V = 49.94
$$

2. When warranty interval is 3 year, imperfect preventive
warranty policy and failure rate relevant coefficient are consi-
dered. Choosing best unit cost-effective $T$, the initialized
warranty cost, availability and unit cost-effective are as follows:

$$
T=120, W=1080 \text{ (years)}, C_{min} = 39236, A = 0.9699, V = 37.46
$$

3. When warranty interval has different years, imperfect
preventive warranty policy and failure rate relevant coefficient
are considered, the best warranty data are as follows:

4.2. Result analysis

1. The proposal one and two are the operation outcome
when using general corrective maintenance warranty policy
and imperfect preventive warranty policy respectively. After
a comparative analysis of the two proposals, when the warranty
interval is three years, the warranty cost is relatively high and
the availability low based on the general corrective maintain-
ance warranty policy. Compared with not adopting the imperfect
warranty policy not considering failure interaction, the warrant-
ity cost of the diesel will have a decrease of sixteen percent, and
availability an increase of twelve percent, when adopting the
imperfect warranty considering failure interaction.

2. Table 1 includes various corresponding data for the war-
ranty cost and availability and unit cost efficient in different

![Fig.1. Cost model](image-url)
warranty intervals, through which we can easily find that, in the premise of the guarantee for pump for initial unit efficient cost (that is the unit efficient cost without preventive warranty), the warranty interval can be extended to nine years if using imperfect preventive warranty.

(3) Table 1 includes the balance of the needs and interests between the manufacture and the users of different warranty intervals, the corresponding information data as guarantee cost, the availability and unit efficient cost, which can provide a available scientific information for equipments using department about warranty cost and the availability in addition, the procurement department can also select the standard warranty interval based on the reference information data and actual needs.

<table>
<thead>
<tr>
<th>Serial number</th>
<th>W/day</th>
<th>T/day</th>
<th>C/yuan</th>
<th>A</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1080(3 years)</td>
<td>120</td>
<td>39236</td>
<td>0.9699</td>
<td>37.46</td>
</tr>
<tr>
<td>2</td>
<td>1440(43 years)</td>
<td>126</td>
<td>52859</td>
<td>0.9501</td>
<td>38.64</td>
</tr>
<tr>
<td>3</td>
<td>1800(53 years)</td>
<td>132</td>
<td>65954</td>
<td>0.9302</td>
<td>39.39</td>
</tr>
<tr>
<td>4</td>
<td>2160(63 years)</td>
<td>136</td>
<td>83223</td>
<td>0.9098</td>
<td>42.35</td>
</tr>
<tr>
<td>5</td>
<td>2520(73 years)</td>
<td>140</td>
<td>99822</td>
<td>0.8978</td>
<td>44.12</td>
</tr>
<tr>
<td>6</td>
<td>2880(83 years)</td>
<td>146</td>
<td>118337</td>
<td>0.8777</td>
<td>46.81</td>
</tr>
<tr>
<td>7</td>
<td>3240(93 years)</td>
<td>146</td>
<td>136738</td>
<td>0.8614</td>
<td>48.99</td>
</tr>
<tr>
<td>8</td>
<td>3600(103 years)</td>
<td>150</td>
<td>152833</td>
<td>0.8485</td>
<td>50.03</td>
</tr>
</tbody>
</table>

Fig. 2. Availability model

Fig. 3. Cost-efficient function of unit time
5. Conclusion

This article mainly aimed at the study of multi-component with failure interaction, and analysis on the failure interaction in multi-component system with the point of relevant coefficient on failure rate, based on the improving imperfect preventive warranty policy. A model for cost and availability is established and make analysis on the model. At last combined with cases, proposal on the multi-component system with failure interaction is put forward, for which the article provide analytical validation.

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6. References