Summary: This paper presents the comparison among the control algorithms of a three phase, three wire shunt active power filter. The control algorithms are based on: 1) the current’s physical components theory, 2) the $p-q$ power theory. The active power filter operation under the distorted supply voltage at the point of common coupling condition is considered. The comparison concerns the aspects of: filtering current harmonics, reactive power compensation (defined in the fundamental harmonic domain), supply current balancing and dynamic performance. The results of the laboratory investigation, using the fast prototyping system dSPACE, are given in the paper. This paper confirms that the current’s physical components theory allows developing the control algorithm of shunt active power filter and its practical implementation. The investigation allows the conclusion that this control algorithm enables achieving the supply system current shape close to the sinusoidal, despite shunt active power filter operation under the distorted supply voltage. A sinusoidal supply current cannot be attained by the control algorithm based on the $p-q$ power theory without some additional control elements, such as: PLL, signal filters, etc. The experiment also has shown that the control algorithm based on the $p-q$ power theory is easier to implement and less demanding in terms of hardware than the control algorithm based on the current’s physical components theory.

1. INTRODUCTION

The active power filters (APFs) have been known as one of the best tools for harmonic mitigation as well as reactive power compensation, supply current balancing, voltage regulation and voltage flicker compensation etc. This wide range of objectives is achieved either individually or in combination, depending upon the requirements, a control algorithm (CA) and a configuration which have to be selected appropriately. One of the most popular APFs is shunt APF (SAPF), which is controlled to generate the required compensating currents. The control algorithm (CA) is one of the subsystems of the APF control system. The task of control algorithm is evaluation of reference signals on the grounds of information about voltages and currents at the point of common coupling (PCC). To obtain elimination of undesirable current components the applied control algorithm of SAPF has to evaluate properly the current components corresponding to higher harmonics, reactive power (defined in the fundamental harmonic domain) and the negative sequence components of the load current. The presence of distorted voltage at the PCC could affect evaluation of correct current components values. It depends on the chosen control algorithm. There are a number of methods that enable implementation of the control algorithm of SAPF, in a time or frequency domain. They are based on one of the power theories or the other approaches. This paper presents the comparison of the two control algorithms of SAPF under conditions of distorted voltage. They were designed on the basis of: the $p-q$ power theory (PQT) and the current’s physical components theory (CPCT). The first one was proposed by H. Akagi, A. Nabae and Y. Kanazawa [1–4], the second one was proposed by L.S. Czarnecki [9–12]. The PQT was developed in time domain, whereas the CPCT was developed in the frequency domain. The PQT is a well-known modern power theory and it is widely used in the field of active filtering, whereas the CPCT has been used in this area only once [9].

2. DESCRIPTION OF THE INVESTIGATED SYSTEM

The block diagram of the investigated three-phase three-wire SAPF system is shown in Figure 1. A thyristor rectifier, with RL load was used as a non-linear load. The resistor $R_{urs}$ was connected between the phases $R$ and $S$ in order to cause load currents asymmetry. SAPF configuration is based on a force-commutated pulse-width modulated voltage-source inverter (PWM-VSI) connected to a dc capacitor $C_{APF}$. The elimination of undesired load current components is achieved by injecting equal but of opposite sign compensating currents $i_{APF}$, which are generated by SAPF according to the chosen control algorithms. SAPF operation with the source non-zero internal impedance $Z_S$ and for distorted voltage at the point of common coupling, resulting from the influence of other loads connected at the PCC, has been considered.

The current controller ensures constant switching frequency $f_{sw}$. It is essential considering the structure and effectiveness of the switching frequency filter, which eliminates the switching frequency component of SAPF current. A current controller, whose structure is presented in Figure 2, has been applied for this purpose [17, 18].
3. CONTROL ALGORITHMS

According to the PQT, the current of the three-phase, unbalanced, non-linear load has been expressed into four components (Phase currents and voltages are expressed in the form of column vectors \( i = i(t) \) and \( u = u(t) \), where \( i = [i_r,i_s,i_t]^T \), \( u = [u_r,u_s,u_t]^T \)):

\[
i = i_p + i_q + i_h + i_{2f(1)}
\]

where:

\[
p = \bar{p} + \tilde{p}_h + \tilde{p}_{2f(1)} \quad \text{— instantaneous real power,}
\]

\[
q = \bar{q} + \tilde{q}_h + \tilde{q}_{2f(1)} \quad \text{— instantaneous imaginary power}
\]

(\( \bar{p}, \bar{q} \) — average components, \( \tilde{p}_h, \tilde{q}_h \) — oscillating components (\( h \) — "harmonic") and \( \tilde{p}_{2f(1)}, \tilde{q}_{2f(1)} \) — oscillating components (\( 2f(1) \) — with double frequency of the fundamental component frequency))

\( i_p \) — is associated with \( \bar{p} \), i.e. with the active power \( P \) defined in the traditional way;

\( i_q \) — is associated with \( \bar{q} \), i.e. with the reactive power \( Q \)

(In the case of a symmetrical sinusoidal supply voltage and balanced load with or without harmonics, \( \bar{q} \) is equal to the reactive power \( Q \) defined in the traditional way in the fundamental harmonic domain);

\( i_h \) — is associated with \( \tilde{p}_h \) and \( \tilde{q}_h \), i.e. with the presence of harmonics in voltage and current waveforms;

\( i_{2f(1)} \) — is associated with \( \tilde{p}_{2f(1)} \) and \( \tilde{q}_{2f(1)} \), i.e. the unbalance load currents. The block diagram of the tested CA-PQT is shown in Figure 3a.

According to the CPCT, the current of the three-phase, unbalanced, non-linear load has been decomposed into five components:

\[
i = i_a + i_s + i_r + i_u + i_B
\]

where:

\( i_a \) — active current,
where: the subscript (h) means the harmonics without the first harmonic. The subscript (1) means the first harmonic. The block diagram of the tested CA-CPCT is shown in Figure 3b.

The aim of SAPF is the elimination of all unwanted components of the load current from the point of view of the presented above current decompositions (1), (2). To achieve SAPF control objectives, the CAs have to generate the following reference current signals:

— according to the PQT (the control algorithm PQT – CA-PQT):

\[ i_{refPQT} = i_\pi + i_s + i_{2f(1)} \]  

(4)

— according to the CPCT (the control algorithm CPCT – CA-CPCT):

\[ i_{refCPCT} = i_{a(0)} + i_s + (i_{r(1)} + i_{r(h)}) + (i_{u(1)} + i_{u(h)}) + i_B \]  

(5)

4. EXPERIMENTAL RESULTS

The complete control system of SAPF consists of: a control algorithm unit, a current controller, a dc voltage controller, a gating signals generator, basic automation functions, etc. has been designed using the rapid prototyping system dSPACE and Matlab/Simulink environment. The developed SAPF control algorithms (CA-PQT, CA-CPCT) have been implemented and tested in the laboratory setup. The topology of the source–load path and SAPF is shown in Figure 1. The control systems of SAPF have been developed using dSPACE rapid prototyping system, consisting of:

— DS1005 processor board, with signal processor Motorola PowerPC 750, 480MHz,
— DS2002 Multi-Channel A/D board, 32 x 16-bit MUX, 2 ADCs, sim. S&H,

\[ i_s \] — scattered current,
\[ i_r \] — reactive current,
\[ i_u \] — unbalanced current
\[ i_B \] — load generated current.

Each component is associated with a different power phenomenon and is orthogonal with respect to the others. Thus the current decomposition (2) reveals five different physical phenomena, which determine the load current value, namely: active energy transmission to the load – \( i_{\pi} \); change of the load conductance \( G_{\pi} \) with frequency – \( i_s \); reciprocating flow of energy – \( i_u \); load currents unbalance – \( i_u \); active energy transmission back to the source – \( i_B \). If harmonic currents are generated by the supply source, due to the distorted voltage, then the current components \( i_\pi, i_s, i_u, i_B \) may contain, besides the fundamental, also high-order harmonics of the orders already present in the voltage (This is decided by the sign of the active power calculated for the given harmonic [11, 12]). Thus, each of the components \( i_\pi, i_s, i_u, i_B \) can be additionally decomposed into the fundamental and high-order harmonics, e.g.:

\[ i_a = i_{a(1)} + i_{a(h)} \]  

(3)
conversion time for a single channel is minimum 3.8 \mu s, and for 4 channels supported it gives 15.2 \mu s (in the experiment 8 channel have been used, 4 per one converter). Also the time required for all computations associated with the control algorithm, as well as with the other elements of the control system, such as: current controller, dc voltage controller, basic automation functions, etc. should be accounted for. The resulting sampling frequency was \( f_s = 25.6 \) kHz. For the obtained sampling frequency, a four times lower switching frequency has been defined \( f_{sw} \); \( f_{sw} = 4 \). The control system CS-PQT has been developed using the Simulink standard blocks. The CS-CPCT has been developed using, apart of the Simulink blocks, also the DSP Blockset blocks. The time required to perform all computations for CS-PQT is successively ca. 26 \mu s, whereas for CS-CPCT this time is ca. 25.4 \mu s, depending on the ‘quality’ of the compilation carried out by dSPACE software. The computations for CS-PQT was performed in one time-loop, but for CS-CPCT in two time-loops: a master loop (\( T_{\text{m1}} \)), identical for both control systems, restricted by the sampling time \( T_s = 39.0625 \mu s \), and the slave loop (\( T_{\text{m2}} \)), which performs computations related solely to the CA-CPCT. Because of time restrictions, the number \( k \) of points DFT has been limited to \( k = 32 \). Thus, in the slave loop, for CA-CPCT, the value of reference current is updated every 625 \mu s (20 ms/32) – during this time it is held at the constant value (Zero-Order-Hold block). The real time needed for all computations to be carried out in the slave loop is 325 \mu s, 300 \mu s still remaining. However, the DSP Blockset toolbox DFT algorithm requires 2\( k \) points. Increasing the number of points to \( k = 64 \), increases the duration of calculations and at the same time reduces the duration of calculations in the slave time-loop to 312.5 \mu s (20 ms/64). This was beyond the capacity of the available dSPACE set.

The analysis of APF operation was carried out under conditions of a ‘weak’ supply system. Figure 4 shows the waveform and spectrum of the voltage \( u_S \) phase \( R \), as observed at the terminals of unloaded supply source. The voltage \( u_S \) is distorted due to the adverse impact of other non-linear loads connected to the supply system. At the considered PCC this distortion varies within 4–6\% \( THDU_S \).

Figure 5 shows in succession the waveform of the supply voltage \( u_S \) and the waveforms of the load current \( i_L \) in steady state and dynamic state (step increase, \( \alpha_1 = 50^\circ \Rightarrow \alpha_2 = 0^\circ \). The load current is asymmetrical, distorted and shifted in phase with respect to the voltage \( u_S \). The unbalance factor \( UFI_L \) is at the level 20.4% for \( \alpha_1 = 50^\circ \) and 12.0% for \( \alpha_2 = 0^\circ \). The structure and parameters of the current controllers and the \( UC_{APF} \) voltage controllers were identical for both control algorithms. The values of the basic parameters of the investigated source-load path are given in Table 1.

The effects of SAPF operation in both the steady and dynamic states, for the developed SAPF control algorithms

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**Table 1. The values of the parameters before application of SAPF (UFI – unbalance factor).**

<table>
<thead>
<tr>
<th>phase</th>
<th>( \alpha_1 = 50^\circ )</th>
<th>( \alpha_2 = 0^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R )</td>
<td>( S )</td>
</tr>
<tr>
<td>( THD_{i_L} )</td>
<td>%</td>
<td>24.84</td>
</tr>
<tr>
<td>( I_L )</td>
<td>A</td>
<td>8.50</td>
</tr>
<tr>
<td>( DPF )</td>
<td>–</td>
<td>0.887</td>
</tr>
<tr>
<td>( PF )</td>
<td>–</td>
<td>0.856</td>
</tr>
<tr>
<td>( UFI_L )</td>
<td>%</td>
<td>20.40</td>
</tr>
</tbody>
</table>
CA-PQT and CA-CPCT, are shown in Figure 6. Oscilloscope screen images show, consecutively, the waveforms of phase supply currents: $i_{SR}$, $i_{SC}$, $i_{ST}$.

Enlarged fragments of steady-state waveforms of the current $i_{ST}$ (phase $T, \alpha_2 = 0^\circ$), with its spectrum, are presented in Fig. 7 (left column). Subtracting the reference current $i_{ref}$, computed by the given CA, from the load current $i_L$ we obtain the reference signal $i_{SRref}$ for the current at the supply system side. The appropriate waveforms of the current $i_{SRrefT}$, with its spectrum, are presented in Figure 7 (right column).

$$i_{SRref} = i_L - i_{ref} \quad (6)$$

The values of basic parameters of the investigated SAPF system at the supply source side are given in Table 2.

From comparing the obtained waveforms of $i_S$ current and the values presented in Table 2 it can be concluded that:

- the wave shapes of $i_{SPQO}$ and $i_{SCPC}$ are close to each other and $THDi_S$ values are also close to each other.
- both CAs give the similar results in the supply current balancing.
- SAPF dynamic performance is closely comparable for both CAs. The obtained settling time in response to a step increase of the load current $i_L$ is to 40 ms for both CAs.
- a difference occurs in the results of reactive power compensation. In the event of CA-CPCT, a phase shift occurs between the voltage $u_S$ and current $i_S$ due to the delay caused by the time needed for computing (slave time-loop: 625 $\mu$s $\Rightarrow$ 11.25° $\Rightarrow$ DPF = 0.981). Enlarged fragments of steady-state waveforms of the supply voltage $u_{SR}$, the supply current $i_{ST}$ and the reference current $i_{SRrefT}$ for CA-PQT and CA-CPCT are presented in Figure 8.

![Image](image1.png)

Fig. 6. The waveforms of current $i_S$ for CA-PQT and CA-CPCT – experimental results (current scale × 2)

Table 2. The values of the parameters after application of SAPF ($UF_i$ – unbalance factor)

<table>
<thead>
<tr>
<th>phase</th>
<th>$\alpha_1 = 50^\circ$</th>
<th>$\alpha_2 = 0^\circ$</th>
<th>$\alpha_2 = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$S$</td>
<td>$T$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CS-PQT</strong> – turnaround time: $T_{\alpha_1} = 26\mu$s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$THDi_S$</td>
<td>%</td>
<td>18.15</td>
<td>16.52</td>
</tr>
<tr>
<td>$I_S$</td>
<td>A</td>
<td>7.27</td>
<td>7.30</td>
</tr>
<tr>
<td>$DPF$</td>
<td>–</td>
<td>-0.997</td>
<td>-0.998</td>
</tr>
<tr>
<td>$PF$</td>
<td>–</td>
<td>-0.966</td>
<td>-0.957</td>
</tr>
<tr>
<td>$UF_i$</td>
<td>%</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td><strong>CS-CPCT</strong> – turnaround times: $T_{\alpha_1} = 25.4\mu$s, $T_{\alpha_2} = 325\mu$s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$THDi_S$</td>
<td>%</td>
<td>17.80</td>
<td>16.13</td>
</tr>
<tr>
<td>$DPF$</td>
<td>–</td>
<td>0.981</td>
<td>0.978</td>
</tr>
<tr>
<td>$PF$</td>
<td>–</td>
<td>0.950</td>
<td>0.938</td>
</tr>
<tr>
<td>$UF_i$</td>
<td>%</td>
<td>0.65</td>
<td></td>
</tr>
</tbody>
</table>

A. Firlit: Current’s Physical Components Theory and p-q Power Theory in the Control...
Fig. 7. Enlarged fragments of steady-state waveforms of current $i_{SF}$ (left column) and $i_{SRef}$ (right column), with their spectra, for CA-PQT and CA-CPCT – experimental results.

The reference current signal $i_{SRef}$ consists of the desired component of the load current $i_L$ and the $i_{APF_c}c$ current, needed to replenish the $C_{APF}$ capacitor energy, therefore:

— according to the CS-PQT:

$$i_{SRefPQ} = i_p + i_{APF_cPQ}$$  \(7\)

— according to the CS-CPCT:

$$i_{SRefCPC} = i_{a(1)} + i_{APF_cCPC}$$  \(8\)

Figure 7 (right column) presents the example waveforms and spectra of currents $i_{SRef}$ from relationships (7)–(8). Differences do occur between the currents $i_L$ at the source side after applying SAPF and the currents $i_{SRef}$ generated by the control systems for the given CA. This is a consequence of a relatively low switching frequency $f_{sw}$ and the lack of passive filtering of the switching frequency component in SAPF current. The currents (7)–(8) differ between themselves, as well. The differences occur in both the wave shapes and in spectra. The current $i_{SRefPQ}$ is distorted because of the voltage $u_S$ distortion. In the current $i_{SRefPQ}$ the 7th harmonic is dominant, though the 5th, 11th, 19th, etc. also occur. The
\(I_{\text{ref CPC}}\) current waveform approximates the sine-wave with the accuracy determined by the hardware capability. This is related to the above described method of CA-CPCT, where each subsequent value, computed by dSPACE, is updated every 625 \(\mu\)s. Hence, in the \(I_{\text{ref CPC}}\) spectrum, beside the first harmonic, also the 31\textsuperscript{st} and 33\textsuperscript{rd} harmonics occur. This is the cause of the \(I_{\text{ref CPC}}\) current distortion. Therefore the main cause of the \(I_{\text{ref CPC}}\) current distortion is not the supply voltage \(u_d\) distortion, but the hardware limitations. It ought to be emphasized that the CA-CPCT worked out the correct reference current \(I_{\text{ref CPC}}\) in spite of the distorted supply voltage at the PCC. It was achieved without using the additional components of CA, such as PLL circuit, signal filter or others. The CA-CPCT was elaborated only on the grounds of the proposed physical interpretation of the phenomena occurring in electrical circuits and the mathematical relationships formulated within CPCT by L.S. Czamecki.

### 5. CONCLUSIONS

In this paper the comparison of two control algorithms (CAS) of SAPF was presented. The CAS were designed on the basis of: the \(p-q\) power theory (PQT, CA-PQT) and the power theory based on the current’s physical components theory (CPCT, CA-CPCT). The power theory based on the current’s physical components took its final form in 1994 [12]. Until now, only its Author has presented proposals for this theory’s practical application [9, 10]. This paper confirms that the power theory based on the current’s physical components theory allows developing the control algorithm (CA-CPCT) of SAPF and its practical implementation. The CAS were implemented and tested using the fast prototyping system dSPACE. This hardware and software platform turned out to be a very useful tool for building and developing the control algorithms of SAPF. The laboratory results, with regard to the filtering current harmonics and supply current balancing, are similar for both control algorithms at the given switching frequency of SAPF \(f_{sw}\) = 6.4 kHz. Likewise the dynamic performance is comparable in the case of both CAS. The difference occurs in the results of reactive power compensation. In the event of CA-CPCT, a phase shift occurs between the voltage \(u_S\) and current \(i_L\) due to the delay caused by the time needed for computing (slave time-loop – 625 \(\mu\)s). Significant differences occur between the reference current signals \(i_{\text{ref}}\) generated by the CAS. The reasons for the differences between the real \(i_L\) and the reference \(i_{\text{ref}}\) currents at the source side after applying SAPF are that the laboratory setup has not been equipped with a passive filter for switching frequency elimination and a relatively low switching frequency \(f_{sw}\). The investigation allows the conclusion that the developed algorithm CA-CPCT enables achieving the supply system current shape close to the sinusoidal, despite SAPF operation under the distorted supply voltage. A sinusoidal supply current cannot be attained by the CA-PQT without some additional control elements, such as: PLL, signal filters, etc. The experiment has shown that testing the capability of SAPF controlled by the CA-CPCT algorithm to its full extent requires a controller of a larger computing power.

This concerns both the A/D conversion time and harmonic analysis. An alternative solution is the use of a method for harmonic analysis of current and voltage signals different from the discrete Fourier transform DFT. The control algorithm CA-PQT, is certainly easier to implement and less demanding in terms of hardware.

### LIST OF ABBREVIATIONS AND MAIN SYMBOLS

- **APF**: active power filter
- **CA**: control algorithm of shunt APF
- **CA-CPT**: control algorithm of shunt APF based on the current’s physical components theory
- **CA-PQT**: control algorithm of shunt APF based on the \(p-q\) power theory
- **CPCT**: control’s physical components theory
- **CS-PQT**: control system of shunt APF with CA-PQT
- **CS**: control system of shunt APF
- **CS-CPCT**: control system of shunt APF with CA-CPCT
- **PCC**: point of common coupling
- **PQT**: \(p-q\) power theory
- **SAPF**: shunt APF
- **CPC**: in subscript – designation of CPCT, for example \(i_{\text{ref CPC}}\)
- **PQ**: in subscript – designation of PQT, for example \(i_{\text{ref PQ}}\)
- **\(R-S-T\)**: in subscript – designation of phases, for example \(i_{LR}, i_{LS}, i_{LT}\) – phase load currents
- **\(DPF\)**: displacement power factor
- **\(i, u\)**: vector of phase currents and voltages
- **\(i = [i_R, i_S, i_T]^T, u = [u_R, u_S, u_T]^T\)**
- **\(i_{\text{APF}}\)**: SAPF current (generated by SAPF)
- **\(i_{\text{APFdc}}\)**: current needed to replenish the \(C_{\text{APF}}\) capacitor energy
- **\(i_L\)**: load current
- **\(i_{\text{ref}}\)**: reference current (generated by CA)
- **\(i_S\)**: supply current (current at the supply source side)
- **\(i_{\text{ref}}\)**: reference signal for the current at the supply system side \(i_{\text{ref}} = i_L - i_{\text{ref}}\)
- **\(PF\)**: power factor
- **\(THD\)**: total harmonic distortion
- **\(UF\)**: unbalance factor
- **\(u_S\)**: supply voltage

### REFERENCES


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