Sampled Method of Active Power Filter Control (Part I)

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Summary: The aim of the paper is to demonstrate how to use sampled method of control of a shunt active power filter to maintain invariable power of a supplying voltage source, even though the supplied load varies randomly. The definition by S. Fryze of load current components is used as a basis for the construction of a family of shunt active power filters. The under discussion family of filters is designed to work synchronically with a time period \( T \). The period \( T \) may be regulated in some range. The distinctive property of these filters is maintaining non-deformed source current in every one individually considered period \( T \).

Main features of the family of filters are described in the first part of the paper, which is devoted to DC circuit. Then in the second part, the results, which are obtained in DC circuit, are implemented to a single- and then to a three-phase circuit. Additionally, some new features, which may be applied only for AC circuit, are discussed in the second part. All the presented waveforms are obtained using computer simulation tools.

1. INTRODUCTION. ACTIVE FILTER FOR DC CIRCUIT. BASIC EQUATIONS AND FILTER STRUCTURE

Electric systems are used for transmitting signals or as a medium for transportation energy from a place where it is produced (a source) to the places, where the energy is transferred into work (a load). The paper deals with the second question. Not only AC systems are used for this purpose, DC systems may be used as well.

This is well known, that in AC systems may appear non-active currents, which cannot transfer active power from the source of energy to the load. Can non-active current appear in a DC circuit? The answer is: yes. And its flowing is as well useless in DC as in AC systems. In the paper consideration on this problem is chosen as a platform for discussion on development of a family of shunt active power, which may work as well in DC as in AC circuits.

Let us consider an elementary electric circuit consisting of a DC voltage source and a varying load. A constant time period \( T \) may be designated for the circuit. During any \( n \)-th period \( T_n \) the active power \( P(T_n) \) of the load is:

\[
P(T_n) = \frac{1}{T} \int_{nT}^{(n+1)T} U_S i_{L_s} dt = U_S \frac{1}{T} \int_{nT}^{(n+1)T} i_{L_s} dt = U_S I_{AV(T_n)}
\]

where:

\( n = 0, 1, 2, ..., N \)

\( U_S \) is the voltage across the load,

\( i_{L_s} \) is the current flowing through the load, and

\( I_{AV(T_n)} \) is the average value of current flowing through the load during period \( T_n \).

According to proposition by S. Fryze [1], during any period \( T_n \) the current of load \( i_{L_s(T_n)} \) consists of two components: an active component \( i_{p(T_n)} \) and a non-active one \( i_{q(T_n)} \).

\[
i_{L_s(T_n)} = i_{p(T_n)} + i_{q(T_n)}
\]

At the core of the definition is the idea of an equivalent conductance \( G_{(T_n)} = P_{(T_n)} / U_s^2 \) of the load. The conductance is used for calculating the active current component:

\[
i_{p(T_n)} = G_{(T_n)} U_s = \frac{P_{(T_n)}}{U_s^2} U_s = \frac{U_S I_{AV(T_n)}}{U_s^2} U_s = I_{AV(T_n)}
\]

From Expression (1.2) and (1.3):

\[
i_{q(T_n)} = i_{L_s(T_n)} - i_{p(T_n)} = i_{L_s(T_n)} - I_{AV(T_n)}
\]

The non-active component \( i_q \) cannot transfer active power \( P \) to the load, and the average value of the non-active current component is equal to zero. Taking into account Expression (1):

\[
\frac{1}{T} \int_{nT}^{(n+1)T} U_s i_{q(T_n)} dt = U_S \frac{1}{T} \int_{nT}^{(n+1)T} i_{q(T_n)} dt = U_s \cdot 0
\]

Figure 1 shows how the currents may exist in an example series circuit, consisting of a voltage source, a chopper (acting with the period \( T \)) and a resistor.

Active filter for DC circuit. Each member of the family of active filters is constructed based on a full-bridge voltage-source inverter (see Fig. 2 and Fig. 3). Let us connect the inverter (which acts as a current source and is here called the shunt active power filter) in parallel to the load (Fig. 3) and using the inverter force an additional non-active component \( i_F \) through the source. The reference for the additional component \( i_F \) should be the load non-active component \( i_{q(T_n)} \) in order to satisfy the equation:

\[
i_q + i_F = 0
\]

In this situation the inverter cancels the non-active current component of load in the source, and the source provides...
starting at the beginning of the next period \( T \). So, the transfer of energy from the source to the load has to be delayed for one period \( T \) (see waveform 2 in Fig. 4):

\[
i_{p,nT} = G_{(n-1)T}U_s
\]

This way of operation implies, that the active filter acts as a buffer of energy, which participates in transferring energy between the source and the load.

The first synchronization period \( T_1 \) is a particular period, for during it the load is supplied not by the source but exclusively by energy stored in the active filter, mainly by energy stored previously in the capacitor of the filter:

\[
P(T_1) = \frac{C(U_{C0} - U_{C,T1}^2)}{2}
\]

Taking into account energy stored also in the inductor \( L \), the active component calculated for any \( n \)-th period \( T \) is as follows:

\[
i_{p,T(n)} = G_{F(n-1)}U_s = \frac{C(U_{C0}^2 - U_{C,T(n-1)}^2) - LI_{L,T(n-1)}^2}{2TU_s^2}U_s
\]

where:

- \( U_{C0} \) is an initial voltage across the filter capacitor,
- \( U_{C,T(n-1)} \) is its voltage at the end of \( (n-1) \)-th period \( T \), and
- \( I_{L,T(n-1)} \) is reactor current at the end of \( (n-1) \)-th period \( T \).

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**Fig. 1.** Load current (the same as source current, waveform 1), non-active component \( i_q \) (waveform 3), active component \( i_p \) (shown as \( I_{Active} \), waveform 2)

only active current \( i_p \) now. Of course, the apparent power of the source is diminished.

The active filter observes the current flowing through source and is controlled to maintain only active current through the source. The equivalent conductance of the load, if only known, may be used to calculate the active current (Expr. 3). In respect of the whole family of presented filters, the load equivalent conductance \( G_T \) is obtained in the same way. The equivalent conductance, and then the active current component, is calculated at the very end of each time period \( T \), as a function of the amount of energy stored in the reactance elements: the capacitor \( C \) and the inductor \( L \) of the inverter. The period \( T \) can be named the synchronization period. As the conductance for any period \( T \) can be found only at the very end of this period, it cannot be valid for this period. The last obtained conductance may be applied only

**Fig. 2.** Functional diagram of presented active filter
If the load varies, the active filter balances changes of power of load, because source power cannot change during any synchronization period. As is shown in Figure 5 magnitude of voltage across the filter capacitor at the end of period $T_n$ contains almost complete information on energy, which is consumed by the load. The capacitor voltage contains also information on energy consumed by the active filter. The energy stored in the filter reactor can be skipped. Then the equivalent conductance can be calculated using only voltage across the filter capacitor, and applied in the next period $T_{n+1}$ as the new conductance $G_{T(n+1)}$.

Figure 2 shows implementation the Expr. 7 to the filter structure. Energy stored in the Filter capacitor balances between the capacitor and the Source via the $H$-bridge inverter. Simultaneously, square of the capacitor voltage $u_c^2$ is compared with square of the Initial capacitor voltage $U_p^2$. The difference of the both signals is multiplied by magnitude of the Filter regulator $K_u$ (see Expr. 11a). As the result, the signal of instantaneous equivalent conductance $g(t)$ is calculated. The Sample-and-Hold device memorizes the signal of conductance at the end of each synchronization period $T$ (using the Synchronizer), and maintains the signal constant till the next synchronization moment. Then the constant signal of the conductance is multiplied by the signal of the source voltage $u_s$, so the signal of the load active current $i_p$ reference is calculated. The reference current is compared with the signal of source current. The result, the current error, is compared with the signal Tolerance band $\Delta$. The output of the Comparator controls switches of the $H$-bridge inverter to maintain the source current $i_s$ equal to the load active current $i_p$ with maximum error no bigger than $\Delta$.

Figure 3 presents the basic power circuit diagram of the active filter.

The Equations 8 and 9 describe currents and voltages in Figure 3.

\[
\frac{di_p}{dt} = \frac{u_s + \sigma u_c}{L} + \frac{di_s}{dt}
\]

\[
\frac{du_c}{dt} = \frac{\sigma (i_s - i_p)}{C}
\]

The introduced coefficient $\sigma$ is equal to +1 when the pair NA-PB of filter switches is in state ON (the pair PA-NB is then OFF), and is equal to −1 when the pair PA-NB is ON (NA-PB is OFF). The current of the active filter is shaped as follows: The source current is compared with a tolerance band $\Delta$ around the active current $i_p$. The pair PA-NB is turned ON (σ is equal to +1) if the actual source current rises beyond $(i_p + \Delta)$. The pair NA-PB is turned ON (σ is equal to +1) if the actual source current falls below the lower tolerance band.

Figure 4 shows example waveforms, which may relate to the discussion. The constant parameters in the Expression 7 are chosen as follows: $T = 10$ms, $C = 4$mF, $L = 2$mH, $U_s = 100$V, $U_c = 300$V.

Parameters of the load current (waveform 1, starting from $t = 0$ms till 400ms) are as follows: RMS 13.9A, mean 9.6A, standard deviation 10.1A. The same parameters for the source current (waveform 2, time “shifted” for one synchronization period: $t = 10$ms – 410ms) the parameters are 10.1A, 9.9A and 2.2A, respectively. From these parameters results, that there is still some space for improving the quality of the filter action. For the filter current (waveform 3, $t = 10$ms – 400ms) the parameters are 10.4A, 0.4A and 10.4A, respectively.

The equation 7 may be rewritten as follows:

\[
i_{p,T(n)} = G_{T(n-1)}u_s = K_u\left(U_s^2 - U_c^2\right) - K_iI_{L,T(n-1)}
\]

where: $K_u = \frac{C}{2TU_s^2}$: “voltage” coefficient, (11a)

$K_i = \frac{L}{2TU_s^2}$: “current” coefficient (11b)

We can say, that the equivalent conductance $G$ consists of two components: the “voltage” component $K_u(fU_c)$ and the “current” one $K_i(I_L)$. Let the coefficients $K_u$ and $K_i$ calculated without any modification exactly from Expressions 11 are called nominal. For the synchronization period $T = 10$ms, filter capacitor $C = 4$mF, filter inductor $L = 2$mH and source voltage $U_s = 100$V, the nominal coefficients are:

\[
K_u = 20 \cdot 10^{-6} \frac{A}{V^3}
\]

\[
K_i = 10 \cdot 10^{-6} \frac{1}{A \cdot V}
\]

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For the example shown in Figure 4, the range of the capacitor voltage is 300V – 288V and the range of the current of the inductor is about –15A, +15A. Taking into account Expression 10, the ratio of the “voltage” to the “current” component of the actual instantaneous equivalent conductance \( g(t) \) can be displayed as in Figure 5. We can see, that the “voltage” component is much bigger than the “current” one. The “current” component is almost always less than about 3% of the “voltage” one. So one can say, that the impact of the “current” component on the whole equivalent conductance \( G_{eq} \) may be neglected.

2. DEVELOPMENT OF FILTER’S PROPERTIES

If the power of load changes, the actual equivalent conductance \( G_{eq} \) may not equal to the previous conductance \( G_{(n-1)} \). As a result the actual power of load may be different from the actual power of source. But the presented active filter can balance the energy flowing from the source to the load. From this perspective, the filter is engaged in transferring active power from the source to the load. As a consequence, the source of energy which feeds the load (for example a battery, a fuel cell, a photovoltaic, etc.) operates with lowered apparent power and in much more stable conditions. Some possibilities of developing of the filter structure and regulation of the filter action, in order to diminishing the non-active current component flowing through the supplying source, are described in this section.

2.1. Properties of coefficient \( K_u \)

The nominal value of the “voltage” coefficient \( K_u \) is determined with the assumption, that the active filter balances every change of load power within each individually observed single period \( T_p \). Figure 6 illustrates process of switching ON an invariable load at \( t = 10 \) ms. The figure shows voltage across the filter’s capacitor (waveform 1), and the source current (waveform 2). Period \( T \) is equal to \( 20 \) ms, and the coefficient \( K_u \) is set to the nominal magnitude. We can see, that the source power reaches steady state in a single synchronization period \( T \) (see Expressions 6 and 7).

Figure 7 shows the same runs, but this time the coefficient \( K_u \) is set to half of its nominal value.

One can see in Figure 7, that the filter action is “slowed”: the source current approaches the steady-state step by step. For the reason that \( K_u \) is reduced by half, the source power approaches the steady-state in fifty percent between each step (each period \( T \)). It should be also noticed, that the filter acts with much lower capacitor voltage when coefficient \( K_u \) is below its nominal magnitude. This effect is obvious, but it makes worse the ability of the filter to compensate sudden changes of load power. This problem will be considered in one of the following sections.

The “slowing” of the filter action may be used for averaging changes of power of the source. Reaching the steady state is spread on several subsequent periods of source cycle after each change of power of the load. If during these several periods the load power changes in the opposite direction, the equivalent conductance of the load is averaged in some way. The capacitor of the filter acts as an energy buffer (between source and load) in this situation. Consequently, the RMS of source current is lowered. There is no change of the load power, what is obvious.

Figure 8 illustrates the effect of reducing the coefficient \( K_u \) in a more general case. For the waveform 1 the \( K_u \) is set nominal, and for the waveform 2 is reduced by half. The load current is the same as presented in Figure 4. The RMS for the waveform 1 for \( t \in (50 \text{ms} - 410 \text{ms}) \) is 10.1A, its mean is 9.8A, and its standard deviation is 2.2A. The RMS for the waveform 2 for \( t \in (50 \text{ms} - 410 \text{ms}) \) is 9.9A, its mean is 9.8A, and its standard deviation is 0.95A.

2.2. Running adjustment of the coefficient \( K_u \)

Reducing of the coefficient \( K_u \) leads to averaging of changes of equivalent conductance of load. The more the coefficient \( K_u \) is reduced, the less fluctuates source power. But from the other hand, the filter acts at lower voltage of the
filter capacitor. To reconcile the contradiction, the coefficient $K_u$ may be adjusted dependently on the load changeability. Let the load changeability be characterized by a new coefficient $\lambda(t)$, so adjusting of the coefficient $K_{uvar}$ is as follows:

$$K_{uvar} = \lambda(t)K_u$$

(13)

The coefficient $\lambda$ should be adjusted in the follow-up way. It should equal unity if the load is constant, and should be reduced if load varies. The $l$ may be defined on many ways. For example, it may depend on amplitude of load current compared to active current component:

$$\lambda(t) = (\Delta I_{Load})/I_{P(T(n)}}$$

(14)

The variable $\Delta I_{Load}$ is easy to obtain, but there is an inconvenience if the RMS of load current changes, but the average value of the current remains constant (in other words, if there is a change of load apparent power without any change of load active power). It causes needless change of the coefficient $K_{uvar}$ and, as a consequence, needle change of the voltage across the filter’s capacitor (Fig. 9, waveforms 2 and 3). The mentioned inconvenience may be avoided if the coefficient $\lambda$ is dependendent on successive changes of load active power compared to the averaged active power of load in some range of periods $T$.

$$\lambda_p(t) = (\Delta P_A)/P_A$$

(15)

To avoid the positive feedback between the coefficients $K_u$ and $\lambda$, the changes of load active power must be obtained not from changes of the filter’s capacitor voltage, but by observation of load. Waveforms of coefficient $\lambda$ (waveform 1 for $\lambda_p$ and waveform 2 for $\lambda_1$) and capacitor voltage (waveforms 4 and 3, respectively) are compared in Figure 9 for the both methods. To show the difference, the load is modified in some way to increase the RMS of load current, without any change of its average value, within the time period $T \in (150ms, 350ms)$.

The next example illustrates using of changeable coefficient $K_{uvar}$ instead of invariable $K_u$. Figure 10 presents the active current to be realized by the filter through the source. First the coefficient $K_u$ is set to the nominal level. The parameters of waveform 1, within time period 10ms-410ms, are as follows: its RMS is 10.1A, average is 9.9A, and standard deviation is 2.2A. Then the variable $K_{uvar}$ is applied, and the coefficient $\lambda_p$ is used. The RMS is 10.3A, average 10.2A, and standard deviation is 1.6A (waveform 2, within time period 10ms-410ms). Waveform 3 presents the coefficient $K_{uvar}$ and corresponds to the waveform 2.

The effect of reducing the coefficient $K_p$, that means averaging of changes of source power and increasing of filter’s capacitor voltage, is clear visible when $T \in (50ms, 100ms)$ and $T \in (240ms, 300ms)$.

2.3. Changes of synchronization period

Lengthening of the synchronization period $T$ may average and smooth the current of the source as well as diminishing the coefficient $K_u$. Figure 11 shows source current when period $T$ is set to 50ms (waveform 1), and then when $T$ is set to 10ns (waveform 2). The coefficient $K_u$ is set to the nominal magnitude for the both experiments, respectively (see the first of Expression 11). Load current (waveform 3) is exactly like in foregoing examples (RMS is 14.1A, average is 9.6A, and standard deviation is 10.1A). There are following parameters for the waveform 1: RMS is 9.9A, average is 9.9A, standard deviation is 0.9A for $T \in (50ms-450ms)$; and then RMS 10.1A, average 9.9A, and standard deviation 2.2A for
the waveform 2 for \( t \in (10\text{ms} - 410\text{ms}) \). Looking at the example we can say, that the power of the load is sufficiently characterized during the first period \( T = 50\text{ms} \), so there is no need to correct source power in successive periods. The length of synchronization period \( T \) if set to 10ms is too short to reach the like effect.

The sufficient length of period \( T \) must be calculated with respect to band spectrum of load power. The bigger is the difference between the spectrum and the frequency \( 1/T \), the better the filter can stabilize the source power. The related example is presented in Figure 12.

The active filter acts with lower capacitor voltage when the period \( T \) is lengthen, see Figure 13 (or when coefficient \( K_C \) is reduced, see Figure 7). The voltage waveforms in Figure 13 correspond to current waveforms in Figure 11.

When capacitor voltage falls down the ability of the filter to compensate sudden changes of load power worsen. Figure 14 shows the pulse of source current caused by rapid change of load power. The Figure 14 is an enlargement of the area around \( t = 56\text{ms} \) in Figure 11. The tags of current waveforms correspond to those shown in Figure 13.

### 2.4. Supplementing of filter capacitor voltage

As mentioned in section 1, the filter capacitor operates as a “sensor” of load equivalent conductance and, simultaneously, as an energy store used to compensate non-active current component in the source. The “sensing” function leads the capacitor to lowering its voltage, which is necessary for obtaining the equivalent conductance of load. Stabilizing of the source power also leads to the capacitor voltage decreasing, as is shown in section 2.1, 2.2 and 2.3.

But from the other hand the decreasing of the voltage is unfavourable due to the following: the “allowed” magnitude of the load active power becomes limited at a lower level, because the capacitor voltage must be much higher then the source voltage. Additionally, the dynamic properties of the filter are worse, because the speed of changes of the filter current, which determines the ability to follow rapid load changes, depends on the filter capacitor and the source voltage difference.

A supplementing-capacitor-voltage procedure may be used in order to reduce these inconveniences. The operation principle of the procedure may be memorizing information on successive filter’s capacitor voltage changes. If the procedure is applied, the capacitor of the active filter does not have to perform the task of indicating the existing load power. Its charge can be supplemented now from the source. This happens due to the fact, that the supplementing procedure forces the doubling of the source current change in the next period \( T \) after the load change. The need that the capacitor voltage must be considerably higher then the peak source voltage should be referred now not to the total active power of the load, but to the maximal change of the load power during a single period \( T \) (see also Expr. 6).

The effect of using the supplementing procedure on the capacitor voltage is presented in Figure 15 (the shown load current is the same as in previous examples, period \( T \) equals 50ms). It should be mentioned that – if the procedure is
Fig. 15. Supplementing of filter capacitor voltage. Load current (waveform 1), source current (waveform 2), and filter capacitor voltage (waveform 3)

Fig. 16. Supplementing-capacitor-voltage procedure is applied, but synchronization period \( T \) is chosen improperly. Synchronization period \( T \) is set to 10ms. Load current (waveform 1), source current (waveform 2), capacitor voltage (waveform 3)

Fig. 17. Eliminating of useless energy flow if supplementing-capacitor-voltage procedure is applied. Load current (1), source current (2), capacitor voltage (3)

As mentioned above, there is a needless additional flow of energy after the load is off if the supplementing-capacitor-voltage procedure is applied (Fig. 15, waveforms 2 and 3). The additional flow of energy results from delayed (for one synchronization period \( T \)) realization by the filter of the reference (active) current through the source (see Expr. 7).

This useless flow of energy may be eliminated if the shutting down of the filter is done synchronously with the shutting down of the load. The shutting down of the filter can be done by switching off all switches of the active filter. In this way the filter returns to its initial state: voltage across the filter’s capacitor is near its initial value \( U_{C0} \) and the energy stored in the filter’s inductor discharges. Such effect is presented in Figure 17 (see also Fig. 15).

2.5. Reduction of pulse source current distortion

When the load current changes rapidly, pulse distortions occur in the source current. The distortions appear when the load current rate of change is greater then the maximal rate of change of the filter current (see Expr. 8):

\[
\frac{u_s + \sigma u_c}{L} < \frac{dI_s}{dt} \tag{16}
\]

Supplementing the voltage of the capacitor to the initial level \( U_{C0} \) can decrease the duration and the energy of each pulse source current distortion (Expr. 16, see also Fig. 14). The other way of reducing the distortions is decreasing inductance of the filter reactor \( L \). However, decreasing the inductance causes increasing frequency of the filter switches action. But there is a possibility of applying a “small” reactor when fast change of load current occurs (to reverse the inequality 16), and a “large” reactor when the rate of change of the filter current is sufficient compared to the rate of change of load current. This idea may be satisfied by the active filter with modified structure [6], shown in Figure 18.

The former reactor \( L \) may be divided into two parts \( L = L_1 + L_2 \), \( L_1 < L_2 \), and linked to the additional leg consists of two switches \( PC \) and \( NC \). The switches “disconnect” the “large” reactor \( L_2 \) is every time the pulse source current distortion occurs.
3. CONCLUSION

The H-bridge inverter is used as a tool for compensating non-active current, which may flow through the electric source. Changes of its DC output capacitor voltage are used for calculating reference run for the active filter current. The filter structure is “flexible”, there are many possibilities for improving quality of the active filter work.

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