ANALYSIS OF THE ROUNDNESS DEVIATION MEASUREMENT ERROR WITH A DOUBLE V-BLOCK SUPPORT CONNECTED WITH ERRORS OF DETERMINING THE METHOD PARAMETERS

Key words
Measurement errors, roundness deviation, double V-block support, method parameters.

Summary
This article presents the results of an analysis of the error arising in measurements of roundness deviation in a double V-block support of a cylindrical object, connected with errors in determining the method parameters. The results lead to a conclusion that the effect of this error on the total random error of the method is minor and the best measurement systems for the error to be minimised are those with the V-block angle $\gamma = (30–60)°$.

Introduction
Common applications of a V-block as an element for supporting cylindrical components of machines result from such features as their simple design, the possibility of supporting large dimension objects, and free rotation of these objects during object machining or performing measurements. In theoretical considerations on the accuracy of supporting an object in V-blocks, it is assumed that, during measurements, the direction of shifting the gauge measuring pin contacting the cylindrical surface of the object being measured runs through the
geometric centre of the measured shape profile [1, 2, 3]. Practically, due to changes in the profile shape and errors in determining the parameters of the measurement system (angles $\alpha$ and $\gamma$, Fig. 1), the object centre during its rotation will change its position in an unknown direction, causing errors of roundness deviation measurements. An analysis of this error will allow for quantitative assessment of the extent the errors of method parameter determination and the measured profile itself affect the value of the measured roundness deviation.

Fig. 1. Auxiliary diagram for the determination of roundness deviation measurement error made due to errors of method parameter (angles $\alpha$ and $\beta$) determination
1. The theoretical basis for the determination of the roundness deviation measurement error connected with errors of method parameters determination and with the tolerance of a double V-block based object

In order to define indispensable mathematical relationships, e.g. [3], an auxiliary diagram was used as shown in Figure 1. The measured roundness deviation is illustrated by the vector $\Delta F'$, while the real roundness deviation is reflected by the vector $\Delta F$. The relative roundness deviation measurement error in $\Delta F$, with the error of gauge positioning $\Delta \alpha$ taken into account, can be written as follows:

$$ W_{\Delta F} = \frac{\Delta F' - \Delta F}{\Delta F} = \frac{1}{\cos(\Theta + \Delta \alpha)} - 1 $$  \hspace{1cm} (1)

Taking into consideration both the error of V-block angle setting $\Delta \gamma$ and the tolerance of measured object $\Delta R$, the relative error $W_{\Delta F}$ caused by the displacement of the measured profile centre, after transformations, can be expressed in this way:

$$ W_{\Delta F} = \frac{1}{\cos\left(\arcsin\left[\frac{a_3 \sin(\alpha - \arctan \frac{x_3}{y_3})}{R_3}\right] + \Delta \alpha\right)} - 1 $$  \hspace{1cm} (2)

where:

- $\alpha$ – measuring gauge positioning angle;
- $\Delta \alpha$ – gauge angular positioning error;
- $R_3$ – theoretical radius of the measured object;
- $x_3, y_3$ – coordinates of the position of the defined roundness profile centre, written down by the equations (3) and (4);
- $a_3$ – root of the sums of squared coordinates of the defined roundness profile centre.

$$ x_3 = \left[ \frac{R_2 \sin \Delta \gamma_2 + \Delta R_2 \sin(\gamma + \Delta \gamma_2)}{\sin \gamma} \right] + \left[ \frac{R_1 \sin \Delta \gamma_1 + \Delta R_1 \sin(\gamma + \Delta \gamma_1)}{\sin 2\gamma} \right] + $$  \hspace{1cm} (3)

$$ \left[ \frac{R_2 \sin \Delta \gamma_2 + \Delta R_2 \sin(\gamma + \Delta \gamma_2)}{\sin \gamma} \right] \frac{L_1}{L} $$
In formulas (3) and (4) the radiiuses \( R_1 \) and \( R_2 \) as well as their tolerances \( \Delta R_1 \) and \( \Delta R_2 \) are nominal radiuses of roundness profiles contacting the V-block setting points. The V-blocks are positioned at a distance \( L \) from each other, while the measured profile position is determined by the coordinate \( L_1 \).

Since the actual centre of the measured profile may not be concentrically placed in relation to the defined roundness profile centre, the actual measured profile centre will move relative to the defined profile centre in an eccentric path \( e \) (Fig. 1).

At the same time:

\[
R_3 = e \cos(\phi + \omega) + \sqrt{R_{30}^2 - e^2 \sin^2(\phi + \omega)} = e \cos(\phi + \omega) + R_{30} \cos \left[ \arcsin \left( \frac{e \sin(\phi + \omega)}{R_{30}} \right) \right] \tag{5}
\]

We can assume that \( R_1 = R_2 = R_{30} \).

Finally, after transformations, the error value \( W_{\Delta F} \) can be written as follows:

\[
W_{\Delta F} = \frac{1}{\cos \left[ \arcsin \left( \frac{\sin \alpha \cdot x_1 - \cos \alpha \cdot y_1}{t_e \cos(\phi + \omega) + \sqrt{1 - t_e^2 \sin^2(\phi + \omega)}} \right) + \Delta \alpha \right]} - 1 \tag{6}
\]

Relationship (6) was obtained from the relationships (2), (3), (4) and (5) by expressing tolerances \( \Delta R_1 \) and \( \Delta R_2 \) and the eccentric path \( e \) in the form of relative radius \( R_{30} \) (\( \Delta R_1 = t_1 \cdot R_{30} \), \( \Delta R_2 = t_2 \cdot R_{30} \), and \( e = t_e \cdot R_{30} \), then also \( x = \frac{x}{R_{30}} \), and \( y = \frac{y}{R_{30}} \)).

Therefore, the value of the relative error of roundness deviation \( W_{\Delta F} \) measurement changes depending on the angle of object rotation \( \phi \).

The examination of changes in the function described by relation (6) has shown that the maximum value is reached for the angle \( \phi = \pi - \omega + 2k\pi \). The
effects of the individual parameters of the method and the errors of their setting and relative tolerance of the object on the changes in error values $W_{\Delta F_{\text{max}}}$ were examined by computer simulations in the Matlab environment. This examination has shown that the error value for the adopted ranges of changes in influential quantities reaches its maximum between $(2.9925 \cdot 10^{-9} - 0.003)$ and only slightly affects the accuracy of roundness deviation measurements.

This is confirmed by spatial graphs displayed in Figures 2, 3, and 4, showing changes in the error $W_{\Delta F_{\text{max}}}$ as a function of method parameters and errors of their setting.

Figure 2 presents the effect of method parameters (angles $\alpha$ and $\gamma$) on the error $W_{\Delta F_{\text{max}}}$ with the adopted constant values $\Delta \alpha = 0.5^\circ$, $\Delta \gamma_1 = 0.2^\circ$, $\Delta \gamma_2 = 0.5^\circ$, relative tolerances $t_1 = 1\%$, $t_2 = 2\%$ and relative eccentric $t_e = 2\%$, $\omega = 82^\circ$, $L_1 = 1100$, $L = 3000$. The error $W_{\Delta F_{\text{max}}}$ for the adopted range of method parameters changes $\alpha \in (0 - 360^\circ)$ and $\gamma \in (10 - 70^\circ)$ yields the value range $(2.56741 \cdot 10^{-8} - 0.003)$.

![Fig. 2. Chart illustrating the effect of method parameters (angles $\alpha$ and $\gamma$) on the value of relative error $W_{\Delta F}$ of roundness deviation measurement](image)

Figures 3 and 4 present, respectively, examples of changes in the relative error values $W_{\Delta F_{\text{max}}}$ depending on the angle $\alpha$ and its setting error $\Delta \alpha$ (for $\gamma = 60^\circ$, $\Delta \gamma_1 = 0.2^\circ$, $\Delta \gamma_2 = 0.5^\circ$, $t_1 = 1\%$, $t_2 = 2\%$, $t_e = 2\%$ and $L_1 = 1100$, $L = 3000$ and $\omega = 82^\circ$) and the change in the relative error value $W_{\Delta F_{\text{max}}}$ depending on the angle $\gamma$ and its errors $\Delta \gamma_1 = \Delta \gamma_2 = \Delta \gamma$ (for $\alpha = 80^\circ$, $\Delta \alpha = 0.5^\circ$, $t_1 = 1\%$, $t_2 = 2\%$, $t_e = 2\%$ and $L_1 = 1100$, $L = 3000$ and $\omega = 82^\circ$). Additionally, it can be
stated that, from the point of view of error $W_{AF}$ minimisation, it is useful to use measurement systems with angles $\gamma \in (30^\circ \div 60^\circ)$ and any angle $\alpha$. Since the character of changes in the position of object’s centre is not known during the measurements, the error of deviation measurement $\Delta F$ caused by shifts in this centre will be accidental and automatically will affect the accidental method error.

Fig. 3. Chart illustrating the effect of angle $\alpha$ and its setting error $\Delta \alpha$ on the value of relative error of roundness deviation measurement

Fig. 4. Chart illustrating the effect of angle $\gamma$ and its setting error $\Delta \gamma$ on the value of relative error of roundness deviation measurement
Conclusions

This analysis of the measurement error of roundness deviation in a double V-block based cylindrical object connected with errors of determining (setting) the method parameters has shown that it only slightly affects the accuracy of roundness deviation measurements. The changes in the value of this error are random; therefore, they will have a minor effect on the total accidental error of the method. The best measurement systems, from the viewpoint of minimum error connected with errors of method parameter determination, are those with V-block angles $\gamma = (30–60)°$ and any angles $\alpha$ at which the gauge is placed.

References


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Analiza błędu pomiaru odchyłki okrągłości przy podparciu dwupryzmatowym związanego z błędami ustalenia parametrów metody

Słowa kluczowe

Błędy pomiaru, odchylka okrągłości, podparcie dwupryzmatowe, parametry metody.

Streszczenie

W artykule przedstawiono wyniki analizy błędu pomiaru odchyłki okrągłości przedmiotu cylindrycznego podpartego dwupryzmatowo, związanego z błędami ustalenia parametrów metody. Otrzymane rezultaty pozwalają wnosić, że wpływ tego błędu na sumaryczny błąd przypadkowy metody jest niewielki, a najkorzystniejsze układy pomiarowe z punktu widzenia minimum tego błędu, to układy pomiarowe o kącie rozwarcia pryzmy $\gamma = (30–60)°$. 