Krzysztof Naus*

Accuracy in Fixing Ship’s Positions by CCD Camera
Survey of Horizontal Angles

1. Introduction

Navigator identifies beacons being within his sight visually when conducting maritime coastal navigation. He measures angles and distances in respect to the beacons. Then, based on the measurements’ results, he determines coordinates of the ship position with analytical or graphical methods applied.

In general, it may be concluded that, in this process, he acts as a measuring device processing visual signals with information about the beacons coming from nautical publications onto coordinates of the position.

So far, attempts at automating navigators’ activities have not been undertaken, however there has been such equipment available as: high-resolution visual cameras – capable of performing identification and measurements, electronic navigational charts – containing information about the beacons in digital form, computers of high processor capacities – allowing processing of visual images in real time.

However, from scientific point of view, this new situation implicates interesting questions.

Is it possible to elaborate digital methods for automatic identification of beacons based on a sequent of coast’s images and of the electronic navigational chart?

What accuracy of determination of the ship’s position coordinates can be obtained from results of measurements of horizontal angles executed with the high-resolution CCD cameras?

Answers to these questions may justify purposefulness of further research conduction with a use of optical systems for automation of the maritime coastal navigation performance.

* Polish Naval Academy, Institute of Navigation and Hydrography, Gdynia, Poland
Recognition of images and using them as a basic source of information about environment has been a subject of many research in the last years, particularly in photogrammetry and robotics – an interdisciplinary branch of science dealing with mechanics, automatics, electronics, cybernetics and computer science.

Researches regarding the following areas of concern are worth special attention:

- active analyses of visual information, conducted by the Robotics Research Group at University of Oxford [5, 14] and University of Texas [23, 25],
- automation of process of external orientation elements’ determination, run within a scope of the OEEPE test [13, 20],
- construction of environment’s model [17, 29],
- SLAM (Simultaneous Localization and Mapping) techniques, used by the Laboratory of the Carneggie Mellon University [16, 24, 26],
- localization based on the Monte-Carlo method [21],
- research connected with application of omnidirectional camera [2, 28],
- increase of resistance of the localization process to errors [1, 3].

Needs of the maritime navigation in the domain of computational image processing are only partially satisfactory these days and they refer to the ENC (Electronic Navigational Chart) visualisation in ECDIS (Electronic Chart Display and Information System), comparative navigation conducted based on radar images, sonar presentations of sea bottom pictures.

Few researches connected with the image processing refer only to automation of the process of conducting a vessel along the fairway, navigation line, in particular a light sector or to integration with different navigation systems [6, 7, 22].

There are no scientific investigations done about optical-comparative system destined for automation of the process of a ship position coordinates’ determination when manoeuvring in a coastal zone there.

Initial research over the optical-comparative system, regarding possibility of applying CDD cameras to execute optical measurements from board of a ship in respect to beacons is presented in the article. Accuracy of the position’s coordinates determination based on results of horizontal angles’ measurements executed with a Rolleiflex 6008 camera have been evaluated [31].

2. Description of the Research

Navigator planning the ship’s itinerary usually has to face a problem of determining accuracy of the position coordinates not in selected points but on the entire water region of the planned route. Thus, it becomes important to know the re-
region on which accuracy of the position coordinates determination shall be better than the set one.

Depending on a method of the position determination, limits of the accuracy zone are determined with various ways, but based on the same assumptions, i.e. determination of geometric positions of points representing identical accuracies of their positions coordinates.

For the angle measurements taken with camera, mean error or area of the figure errors may be parameters for evaluation of accuracy of the position coordinates, determined with an angle reverse intersection method applied. Values of these parameters may be determined based on evaluated accuracy of the angle measurement made with the camera and on positions of reference points – permanent, optical beacons present in the water region (to be more precise – on a shape of the geometric measurement structure).

If values of these parameters are known in every point of the water region in question, one may determine contour-lines marking the accuracy areas off, which – if applied on a maritime navigational chart – inform whether accuracy of the position coordinates under determination shall be higher than the required one in the phases of coastal, ports approaching and port navigation.

2.1. Method of Calculating Mean Error Measurement Value

As a matter of fact, determination of the horizontal angle value with the camera consists in measurement of a distance between two objects seen on a picture registered on a CCD matrix (Fig. 1).

Accuracy of such a measurement may be characterized by means of a mean error. According to the fig. 1, its value mainly depends on length of focal distance $f$ of the camera’s optical system and on size of pixel $r$ on the CCD matrix.

With the mean errors of independent variables $f$, $x_1$, $x_2$ of a single measurement result function known:

$$\alpha = \arctg \frac{x_2}{f} - \arctg \frac{x_1}{f}$$

(1)

and applying the mean error propagation law, one may easily write down a mean error equation:

$$m_\alpha = \left[ \left( \frac{\partial \alpha}{\partial f} \cdot m_f \right)^2 + \left( \frac{\partial \alpha}{\partial x_1} \cdot m_{x_1} \right)^2 + \left( \frac{\partial \alpha}{\partial x_2} \cdot m_{x_2} \right)^2 \right]^{1/2}$$

(2)
which – after determination of partial derivatives – shall take a form of:

\[
m_{a} = \left[ \left( \frac{x_1}{f^2 + x_1^2} - \frac{x_2}{f^2 + x_2^2} \right)^2 \cdot m_f^2 + \left( -\frac{f}{f^2 + x_1^2} \right)^2 \cdot m_{x1}^2 + \left( -\frac{f}{f^2 + x_2^2} \right)^2 \cdot m_{x2}^2 \right]^{1/2}
\]

(3)

where:

- \( m_f \) – mean error of focal distance length measurement,
- \( m_{x1} = m_{x2} \) – mean error of distance measurement on CCD matrix.

![Diagram of horizontal angle measurement made with camera](image)

**Fig. 1.** Idea of horizontal angle measurement made with camera

### 2.2. Methods of Calculating Position Coordinates’ Mean Error and Area of Figure Errors

Coordinates of the ship position may be determined based on results of horizontal angle measurements and known coordinates of the beacons’ positions – with a method of reverse angle intersection. Accuracy of determining the position from two \( \alpha_1 \) and \( \alpha_2 \) angles may be evaluated based on the mean error, as well as
on a size of the so-called figure of errors area. Picture of the errors figure of coordinates for a position determined from two horizontal angles is presented on figure 2.

![Figure 2. Figure of errors of coordinates for position determined from two horizontal angles](image)

Its surface area is calculated with a dependence of [12, 15]:

$$P_e = \frac{4 \cdot e_{a_1} \cdot e_{a_2}}{\sin \theta} \quad (4)$$

where:

- $e_{a_2}, e_{a_1}$ – widths of strips of oscillations:

$$e_{a_2} = \frac{D_p^{z_2} \cdot D_p^{z_1}}{D_p^{z_2} \cdot m_{a_2}}, \quad e_{a_1} = \frac{D_p^{z_2} \cdot D_p^{z_1}}{D_p^{z_2} \cdot m_{u_1}}$$

- $\theta$ – angle of intersection of oscillation strips’ axes:

$$\theta = \Psi_{z_0} + \gamma_{z_2} = 360^\circ - (\beta + \alpha_1 + \alpha_2)$$

- $m_{u_1}, m_{u_2}$ – mean errors of first and second measurements of horizontal angles (calculated with dependence [3]).
Determination of position from three and more horizontal angles requires application of a measurement distribution method, in order to set optimal coordinates of the ship position. Classic adjustment problem may be formulated in the following way [27]:

\[
\begin{align*}
\mathbf{V} &= \mathbf{A}\hat{\mathbf{d}}_x + \mathbf{L} - \text{functional model} \\
\mathbf{C}_x &= \delta_0^2 \mathbf{P}^{-1} - \text{statistical model} \\
\mathbf{V}^T \mathbf{P} \mathbf{V} &= \min - \text{adjustment criterion}
\end{align*}
\]

where:

\( \mathbf{A} \) – design matrix,

\( \hat{\mathbf{d}}_x \) – estimator of measured values,

\( \mathbf{L} \) – vector of observations,

\( \delta_0^2 \) – variance coefficient,

\( \mathbf{P} \) – weighting matrix.

The following dependence is used to calculate mean error of the adjusted ship position:

\[
m_{\text{pos}} = (m_{\xi_S}^2 + m_{\eta_S}^2)^{1/2}
\]

where stand for mean errors of adjusted coordinates of the ship position. Values, are obtained by determining estimator of covariance matrix:

\[
\hat{\mathbf{C}}_x = \delta_0^2 (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} = \begin{bmatrix}
m_{\xi_S} & \text{cov}(\hat{X}_S, \hat{Y}_S) \\
\text{cov}(\hat{Y}_S, \hat{X}_S) & m_{\eta_S}
\end{bmatrix}
\]

where:

\[
\delta_0^2 = \frac{\mathbf{V}^T \mathbf{P} \mathbf{V}}{n-k}
\]

\[
\mathbf{P} = \begin{bmatrix}
\frac{1}{m_{\alpha_1}^2} & 0 & \ldots & 0 \\
0 & \frac{1}{m_{\alpha_2}^2} & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \frac{1}{m_{\alpha_n}^2}
\end{bmatrix}
\]
\[
\begin{bmatrix}
    \frac{\Delta y_p^Z_1}{(D_p^Z_1)^2} - \frac{\Delta y_p^Z_0}{(D_p^Z_0)^2} & \frac{\Delta x_p^Z_1}{(D_p^Z_1)^2} + \frac{\Delta x_p^Z_0}{(D_p^Z_0)^2} \\
    \frac{\Delta y_p^Z_2}{(D_p^Z_2)^2} - \frac{\Delta y_p^Z_0}{(D_p^Z_0)^2} & \frac{\Delta x_p^Z_2}{(D_p^Z_2)^2} + \frac{\Delta x_p^Z_0}{(D_p^Z_0)^2} \\
    \vdots & \vdots \\
    \frac{\Delta y_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} - \frac{\Delta y_p^{Z_0}}{(D_p^{Z_0})^2} & \frac{\Delta x_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} + \frac{\Delta x_p^{Z_0}}{(D_p^{Z_0})^2} \\
    \frac{\Delta y_p^{Z_{lzz}}}{(D_p^{Z_{lzz}})^2} - \frac{\Delta y_p^{Z_0}}{(D_p^{Z_0})^2} & \frac{\Delta x_p^{Z_{lzz}}}{(D_p^{Z_{lzz}})^2} + \frac{\Delta x_p^{Z_0}}{(D_p^{Z_0})^2} \\
    \vdots & \vdots \\
    \frac{\Delta y_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} - \frac{\Delta y_p^{Z_{lzz-2}}}{(D_p^{Z_{lzz-2}})^2} & \frac{\Delta x_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} + \frac{\Delta x_p^{Z_{lzz-2}}}{(D_p^{Z_{lzz-2}})^2} \\
    \frac{\Delta y_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} - \frac{\Delta y_p^{Z_{lzz-2}}}{(D_p^{Z_{lzz-2}})^2} & \frac{\Delta x_p^{Z_{lzz-1}}}{(D_p^{Z_{lzz-1}})^2} + \frac{\Delta x_p^{Z_{lzz-2}}}{(D_p^{Z_{lzz-2}})^2}
\end{bmatrix}
\]

\[ (10) \]

and:

- \( \Delta x_p^{Z_i} \) – approximate distance along abscissae’ axes from the ship position prior to the adjustment to the \( i \)-th beacon of known coordinates,
- \( \Delta y_p^{Z_i} \) – approximate distance along ordinates’ axes from the ship position prior to the adjustment to the \( i \)-th beacon of known coordinates,
- \( l_{zz} \) – number of beacons,
- \( m_{si} \) – mean error of the \( i \)-th horizontal angle,
- \( \hat{\delta}_0^2 \) – estimator of variance coefficient (value has been accepted for the calculations of the variance coefficient’s estimator, assuming correctly selected values of mean errors of the horizontal angle measurements, so the weighting matrix \( P \) elements determined based on them),
- \( n \) – number of results of executed measurements,
- \( k \) – required number of measurement results for the calculations,
- \( D_p^{Z_i} \) – approximate distance from the ship position prior to the adjustment to the \( i \)-th beacon of known coordinates.
2.3. Process of Elaborating Accuracy Distribution Charts

Three software applications were used to prepare the accuracy distribution charts (maritime navigational charts with accuracy regions applied). They were, respectively, responsible for the following areas of concern:

- determination of values of the errors figure surface areas and of mean errors of coordinates in nodes of a squares regular GRID, covering the water region under investigation,
- determination of contour-lines limiting accuracy areas based on the prepared GRID,
- preparation of maritime navigational charts with accuracy areas applied.

The first software is a proprietary application. It was worked out specially to execute first part of the research. According to the accepted initial assumptions, it uses electronic navigational charts (coded in accordance with the S-57 standard [8]) as sources of information about permanent optical beacons making a geometric measurement structure of the water region under investigation.

Based on these data and on additional “manually” entered ones, i.e.:

- ellipsoidal coordinates of the lower left \((\varphi_0, \lambda_0)\) and upper right \((\varphi_{\text{max}}, \lambda_{\text{max}})\) corners of the GRID,
- meridional \(\Delta\lambda\) and parallel \(\Delta\varphi\) intervals between the GRID’s nodes,
- limiting distance (radius length) \(r_p\) for searching the beacons,
- option of computational mode, for \(l_z = \text{false}\) – position determined from two horizontal angles, for \(l_z = \text{true}\) – position determined from three and more horizontal angles,
- maximal focal length \(f_{\text{max}}\),
- width of CCD matrix \(b_m\),
- mean error of focal length measurement \(m_f\),
- mean error of measurement of distance on CDD matrix \(m_{r}\),

it executes, according to the algorithm presented on figure 3, an interactive computational process of values in the GRID’s nodes. Results of these calculations are then saved as records in a file.

Each of the records consists of the following four fields describing:

- ellipsoidal coordinates \((\varphi, \lambda)\) of the grid’s node,
- values of mean error \(m_{\text{pzo}}\) for coordinates of the position determined in the node,
- size of errors figure’s surface \(P_F\) area for coordinates of the position determined in the node,
- number of beacons \(l_{zz}\) used when determining position in a node.
Description of sub-process calculating mean error of the horizontal angle measurement result \( m_c \) has been omitted in algorithm presented on figure 3 (on the interleaf) due to its extensiveness (value of this error is determined by the sub-process as a result of function (3) minimization with a simplex method applied [4, 19]).

Main window of the programme during execution of the interaction computational process for the mean error of the grid’s single node coordinates has been presented on figure 4. In this case, for the needs of the calculations, the programme has accepted eleven beacons present in a region limited with a circle of a radius of 12 nautical miles.

![Figure 4. Main window of programme during execution of interaction computational process for mean error of GRID’s single node coordinates](image)

The second software – “VerdicalMapper” – based on created GRID files and with interpolation methods applied, determines limits of the accuracy regions (a method of converse distances was applied in the investigation) which geometric description is then saved in a “mapping” thematic layer as a file of mif type [30].

The next proprietary programme named “Planszet”, based on the earlier obtained mif type files and electronic navigational charts, generates a final vector picture of the chart in Merkator cartographic form, with the thematic layer applied, and saves it as a emf type file [18].

3. Findings of the Research

The research consisted in preparing charts of accuracy distribution of ship position coordinates having results of horizontal angles measurements executed with Rolleiflex 6008 camera. It was conducted based on distances and angles calculated analytically in respect to geometric measurement structure made out of permanent optical beacons present in the Bay of Gdańsk.
The following technical parameters of the camera were accepted for the calculations:

- mean error of the focal length measurement \( m_f = 0.0008 \text{ m} \),
- mean error of the CDD matrix distance measurement \( m_x = 6.8 \text{ \mu m} \) (it was assumed that \( m_x \) corresponds with a size of one pixel on the CCD matrix),
- maximal length of the focal distance \( f_{\text{max}} = 500 \text{ mm} \),
- width of the CCD matrix \( b_m = 49.1 \text{ mm} \),
- and the limiting distance \( r_p \) of the beacons’ search equal 12 nautical miles.

All three, said in point 2.3, software applications were used in the entire process of calculation and preparation of the distribution charts. Results of the executed investigation are presented on figures 5 and 6. First chart of the ship position coordinates’ accuracy distribution was drawn up based on two horizontal angles measurements, while the second one – on the strength of many measurement and after their adjustment.

Analyzing the elaborated charts of distributions of the position coordinates determination accuracy, one should have in mind the fact that values of the determined accuracy parameters is a function of the measurement’s mean error, so it depends on external and internal settings of the camera and – the so-called – geometric factor, next dependent on the ship position in respect to the optical permanent beacons (shape of the geometric measurement structure). Value of the measurement’s mean error changes insignificantly if external and internal optimal settings of the camera are kept constant, so this error may be considered constant and be omitted in further analysis, and attention may be focused only on the geometric factor [19].

The measurement structure for the water region under investigation obtains the most favourable shape when the ship is by the harbour waters and in the harbour itself. Accuracy of the determined position coordinates rises significantly in these zones.

The coordinates mean error goes down to a level of 5 m and the surface area of errors figure reaches 50 m\(^2\) only.

In other water regions important for navigation, i.e. on:

- fairways approaching harbours – extent of the adjusted coordinates’ mean error is kept at the level of 5 m, while area of the errors figure’s surface is increased even to 2000 m\(^2\) for the coordinates obtained only from two horizontal angles;
- coastal waterways – value of the adjusted coordinates’ mean error is lowered to the level of 20 m, while area of the errors figure’s surface is larger than 5000 m\(^2\) for the coordinates obtained only from two horizontal angles.
Fig. 5. Maritime navigational chart with regions of accuracy of positions' coordinates determined from two horizontal angles.
Fig. 6. Maritime navigational chart with regions of accuracy of positions' coordinates determined from two and more horizontal angles (after adjustment)
4. Conclusions

1. Measuring systems built with a use of high-resolution CCD camera available on the market may guarantee high accuracy of the horizontal angle measurement within limits of the beacons’ and navigation lights’ sight distance (at very good visibility, optical range may reach even 30 nautical miles).

2. Accuracy of the angle measurement made with the camera may be improved by appropriate adjustment of the focal distance length and of the camera’s optical axis direction in respect to the geometric measurement structure. Mean error of the measurement executed with a use of camera properly set may be comparable to a measurement done with sextant or gyro-magnetic and magnetic compass.

3. The measuring system allows executing angle measurements from board of a ship onto observed beacons. Results of these measurements, coupled with coordinates of the beacons’ positions read off from the electronic navigational chart may be used for calculation of the ship position’s coordinates.

4. Accuracy of coordinates of such obtained position jest higher than the one required during the coastal, landing and harbour navigations. Basing on GMDSS NAV 47/7/1 (ANNEX 2, p. 5 IMO) requirements, one may conclude that accuracy of determining the position’s coordinates during the coastal navigation should be higher than 100 m \((P = 95\%)\). While, having in mind the Res. 815(19), it may be concluded that the accuracy depends on local conditions during the landing and harbour navigations. However, it is usually accepted at the level of 10 m \((P = 95\%)\) and higher.

5. Although the executed research is promising, one should have in mind the fact that two important factors, having impact on accuracy of the optical measurements, i.e. distortion of lens and astronomical refraction, were omitted in the investigation. Therefore, next research should be conducted on board of a ship manoeuvring in a coastal zone, in conditions of various visibilities, in the daytime and at night. Analysis of connected results of theoretical and practical tests would provide optimal conclusion on usefulness of CCD cameras application in determining ship position.

References


