An Improved Model of a Three phase Induction Motor Incorporating the Parameter Variations

Arunima DEY 1), Anurag TRIPATHI 1), Bhim SINGH 2)
Bharti DWIVEDI 1), Dinesh CHANDRA 3)
1) Institute of Engineering and Technology, India
2) I I T Delhi, India
3) MNNIT, India

Summary: A great deal of work is being done on the betterment of control through simulation of the electric drives used for various high-power purposes. The authenticity of the simulated results is based on the accurate modeling of the various parts of the electric drive system. Three–phase Induction motors form an extremely important part of the modern day electric drive system and their usage is continuously on a rise owing to their inherent properties of ruggedness, minimum maintenance requirements and continually increasing efficiencies. Usually the three-phase Induction motor model used in various research works does not incorporate stator and rotor core losses, stator and rotor stray load losses and magnetizing saturation and rotor conductor skin effects. The present paper aims at developing a three-phase Induction motor model taking the above losses and effects into account. The dynamic linking of the model to a thermal model considering the temperature dependent resistive elements is an added feature. The motor model described in this paper is the extension of the conventional 2-phase lumped-parameter induction motor model. The biggest advantage is that the model is user-programmable in MATLAB environment and can be used for system level transient studies. The simulation results of the developed model, with various parameter variations taken into account and subjected to sudden changes in load, show better torque and speed performances of the motor both in steady state and dynamic conditions.

1. INTRODUCTION
This paper presents a comprehensive model of induction machine and the outcomes of this effort are expected to assist at least the following:
1. Improvement in the control system and mechanical design can be achieved especially during transient conditions through thorough analysis of inverter-driven motor issues.
2. Advanced simulation techniques of new designs for motor and mechanical system can be accurately developed and evaluated.

An accurate and systematic $\alpha-\beta$ model of the induction machine is made in MATLAB/ SIMULINK environment. The torque developed by the machine is worked out during transient conditions and the developed model is put on trial at each stage. This is presented in the following sections. Finally, the integrated model representing the AC propulsion system is also presented. The model is user-programmable in MATLAB and can be used for system level transient studies.

The model is a detailed transient electrical model [1] based on the conventional two-axis lumped parameter $\alpha-\beta$ equivalent circuit of the induction machine. The dynamic linking of the model to a thermal model considering the temperature dependent resistive elements is an added feature. In [2] the parameter sensitivity concerning only temperature rise and magnetic saturation has been considered for vector speed control. The motor model described in this paper is an extension of the conventional 2-phase ($d-q$ or $\alpha-\beta$) lumped-parameter induction motor model. The model is characterized by incorporating transient and dynamic multi-reference frame model, rotor skin effects, spatial dependent magnetizing saturation and losses viz. stator and rotor core losses, stator stray load loss and friction and windage loss. The description of these features of the model is presented in this paper.

2. MODEL FEATURES

2.1. Rotor Skin Effects
Skin effect is a characteristic where the effective area of cross-section of the rotor conductor is reduced as the current flows through the least impedance path of the conductor’s surface. This increases the effective rotor resistance and decreases rotor leakage inductance due to the reduced leakage flux paths. As a result current ripple, torque ripple, and rotor copper loss under inverter excitation increases. This rotor skin effect gets more pronounced with increasing value of slip frequency. The most practiced way of approximating skin effects is by using a ladder network as shown in Figure 1a. An alternative configuration proposed here is a series combination as shown in Figure 1b. The ladder network in Figure 1a. has lower time constants and hence slower in time domain simulations. The series network enables a larger time step to be used for the same solution accuracy.
2.2. Magnetizing Saturation

Saturation of the main magnetizing field is often simply modeled by saturation factors uniformly affecting the magnetizing inductance along both axes of the $\alpha-\beta$ motor model. Such models do not consider cross-saturation, which is a result of a spatial dependency or effective saliency created by the saturated flux paths. He & Lipo in [3] stated that if cross-saturation effects are taken into account then stability of the motor could be more precisely calculated. They devised a simple way of implicitly including the effects of cross-saturation by modeling saturation along $\alpha$-axis in a reference frame aligned with the magnetizing or air gap flux vector. Saturation thus occurs along the air gap frame $\alpha$-axis, while the $\beta$-axis remains unsaturated. The $\alpha$-axis magnetizing inductance is then varied as a function of the air gap flux magnitude.

2.3. Core Loss

Core loss considered in this model consists of iron eddy current loss, iron hysteresis loss, and copper eddy current loss. During steady state sinusoidal operation at excitation frequency, $f_e$, the stator and rotor core loss at steady state can be written as:

$$P_{ei} = k_{hi}f_e\phi_m^2 + k_{ei}f_e^2\phi_m^2 + k_{ecu}f_e^2\phi_m^2$$

$$P_{ri} = k_{hi}sf_e^2f_e\phi_m^2 + k_{ecu}s^2f_e^2\phi_m^2$$

where: $s = \text{slip}$, and $\phi_m = \text{magnetizing flux}$. The last terms in (1) and (2) represents copper eddy current loss generally attributed to eddy current loss in the conductors caused by additional leakage and stray flux crossing the conductors when the core is heavily saturated. Based upon (1) and (2), the instantaneous core loss ($P_i$) for non-sinusoidal excitation can be represented by three resistive elements ($r_{hi}$, $r_{ei}$ and $r_{ecu}$) in parallel with the magnetizing inductance ($L_m$).

Technically, the core loss is a function of the instantaneous voltage of the winding linking the core. Inclusion of core loss resistances in the motor equivalent circuit is cumbersome. Cochran [4] discourages the addition of supplemental resistances to the basic equivalent circuit because the core loss (and stray load loss) resistances do not necessarily have tête-à-tête connection with the physical mechanisms they supposedly represent within the machine. Under transient conditions, significant errors reportedly can occur, particularly true with modeling hysteresis loss.

2.3.1. Iron Eddy Current Core Loss

Instantaneous iron eddy current loss can be modeled by:

$$P_{ei} = k_{ei}\nu_m^2$$

where $\nu_m$ is the instantaneous air gap voltage in the stationary (stator) frame, and is equal to the derivative of the airgap flux; i.e.:

$$\nu_m = p\lambda_m^s$$

Instantaneous iron eddy current loss can thus be accurately described by a constant resistance ($r_{ei}$) in parallel with an inductance for both sinusoidal and inverter excitation:

$$r_{ei} = \frac{1}{k_{ei}}$$

2.3.2. Hysteresis Core Loss

Hysteresis loss is proportional to flux squared. However, since hysteresis loss is proportional to frequency, a resistance representing hysteresis loss must be proportional to the excitation frequency, i.e.;

$$P_{hi} = \frac{k_{hi}}{[\omega_m]}\nu_m^2$$

where $\omega$ is the frequency of the magnetizing flux vector (which is equivalent to the excitation frequency). The equivalent hysteresis loss resistance is then:

$$r_{hi} = \frac{[\omega_m]}{k_{hi}}$$

For sinusoidal excitation and steady state conditions, equation (6) is generally sufficient. However, during transients or with inverter excitation, calculation of instantaneous angular frequency is difficult and can lead to gross errors in the instantaneous core loss calculation. For example, the perceived instantaneous frequency, $\omega_m$, may momentarily go to zero during transients while the air gap voltage remains nonzero, causing the calculated instantaneous hysteresis core loss to approach infinity.

A more robust means of calculating the instantaneous hysteresis loss is from reactive power calculations similar to that proposed by Udayagiri & Lipo in [5]. Hysteresis loss is proportional to the area of the hysteresis loop of a core material; i.e.:

$$P_{hi} = \left[k_{hi}f_Li^2\right] \frac{2}{\pi}$$

where $i$ is the current through an inductor $L$. Since reactive power through an inductor is:

$$Q_L = 2\pi f_L i^2$$

the hysteresis loss can be modeled as being proportional to reactive power; i.e.:

$$P_{hi} = k_{hi}Q_L$$

Fig. 1. 3rd order R-L networks to transiently model rotor conductor skin effects.
2.3.3. Copper Eddy Current Core Loss

From (3–4), the copper eddy current loss is proportional to frequency squared, and to flux to the $n^{th}$ higher power. Since voltage is proportional to flux squared, the copper eddy current loss can be modeled as:

$$P_{ecu} = k_{ecu} \lambda_m^{n-2} \omega^2$$  \hspace{1cm} (10)

The representative resistive element must vary inversely proportional to the flux to the $n-1$ power; i.e.:

$$r_{ecu} = \frac{1}{k_{ecu} \lambda_m^{n-2}}$$  \hspace{1cm} (11)

Since the changes of magnetizing flux are slow relative to the simulation step size, the representation of copper eddy current loss via the variable resistive element in (11) should not introduce significant errors even during transients or with inverter excitation.

2.3.4. Net Core Loss

Net core loss:

$$P_i = P_{el} + P_{hi} + P_{ecu}$$  \hspace{1cm} (12)

If hysteresis loss is calculated via (5) and (6), then the representative core loss resistance is:

$$r_i = \frac{1}{r_{hi} + r_{ei} + r_{ecu}}$$  \hspace{1cm} (13)

Individual core loss resistances can be defined for both the stator and the rotor. The instantaneous rotor core loss must be calculated in the rotor reference frame with an excitation frequency corresponding to the slip frequency.

2.4. Stray Load Loss

Losses associated with leakage flux are considered as stray load loss. Stray load loss is defined to include all losses not accounted for by stator and rotor copper loss, no-load core loss, and friction and windage. Stator stray load loss can be modeled via resistive elements across the stator leakage inductances [6], having two components; hysteresis and eddy current. Due to the complexity of the rotor circuit with skin effects and leakage saturation, the stray load loss is presently modeled in the stator only.

Note that Cochran [4] advocates the modeling of stray load loss via a resistance in series with the magnetizing inductance.

2.5. Friction & Windage Loss

Friction and windage loss of the motor (excluding the load) is modeled as follows:

Friction loss:

$$P_f = k_f \omega_r$$  \hspace{1cm} (14)

Windage loss:

$$P_w = k_w \omega_r^3$$  \hspace{1cm} (15)

Net friction and windage loss:

$$P_{fkw} = k_f \omega_r + k_w \omega_r^3$$  \hspace{1cm} (16)

Net friction and windage torque:

$$T_{fkw} = k_f + k_w \omega_r^2$$  \hspace{1cm} (17)

3. TRANSCIENT model WITH INTERNAL Core AND STATOR STRAY LOAD LOSS

3.1. Equivalent Circuit

The equivalent circuit of the motor electrical model is shown in Figure 2 in complex vector notation.

3.2. State Variables

The following complex state variables are chosen for formulation of the motor model equations:

$$\lambda_{qfs}$$ — stator flux in stator frame

$$\lambda_{qfm}$$ — magnetizing flux in stator frame

$$\lambda_{qfr}$$ — rotor flux in rotor frame

$$\lambda_{qfr1}$$ — rotor leakage flux component in rotor frame

$$\lambda_{qfr2}$$ — rotor leakage flux component in rotor frame

3.3. Voltage Equations

Each state variable defined in Section 3.2 can be described by a voltage equation in the form of an ordinary differential equation as follows:

Stator Voltage:

$$p \lambda_{qds} = v_{qds} - r_{is} i_{qds}$$  \hspace{1cm} (18)

Air Gap Voltage:

$$p \lambda_{qdm} = r_{sl} i_{qdsi}$$  \hspace{1cm} (19)

Rotor Voltages:
3.4. Flux Equations

In addition to the voltage equations, flux linkage equations can be written for each state variable as follows:

Stator Flux:
\[ \lambda_{qS} = \lambda_{qlm} + L_{iqS} i_{qS} \]  (23)

Air Gap Flux:
\[ \lambda_{m} = L_{m} i_{m} \]  (24)

Rotor Flux:
\[ \lambda_{qR} = L_{br} i_{qR} \]  (25)
\[ \lambda_{qR2} = L_{br2} i_{qR2} \]  (26)
\[ \lambda_{qfr} = \lambda_{qlm} - L_{br} i_{qfr} - L_{br} i_{qR} - L_{br2} i_{qR2} \]  (27)

4. INDUCTION MACHINE MODEL

The machine model in MATLAB environment is built using the equations 18-27. The model is shown in the Figure 4 below:

The inputs to the model are Vph, speed, rotor and stator conductor temperature. From the speed, all the other parameters like flux, current and torque are calculated at each stage. The phase voltages to the motor are fed from the output of the inverter. Finally, the electromagnetic torque, and stator and rotor currents are observed in the form of output waveforms.

5. SIMULATED RESULTS

5.1. Steady state performance

The developed model is subjected to various standard cases from [2, 7] for steady state analysis. The model is provided with the steady state equivalent circuit parameters and is supplied with 3 phase pure sine wave voltages. The calculated and simulated values of torque and current are found to be almost the same as obtained from the results of simulation. For example the simulation for a 2250 hp, 2300 V, 1786 rpm machine yielded 9170.50 Nm torque and 469.45 Amp stator current while the calculated values are 9173.50 Nm and 469.56 Amp respectively. These values are based on following machine internal parameters:
\[ r_1=0.029, \quad x_{l1}=0.226, \quad x_m=13.04, \quad x_{l2}=0.226 \] and \[ r_2''=0.022. \]

The waveforms obtained for 2250 hp machine are shown in Figure 5. Note that the initial decaying transients seen in the electromagnetic torque developed are merely due to the transient offset in the stator currents.

5.2. Dynamic performance during sudden change in load

The machine model is now put on test for a step change in load. The torque and speed along with stator and rotor currents responses after the machine settled at synchronous speed are obtained, during dynamic condition, as shown by the waveforms in figures 6–7. The initial torque is zero under
these conditions. A load torque of 9000 N-m is applied at $t = 3.5$ secs and removed at $t = 5.1$ secs. The simulated waveform of electromagnetic torque shows that the torque developed by the motor has transients but dampens out quickly to settle to an equilibrium point and accordingly follow the changes in load torque. The model developed in this paper incorporates such features, which help in improving the performance of the induction machine by reducing the torque transients despite the consideration of various parameter changes.

6. CONCLUSION

The swift control of torque in ac drives has always been a topic of investigation despite of its preference over dc drives in industry. The induction machine model developed in this paper shows better torque and speed response during both the steady state and dynamic conditions even with the incorporation of effects of various parameters like rotor skin effects, temperature dependent resistive elements and magnetic saturation. The model also takes into account stator and rotor core losses including the iron hysteresis loss, iron eddy current loss and copper eddy current loss. Some features viz saturation of stator leakage inductance due to stator flux, temperature dependency on skin depth and winding and saturation spatial harmonics that are not incorporated in the model, if pursued, may form the future work and the model can further be improved for its application in high power applications.
Fig. 6.1. Effect of Load Disturbance on Electromagnetic Torque

Fig. 6.2. Effect of Load Disturbance on Rotor Speed

Fig. 7.1. Effect of Load Disturbance on Stator Currents

Fig. 7.2. Effect of Load Disturbance on Rotor Currents

REFERENCES