FOUR-STATE EXPLOITATION STRATEGIES OF MAXIMIZE OF THE COEFFICIENT OF THE READINESS WITH THE REGARD OF ECONOMIC CONDITIONS

Miroslaw Siergiejczyk, Adam Rosinski

Warsaw University of Technology Faculty of Transport
Koszykowa Street 75, 00-662 Warsaw, Poland
tel.: +48 22 2347040, +48 22 2347038
e-mail: msi@wt.pw.edu.pl, adro@wt.pw.edu.pl

Abstract

Electronic systems work in different conditions of exploitation. Because of specific nature of their application (such as transport for example), they should be characterized by a high level of coefficient of the readiness. When carrying out a proper reliability analysis of systems can be determined such reliability structures, which will be characterized by the appropriate values of rates of reliability. Such an approach provides to improvement of the reliability of the designed systems. However not to the full extent, it is not possible to achieve the system's readiness at the appropriately high level to the full extent. The paper presents a methodology to optimize the process of exploitation these systems, taking into account economic conditions, such as of financial expenses designed on periodical reviews and the effectiveness of their use. There is presented the dependence that maximizes the value of the coefficient of the readiness in the defined reliability (failures rate), exploitation (intensity of repairs, intensity of maintenance service) and economic conditions (financial outlays earmarked for periodic inspections).

Keywords: transportation, exploitation, strategy, economy

1. Introduction

The issue of the rational realization of the process of exploitation of electronic devices used in transport is a significant problem [2, 14]. This is due to the need to ensure continuity of operation of these devices. It is possible by obtaining appropriate indicators of exploitation and reliability. With this range were already established a number of important studies [3, 6, 8, 23], which are known in the scientific community. They cover both issues related to the reliability [7] and exploitation. When carrying out a proper reliability analysis of systems can be determined such reliability structures, which will be characterized by the appropriate values of rates of reliability. This applies to both systems, taken as a whole [4, 13, 15, 17], as well as only the selected subsystems (e.g. the power supply [10, 12, 16, 20] or subsystems of transmission of information [5, 9, 19, 21, 22]). Such an approach provides to improvement of the reliability of the designed systems. However not to the full extent, it is not possible to achieve the system's readiness at the appropriately high level to the full extent. Therefore, it is necessary to make the exploitation analyse, taking into account the operational properties of the selected operating systems (e.g. failure rate, intensity of check-ups [1], financial outlays earmarked for inspections) [11, 18]. The article presents an analysis of the exploitation process of transport telematics systems, in which were used four generic periodic inspections.

2. Four-state exploitation strategy of maximize of the coefficient of the readiness

Generally, coefficient of readiness can be written:

\[ K_g = \frac{T_m}{T_m + T_n}, \]
where:

\( T_m \) – mean correct operation time between failures,
\( T_n \) – mean time to repair.

The given relationship shows that the system can be in one of two state (Fig. 1):
- usage state (\( S_0 \)),
- repair state (\( S_1 \)).

\[ \lambda \]
\[ \mu \]

\( S_0 \quad S_1 \)

Fig. 1. Graph showing switching between usage and repair states; Denotations in figures: \( \lambda \) – failure rate, \( \mu \) – repair rate

The chart of transitions presented in Fig. 1 does not include all possible and actually occurring states (assuming four types of periodic check-ups). Therefore, it should be supplemented by the following four states (Fig. 2):
- state \( S_{001} \) (during which those actions are performed that are required by the scope of 1\(^{st}\) type check-up),
- state \( S_{010} \) (during which those actions are performed that are required by the scope of 2\(^{nd}\) type check-up),
- state \( S_{011} \) (during which those actions are performed that are required by the scope of 3\(^{rd}\) type check-up),
- state \( S_{100} \) (during which those actions are performed that are required by the scope of 4\(^{th}\) type check-up).

\[ \lambda_1 \]
\[ \lambda_2 \]
\[ \lambda_3 \]
\[ \lambda_4 \]
\[ \mu_1 \]
\[ \mu_2 \]
\[ \mu_3 \]
\[ \mu_4 \]

\( S_{000} \quad S_{001} \quad S_{010} \quad S_{011} \quad S_{100} \quad S_{111} \)

Fig. 2. Chart of transitions between the state of operation, repair, check-up of 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\) and 4\(^{th}\) type; Denotations in figures: \( \lambda \) – intensity of damages, \( \mu \) – intensity of repairs, \( \lambda_1 \) – intensity of 1\(^{st}\) type of check-ups, \( \mu_1 \) – intensity of 1\(^{st}\) type of maintenance service, \( \lambda_2 \) – intensity of 2\(^{nd}\) type of check-ups, \( \mu_2 \) – intensity of 2\(^{nd}\) type of maintenance service, \( \lambda_3 \) – intensity of 3\(^{rd}\) type of check-ups, \( \mu_3 \) – intensity of 3\(^{rd}\) type of maintenance service, \( \lambda_4 \) – intensity of 4\(^{th}\) type of check-ups, \( \mu_4 \) – intensity of 4\(^{th}\) type of maintenance service

For the chart of transitions presented in Fig. 2 the following equations can be derived:
Four-State Exploitation Strategies of Maximize of the Coefficient of the Readiness with the Regard of...

\[ \lambda \cdot P_{000} + \mu \cdot P_{111} - \lambda_1 \cdot P_{000} + \mu_1 \cdot P_{001} - \lambda_2 \cdot P_{000} + \mu_2 \cdot P_{010} - \lambda_3 \cdot P_{000} + \mu_3 \cdot P_{011} - \lambda_4 \cdot P_{000} + \mu_4 \cdot P_{100} = 0, \]

\[ \lambda_1 \cdot P_{000} - \mu_1 \cdot P_{010} = 0, \]

\[ \lambda_2 \cdot P_{000} - \mu_2 \cdot P_{010} = 0, \]

\[ \lambda_3 \cdot P_{000} - \mu_3 \cdot P_{011} = 0, \]

\[ \lambda_4 \cdot P_{000} - \mu_4 \cdot P_{100} = 0. \]  

Equation (2)

Obviously:

\[ P_{000} + P_{001} + P_{010} + P_{011} + P_{100} + P_{111} = 1. \]  

Equation (3)

Thus:

\[ P_{000} \cdot \left(1 + \frac{\lambda_1}{\mu} + \frac{\lambda_2}{\mu_1} + \frac{\lambda_3}{\mu_2} + \frac{\lambda_4}{\mu_3}\right) = 1, \]  

Equation (4)

\[ K_{g1} = P_{000} = \frac{1}{\left(1 + \frac{\lambda_1}{\mu} + \frac{\lambda_2}{\mu_1} + \frac{\lambda_3}{\mu_2} + \frac{\lambda_4}{\mu_3}\right)}. \]  

Equation (5)

\[ K_{g1} = P_{000} = \frac{\mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4}{\mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_1 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_3 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_4 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4}. \]  

Equation (6)

Equation (6) allows calculating the probability values of the system staying in its operational state. It numerically corresponds to the value of the readiness coefficient.

Let us introduce a coefficient that shall make the intensity values \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) mutually dependent (if one of those values increases, the other one will decrease). This coefficient is the ratio of the transition intensity to the total of all the intensities of the check-up of 1st, 2nd, 3rd and 4th type. The chart of transitions as illustrated in Fig. 2 will currently take the following form (Fig. 3):

\[
\begin{align*}
\lambda & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\mu & \quad \mu_1 + \mu_2 + \mu_3 + \mu_4 \\
\lambda_2 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\lambda_3 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\lambda_4 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4
\end{align*}
\]

\[
\begin{align*}
\lambda_1 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\mu_1 & \quad \mu_1 + \mu_2 + \mu_3 + \mu_4 \\
\lambda_2 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\lambda_3 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
\lambda_4 & \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4
\end{align*}
\]

Fig. 3. Chart of transitions between the state of operation, repair, 1st, 2nd, 3rd and 4th type of check-up (the adjustment coefficient is included)
Thus carrying out the considerations as before, we can derive as follows:

\[
K_{g2} = P_{000} = \frac{(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4}{(\lambda + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_2 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda_3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 + \lambda_4 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}.
\]

(7)

The relations obtained (7) allows to determine the impact that intensities, assumed for the performance of 1st, 2nd, 3rd and 4th type of check-up, have on the readiness coefficient of a given system (with an assigned failures rate, the intensity of 1st, 2nd, 3rd and 4th type of maintenance service).

3. Optimization of four-state exploitation strategy of maximize of the coefficient of the readiness, taking into account economic conditions

Figure 4 shows the graph of transitions between states mentioned earlier. The transitions between states include factors:

- \(k_1\) – coefficient of first type check-ups - it determines the linear relationship between the intensity of the current review of the first type of check-ups and the optimum intensity of the review of the first type of check-ups for which the value of coefficient of readiness is maximal,
- \(k_2\) – coefficient of second type check-ups - it determines the linear relationship between the intensity of the current review of the second type of check-ups and the optimum intensity of the review of the second type of check-ups for which the value of coefficient of readiness is maximal,
- \(k_3\) – coefficient third type of check-ups - it determines the linear relationship between the intensity of the current review of the third type of check-ups and the optimum intensity of the review of the third type of check-ups for which the value of coefficient of readiness is maximal,
- \(k_4\) – coefficient fourth type of check-ups - it determines the linear relationship between the intensity of the current review of the fourth type of check-ups and the optimum intensity of the review of the fourth type of check-ups for which the value of coefficient of readiness is maximal.

Fig. 4. Chart of transitions between the state of operation, repair, 1st, 2nd, 3rd and 4th type of check-up (included factors inspections \(k\)). Denotations in figures: \(k_1\) – coefficient of the first type check-ups, \(k_2\) – coefficient of the second type check-ups, \(k_3\) – coefficient of the third type check-ups, \(k_4\) – coefficient of the fourth type check-ups
An important issue occurring in practice is disposal by the user a limited resource funding for periodic inspections. This requires identification of the impact of funding allocated to periodic inspections on system coefficient of readiness. Therefore it is introduced the factor $C$, which will determine the available financial resources earmarked for inspections I, II, III and IV type of check-up. Let us assume that:

- $C = 4$ for the optimal intensity of I, II, III and IV type of check-up ($K_g = \max.$ for $\lambda_1 = \lambda_{1\text{opt}}$, $\lambda_2 = \lambda_{2\text{opt}}$, $\lambda_3 = \lambda_{3\text{opt}}$, and $\lambda_4 = \lambda_{4\text{opt}}$; as in equation (8) $k_1 \cdot C = 1$, $k_2 \cdot C = 1$, $k_3 \cdot C = 1$ and $k_4 \cdot C = 1$),
- $C = 0$ for the intensity of I, II, III and IV type of check-up equal to zero (no reviews); as in equation (8) $k_1 \cdot \lambda_{1\text{opt}} \cdot C = 0$, $k_2 \cdot \lambda_{2\text{opt}} \cdot C = 0$, $k_3 \cdot \lambda_{3\text{opt}} \cdot C = 0$ and $k_4 \cdot \lambda_{4\text{opt}} \cdot C = 0$).

After performing, a mathematical analysis was obtained the following relationship:

$$K_g = P_{000} = \frac{\left(\lambda + k_1 \lambda_{1\text{opt}} \cdot C + k_2 \lambda_{2\text{opt}} \cdot C + k_3 \lambda_{3\text{opt}} \cdot C + k_4 \lambda_{4\text{opt}} \cdot C\right) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \left(k_1 \lambda_{1\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \left(k_2 \lambda_{2\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot \mu_4 + \left(k_3 \lambda_{3\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 + \left(k_4 \lambda_{4\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}{\left(\lambda + k_1 \lambda_{1\text{opt}} \cdot C + k_2 \lambda_{2\text{opt}} \cdot C + k_3 \lambda_{3\text{opt}} \cdot C + k_4 \lambda_{4\text{opt}} \cdot C\right) \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \lambda^2 \cdot \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \left(k_1 \lambda_{1\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_2 \cdot \mu_3 \cdot \mu_4 + \left(k_2 \lambda_{2\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_3 \cdot \mu_4 + \left(k_3 \lambda_{3\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_4 + \left(k_4 \lambda_{4\text{opt}} \cdot C\right)^3 \cdot \mu \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}$$

(8)

The obtained relationship (8) allows to specify the impact of adopted to realize the intensity of the I, II, III and IV type of check-up on the coefficient of readiness of the considered system (when known failure rate, intensity of repairs, the intensity I, II, III and IV type of maintenance service, funding factor $C$).

4. Conclusion

The paper presents the method for optimising the process of exploitation of electronic systems (assuming four-type periodic check-ups), which takes into account selected reliability (failures rate), exploitation (intensity of repairs, intensity of maintenance service) and economic (financial outlays earmarked for periodic inspections) properties. It allows determining the intensity of periodic check-ups with which they should be carried out, assuming the optimisation criterion that maximises the value of the coefficient of readiness.

In future studies of this issue, it is planned to develop an author’s computer program that would enable the practical application of the presented considerations by decision-makers deciding about the process of exploitation of electronic devices used for transport.

References


