MINING-INDUCED SURFACE HORIZONTAL DISPLACEMENT: THE CASE OF BW PROSPER HANIEL MINE

The paper presents a wide-ranged analysis of horizontal displacements in the region of underground mining exploitation. Initially, various theories pertaining to the determination of mining-induced horizontal displacement are discussed, followed by a complex study on horizontal displacements measured for a selected example region of the German coal mine BW Prosper Haniel, as well as the determination of displacement factor $B$.

**Keywords:** horizontal displacements, surface deformations, mining exploitation, horizontal displacement coefficient

1. **Introduction**

Underground mining exploitation of coal seams induces deformations within the rock mass structure, which frequently emerge on the terrain surface. The magnitude of these deformations depends on the following parameters: number of seams worked out, their thickness, dimensions
of fields worked out within each seam, exploitation method, liquidation of space worked out, geological and hydrogeological structure of overburden, tectonics, strength and strain properties of rock mass building rock strata.

During longwall panel exploitation the following weak zones are created above it: caving zone of $h_c$ height, fractured zone of $h_f$ height, and deflection zone ($h_u$). When mining is conducted at large depths, when the total height of caving and fractured zones is less than depth of exploitation, only deflection zone, called subsidence trough, emerges on the terrain surface. The trough is characterized by terrain deformation indicators, such as: subsidence $w$, horizontal displacement $u$, horizontal strain $\varepsilon$ and vertical strain $\varepsilon(z)$, slope $T$, radius of curvature $R$.

Despite the fact that many researchers have been dealing with the problem of exploitation influence on rock mass, the problem of proper description of horizontal displacements distribution and their value has not been unequivocally resolved for years. Two groups of theories describing horizontal displacements, dominate in professional literature:

- The theories resulting from the assumption that surface points move towards center of gravity of selected seam element, among others the works of: Keinhorst (1925), Bals (1931/32), Lehmann (et al. 1942), Beyer, Sann (1949). The theories differ from each other only in values of assumed angles of exploitation influences (Tajduś et al., 2011), and assumed functions of influence on the point of elementary reservoir volume within the area of exploitation impact.

- The theories presenting the relationship between the vector of horizontal displacement and the vector of subsidence trough slope. For the flat (2D) model it can be presented as:

$$u(x) = -B \cdot T(x) = -B \frac{dw(x)}{dx}$$  \hspace{1cm} (1)

where:

- $u(x)$ — horizontal displacement of the point located on the surface in a distance $x$ from the longwall panel edge,
- $T(x)$ — slope of the point located on the surface in a distance $x$ from the longwall panel edge,
- $w(x)$ — vertical displacement of the point located on the surface in a distance $x$ from the longwall panel edge,
- $B$ — horizontal displacement coefficient (or displacement factor).

This assumption was proposed in 1947 by Awierszyn. Coefficient of horizontal displacement $B$ plays a significant role in presented formula. According to Awierszyn, it falls within range $B = (0.15 \div 0.18) \cdot H$ and in the main part of subsidence trough, i.e. in the place of maximum strain, $B$ may be adopted as constant value according to its assumption, however its value increases with the increase of distance. Basing of assumptions of Awierszyn, Budryk (1953) determined the theoretical value of $B$ coefficient equal to $B = \frac{r}{\sqrt{\pi}} = 0.564 \cdot r$, while assuming the incompressibility of rock mass. As a matter of fact, the in-situ measurement analysis of materials demonstrated the necessity of making the above-mentioned value more specific. Through the analysis of deformation indicators, Budryk came to a conclusion that the maximum value of horizontal strain does not exceed, as a rule, 60% of the maximum slope value. From this relationship he obtained the following value:
\[ B = \frac{r}{\sqrt{2\pi}} \approx 0.40 \cdot r \]  \hspace{1cm} (2)

Later on, the fundamental research on the scope of \( B \) coefficient value development was carried out by Popiołek and Ostrowski (1978 among others). The regression relationship, obtained through the analysis of in-situ measurement results, has the following form:

\[ B = 0.156 \cdot H + 53.7 \cdot u_{\text{max}} - 17.1 \cdot w_{\text{max}} \]  \hspace{1cm} (3)

The formula can be converted to the form:

\[ B = \frac{0.156 \cdot H - 17.1 \cdot w_{\text{max}}}{1 - 53.7 \cdot T_{\text{max}}} \]  \hspace{1cm} (4)

Popiołek and Ostrowski (1978) proposed a simplification of the regression formula obtained by themselves to the form:

\[ B = 0.16H \]  \hspace{1cm} (5)

and assuming that average value of \( \beta \) angle of the main influences for Silesia region equals to 63.43 (\( \tan \beta = 2.0 \)), they obtained formula (6):

\[ B = 0.32r \]  \hspace{1cm} (6)

which constitutes a base for forecast calculations currently performed in Polish mining industry.

The \( B \) parameter was an object of interest for many authors, among others Akimow (Multi-author work, 1980), who has found that \( B \) depends on the thickness of loose overburden (the Tertiary and Quaternary one), whereas Niedojadło (1984) found that \( B \) coefficient of proportionality value depended on a position of a specific point in relation to an exploitation field and a method of mining exploitation. The original formula for the determination of \( B \) coefficient was given by Drzęźla (1978), who related its value to \( r \) (main influences radius value), the average value of \( \nu \) Poisson ratio for overburden rocks, and the angle of main influences value \( \beta \):

\[ B = \frac{0.665}{2\pi} \cdot \frac{1 - \nu}{\nu} \cdot \frac{1}{\tan \beta} = 0.106 \cdot \frac{1}{\nu} \cdot H \cdot \cot^2 \beta \]  \hspace{1cm} (7)

Extensive works addressing relationships between horizontal displacements and slopes were presented by Kwinta (2003). Basing on the measurements of horizontal displacements carried out for the “Dębieńsko”, “Jan Kanty” and “Jowisz” mines, Kwinta (2003) found that „in spite of correlation existing between \( u \) and \( T \), adoption of \( u(x,t) = -B \cdot T(x,t) \) relationship causes too large simplification”, and this is why he proposed to introduce \( B(x,t) \) coefficient into analysis, which depends on time and spatial position of a point.

In professional literature adequate empirical formulas are most frequently used for the determination of \( B \) coefficient value:

\[ B = \lambda \cdot r \]  \hspace{1cm} (8)

or

\[ B = \mu \cdot H \]  \hspace{1cm} (9)
The established empirical values of $\mu$ and $\lambda$ coefficients can be found in Table 1.

<table>
<thead>
<tr>
<th>Author</th>
<th>$\lambda$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Awierszyn (1947)</td>
<td>$0.30 \pm 0.36$ (for $\tan \beta = 2.0$)</td>
<td>$0.15 \pm 0.18$</td>
</tr>
<tr>
<td>Budryk (1953)</td>
<td>0.40</td>
<td>–</td>
</tr>
<tr>
<td>Sroka (1976)*</td>
<td>0.26</td>
<td>–</td>
</tr>
<tr>
<td>Popiołek, Ostrowski (1978)</td>
<td>0.32 (for $\tan \beta = 2.0$)</td>
<td>0.16</td>
</tr>
<tr>
<td>Niemiec and Radola (1981)</td>
<td>0.37</td>
<td>–</td>
</tr>
<tr>
<td>Flisiak (et al. 1991)**</td>
<td>0.15 (“Jeziórko”) 0.25 (Machów II)</td>
<td>–</td>
</tr>
<tr>
<td>Szpetkowski (1995)</td>
<td>0.20 (for $\tan \beta = 2.0$) 0.8 · cot $\beta$</td>
<td>–</td>
</tr>
<tr>
<td>Popiołek, et al. (1996)*</td>
<td>0.30 (for tensile strains) 0.36 (for compressive strains)</td>
<td>–</td>
</tr>
<tr>
<td>Xueyi Yu and Niedbalski (1998) (for Chinese conditions)</td>
<td>0.20÷0.40</td>
<td>–</td>
</tr>
<tr>
<td>Jura, Niedojadlo (2013)***</td>
<td>0.38÷0.65 (for tensile strains) 0.23÷0.77 (for compressive strains)</td>
<td>–</td>
</tr>
<tr>
<td>RAG</td>
<td>0.30÷0.40</td>
<td>–</td>
</tr>
<tr>
<td>Sroka (for conditions of the Ruhr Basin)</td>
<td>0.15÷0.70</td>
<td>–</td>
</tr>
<tr>
<td>Popiołek et al. (1996)*</td>
<td>0.30 (for tensile strains) 0.36 (for compressive strains)</td>
<td>–</td>
</tr>
</tbody>
</table>

* for copper ore mining; ** for sulphur mining; *** for copper ore mining

To recapitulate, it can be stated that both Keinhorst’s center of gravity theory, and the hypothesis of Awierszyn, indicate the fact of proportionality between horizontal displacement vector and slope vector of subsidence trough profile.

2. Analysis of horizontal displacements and $B$ coefficient for the region of Prosper Haniel mine: case study

A complete and wide-ranged analysis of terrain surface horizontal displacements and distribution of $B$ coefficient has been performed for the exemplary BW Prosper Haniel hard coal mine in Germany.

In 1999 the mine started exploitation of 698 longwall in O/N seam lying at 960 m average depth. The length of worked out longwall panel reached 270 m, while its height ranged from 3.6 m up to 4.3 m (Fig. 1). The longwall panel was started on the 10th of May 1999 and finished on the 23rd of Nov 1999.
The mine set up a measuring network on terrain surface, comprised of distributed land survey points. The measurements were performed with the use of the GPS method, which is characterized with accuracy of determining the coordinates \( x, y, z \) of less than 5mm (Sroka, 2000). The measurements of horizontal displacements and subsidence were performed. The first measurement was executed on April 1st, 1999, for points numbered from 1 to 48, whose location is presented in Fig. 1.

![Fig. 1. Location of measuring points in relation to longwall panel no. 698 in the O/N seam](image)

After the termination of exploitation of the longwall panel no. 698, while the measurements were still underway, the exploitation of longwall no. 682 in P1 seam, located at the depth of 920 m, was commenced. The situation is illustrated in Fig. 2.

Since the exploitation of longwall panel no. 698 in the O/N seam had finished, a break in its exploitation took place, and subsequent 697 longwall panel was commenced only after half a year (i.e. 19th June 2000). The analysis of measuring points subsidence was performed for the period from the moment of exploitation commencement to the moment preceding the exploitation commencement of the longwall 697. The results of the analysis for exemplary points no. 15 and 27 are shown in Figs. 3 and 4.

As indicated by land survey results (Fig. 3, 4), performed from the moment of exploitation commencement of the longwall no. 698 until the exploitation commencement of the longwall 697, the measured subsidence reached steady state for longwall 698. The exploitation of longwall 698, considering its small dimensions and geological conditions (there is a 90-meter thick layer of strong sandstone over the O/N seam), led to the generation of incomplete trough, i.e. the ma-
maximum measured subsidence was 610 mm, the maximum compressive strain was -2.76 mm/m, while the maximum measured tensile strain was 1.27 mm/m.

\[ K = \frac{I_{\text{measured}}^{\text{max}}}{I_{\text{forecast}}^{\text{max}}} \]  

(10)

where:

- \( I_{\text{measured}}^{\text{max}} \), \( I_{\text{forecast}}^{\text{max}} \) — maximum value of strain indicator, measured and forecast one respectively.
When analyzing the results of terrain deformation measurements ratio to forecasts estimated before the commencement of longwall panel no. 698 excavation, one can notice that the completed exploitation resulted only in the range 47%−53% of expected deformation and dislocation values (Table 2).

**TABLE 2**

Differences in measured and forecast deformation indicators (Stocks & Sroka, 2000)

<table>
<thead>
<tr>
<th>Deformation indicator</th>
<th>Max. forecast value*</th>
<th>Max. measured value</th>
<th>Region of measurement</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value of subsidence ( w_{\text{max}} ) [mm]</td>
<td>1,300</td>
<td>610</td>
<td>Point 27</td>
<td>0.47</td>
</tr>
<tr>
<td>Maximum value of compressive strain ( \varepsilon^- ) [mm/m]</td>
<td>-5.5</td>
<td>-2.76</td>
<td>Pt. 27-Pt. 28</td>
<td>0.50</td>
</tr>
<tr>
<td>Maximum value of tensile strain ( \varepsilon^+ ) [mm/m]</td>
<td>+2.4</td>
<td>+1.27</td>
<td>Pt. 31-Pt. 32</td>
<td>0.53</td>
</tr>
</tbody>
</table>

* The forecast was completed with the following assumptions of the longwall geometrical parameters: thickness (3.9 m); longwall panel width (270 m); panel run (990 m), and parameters of Knothe theory: subsidence factor (0.9); \( \beta = 59^\circ \) (\( \tan \beta = 1.66 \)); panel fringes (30 m).

It should be noticed that after the exploitation of panel 697 the total deformations of two longwall panels corresponded to the forecast value. Therefore, in order to carry out a more accurate analysis of underground exploitation impact on horizontal displacements of terrain surface, the measurements performed until the 15th of August 2001 were utilized. At that time, within the analyzed region of the mine, the longwalls no. 698 and no. 697 in the O/N seam, with the thickness of 3.8 m and 4.0 m respectively, as well as the longwalls no. 682 and no. 683 in the P1 seam, with the thickness of 1.6 m, were worked out (Fig. 5). As indicated by the results of the measurements, only the exploitation of longwalls no. 698 and 697 led to the deformations characteristic for caving exploitation.
The exploitation of longwalls 698 and 697 was conducted within the region weakly disturbed by earlier mining operations. The mining situation within the region of these longwalls is illustrated in figure 5, where only within the R seam, located approx. 110 m over the O/N seam, gobs are located, while two longwall panels (682 and 683) in their immediate vicinity were worked out in P1 seam. The exploitation of these longwalls was conducted during the execution of land survey for 698 and 697 longwalls. The completed measurements provided the values of strain indicators presented in figures 6 and 7.

Fig. 8 presents the map of distribution of horizontal displacements maximum value $u_{\text{max}}$ ratio to measured value of vertical displacements $w$.

As the results of the analysis indicate, the minimum values of $|u_{\text{max}}|/w$ ratio for the exploitation of O/N seam 697 and 698 longwalls are reached in the central part of worked-out space, while the value of the ratio increases with distance from the edge, up to the value over unity. It means that for some distance from the exploitation field, the value of horizontal displacement is over two times higher than the value of vertical displacement.

For the purpose of $B$ coefficient analysis for the Prosper Haniel mine region, twelve computational cross sections were done, perpendicular to the 697 and 698 longwall panel overrun, each of which comprised of as many as 60 computational points (Fig. 9).

For the listed sections, a graph of horizontal displacement values on direction of $u(\alpha)$ in relation to $T(\alpha)$ slope was prepared (among others, Fig. 10).
Fig. 6. Maps of land survey results\(^1\): (a) for horizontal displacements in X direction [mm], (b) for horizontal displacements in Y direction [mm]

For the sections presented, the functional relationships between horizontal displacements and slopes were determined for linear regression method (Table 3). In the formulas presented in table 3, the values of horizontal displacements are shown in millimeters, while the slope is presented in mm/m, which means that the value of \( B \) parameter is expressed in meters.

\(^1\) Matching was done with the use of Kriging method, which assigns appropriate weights, called coefficients of kriging (weights), to samples located within estimation area (area for searching of samples) in such a way that the mean square estimation error (variation of kriging) is minimized.
Fig. 7. Map of land survey results for vertical surface displacement $w$ [mm]

Fig. 8. Distribution map of maximum horizontal displacement value to vertical displacement value ratio ($u_{\text{max}}/w$) for final measured values
In can be noticed when analyzing the results of linear regression matching for the twelve sections (Table 3) that the results of matching for the cases numbered from 1 to 5 are lower and close to $R^2 = 0.9$ value, while matching for the remaining sections is more accurate.
These conclusions were confirmed by the analysis of sum values for the sections from 1
to 6 (Fig. 11a) (up to middle of the longwall overrun), as well as for the sections from 7 to 12
(Fig. 11b).

For summary sections mentioned above, the following matches were obtained:
- for the sections 1 to 6: \( U(\alpha) = -205.9 \cdot T + 3.5 \), for match \( R^2 = 0.891 \),
- for the sections 7 to 12: \( U(\alpha) = -187.1 \cdot T - 117.1 \), for match \( R^2 = 0.952 \).
For all summary sections, i.e. from 1 to 12, the following formula was obtained:

$$U(\alpha) = -196.4 \cdot T - 57.8$$

for match $R^2 = 0.898$.

The presented values of $B$ parameters require estimation of standard deviation. Such analyses were made for sample results presented in table 3 using formula for corrected sample standard deviation:
where:

- $x_i$ — random values from finite data,
- $\bar{x}$ — the mean value of the observations,
- $n$ — size of the sample.

Taking into consideration a small number of samples, the corrected sample standard deviation was estimated as:

$$\tilde{\sigma} = \frac{S}{c_4} = 28.5$$  \hspace{1cm} (12)

$c_4 = 0.97756$ for 12 samples.

It means that the single mean value of displacement factor $B$ can be forecast with the appropriate-ness equal to 15%.

In subsequent steps, the analysis of $B$ coefficient values distribution for exemplary region of the mine was carried out for three sections parallel to the 697 and 698 longwall panel overruns (Fig. 12).
The diagrams of horizontal displacement values on direction of \( u(\alpha) \) in relation to \( T(\alpha) \) slope for selected cross section were prepared. Figure 13 presents the results for exemplary sections 2 and 3. Next analyses are shown in table 4 for three parallel cross sections. The figure 14 shows the diagram of summary values for three cross sections.

Fig. 13. Diagram of horizontal displacements on direction of \( u(\alpha) \) section in relation to \( T(\alpha) \) slope for section no. 2 (a) and 3 (b)
Formulas of linear regression matching for sections parallel to the 697 and 698 longwall overruns.

<table>
<thead>
<tr>
<th>Number of section</th>
<th>Formula of linear regression</th>
<th>Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Section no. 1</td>
<td>$U(\alpha) = -229.6 \cdot T + 152.1$</td>
<td>$R^2 = 0.948$</td>
</tr>
<tr>
<td>Cross Section no. 2</td>
<td>$U(\alpha) = -243.3 \cdot T + 144.1$</td>
<td>$R^2 = 0.979$</td>
</tr>
<tr>
<td>Cross Section no. 3</td>
<td>$U(\alpha) = -229.4 \cdot T + 70.3$</td>
<td>$R^2 = 0.951$</td>
</tr>
</tbody>
</table>

For summary sections from 1 to 3, parallel to overrun of longwall panels, the following formula of regression match was obtained:

$$U(\alpha) = -234.6 \cdot T + 121.9, \text{ for match } R^2 = 0.955.$$  

While taking into account all the completed sections, both perpendicular and parallel to longwall panel overruns, the following formula was obtained:

$$U(\alpha) = -206.5 \cdot T - 7.9, \text{ for match } R^2 = 0.886.$$  

The formulas of regression straight lines matching to the results of measurements differ from the adopted Awierszyn hypothesis by the value of free term. In order to make these values closer to the solution presented by Awierszyn, it was assumed that straight lines of regression shall intercept in the center of coordinate system. The results of such matching are shown for several exemplary summary sections (Table 5).
TABLE 5

<table>
<thead>
<tr>
<th>Summary section</th>
<th>Formula of linear regression</th>
<th>Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sections perpendicular to front face advance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 1 to 6</td>
<td>$U(\alpha) = -205.7 \cdot T$</td>
<td>$R^2 = 0.891$</td>
</tr>
<tr>
<td>From 7 to 12</td>
<td>$U(\alpha) = -192.7 \cdot T$</td>
<td>$R^2 = 0.867$</td>
</tr>
<tr>
<td>From 1 to 12</td>
<td>$U(\alpha) = -199.9 \cdot T$</td>
<td>$R^2 = 0.882$</td>
</tr>
<tr>
<td><strong>Sections parallel to front face advance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 1 to 3</td>
<td>$U(\alpha) = -229.0 \cdot T$</td>
<td>$R^2 = 0.893$</td>
</tr>
<tr>
<td><strong>Sections perpendicular and parallel to front face advance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>–</td>
<td>$U(\alpha) = -206.9 \cdot T$</td>
<td>$R^2 = 0.886$</td>
</tr>
</tbody>
</table>

3. Conclusions

The analysis of horizontal displacement $B$ coefficient, performed for selected region of Prosper Haniel mine, demonstrated the existence of proportionality between the vector of horizontal displacement and the vector of subsidence trough slope profile.

What originates from the summary sections perpendicular to the advance of exploitation front is that the coefficient of displacement equals to $B = 196$ m. The value can be presented as function of radius of influences range or depth of exploitation, and it is equal $B = 0.34r$ or $B = 0.20H$ respectively. It can be noticed when analyzing the results of a detailed section perpendicular to longwall panel overrun that the minimum values of $B$ coefficient of displacements were determined on the boundaries of exploitation, which are subsequently increasing towards the center of the selected exploitation space. The maximum value of $B = 216$ m ($0.23H$ or $0.37r$) was obtained for the section number 6, which intersects the center of the exploitation field. The lowest values of the parameter were assessed for the sections, in which the longwall started its run ($0.14H$ or $0.23r$). Deviations of matching for extreme sections may result from the fact that the period of measurement for the section having initial numbers was not long enough. The exploitation of 698 longwall panel was terminated on the 23rd of November 1999, while the 697 longwall panel ended its run on the 30th of March 2001. The last land survey carried out on the surface took place on the 14th and 15th of August 2001, i.e. less than five months after the last longwall panel have finished its run. This fact might have unfavorably influenced the results of the analyses for the section within the region of exploitation termination. Furthermore it should be pointed out that the sections of initial numbers are located close to the exploitation operations being led in the adjacent P seam.

The value of horizontal displacement coefficient, obtained for the summary sections parallel to longwall overrun, amounts to $B = 234$ m. The value can be presented as the function of radius of main influences distribution, and is equal to: $B = 0.40r$ or as the function of exploitation depth: $B = 0.24H$. Nevertheless, while analyzing the individual sections, it can be noticed that the $B$ value, both for the section along the 698 longwall overrun and also along the 697 longwall panel, amounts to $B = 229$ m (that is $0.40r$ or $0.24H$). At the same time, the value of displacement coefficient obtained for the section going along the edge common for both of the longwall panels equals to $B = 243$ m, which amounts to $0.42r$ or $0.25H$. 
To recapitulate, it can be stated that the best match has been obtained for the sections intersecting the center of the exploitation field, both for the section perpendicular and parallel to the overrun of longwall panel front. It can be related to irregular arrangement of the exploitation boundaries of the 698 and 697 longwall panels. It was also noticed that the B coefficient of displacement assumed higher values for the section parallel to the front advance than for the perpendicular sections. If such a conclusion is confirmed in further analyses undertaken for other mines, it will mean that the value of horizontal displacement depends also on the shape of selected space and the direction of longwall panel run.

The project was financed from the means of National Science Center granted on the grounds of decision No. DEC – 2011/01/D/ST8/07280

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Received: 24 June 2013