OPTIMIZATION OF UNIT LOAD FORMATION TAKING INTO ACCOUNT THE MASS OF PACKAGING UNITS

Kamil Popiela¹, Mariusz Wasiak²
Warsaw University of Technology, Faculty of Transport, Division of Logistics and Transport Systems
¹e-mail: kamilpopiela@o2.pl,
²e-mail: mwa@wt.pw.edu.pl,

Abstract: This article presents a mathematical formulation of the optimization problem of loading unit formation taking into account the mass of packaging units. Proposed model can be applied to optimize the arrangement of non-uniform cubical loading units in loading spaces. The model ensures possibility of defining various dimensions, masses, resistances of particular packaging units and their vertical axis rotation. Within the constraints of formulating optimization problem, taking into account masses and resistances ensures that all packaging units will rest on a pallet or on other packaging units, and the surface of contact between loading units guarantees stability of units arranged in subsequent layers. The mathematical model was verified. The paper provides an appropriate calculation example.

Key words: optimization, unit loads, formation of loading units

1. Introduction

Forming loading units is one of the most important elements of logistical systems. It has an effect on both the performance and the cost of logistical processes [10]. An appropriate arrangement of packaging units on auxiliary loading equipment (e.g. on a pallet) guarantees the best possible use of available loading space. Additionally, the formed unit loads should be arranged so that packaging units are safe from damage sustained under the weight of other units during transport. This implies that the appropriate arrangement of packaging units should ensure maximizing the use of unit load dimensions, taking into account their mechanical resistance.

The problem of loading unit formation occurs primarily in manufacturing companies and logistics. With respect to manufacturing companies, unit loading formation is an issue which has a one-time solution for each specific type of manufactured product. Often, the individual units of one product type are placed in protective packaging. This becomes the packaging unit. When we combine these units with auxiliary loading units, they are formed into uniform loading units. Taking into account the resistance and mass obtained by the expert or optimization methods, a single solution to the packaging unit arrangement problem has a long term application. In logistics companies, the problem of forming unit loads is more complex. Here, unit loads are formed from heterogeneous packages and in terms of resistance and mass. A one-time solution cannot be applied to forming many loading units (because each of them is different) in this situation. As a result, logistics companies are required to obtain real-time solutions.

Many optimization models of loading unit formation have been described in literature[2], [5], [6], [7], [12], [15], [16]. However, these models have been formulated by taking into account various simplified assumptions. This limits the application of existing models to specific cases of this problem. Also, most of them do not take into consideration the mass and resistance of packaging units [6], [7], [11], [14]. On the other hand, in tasks which include the mass and resistance of packaging units, the contact requirement of bearing surfaces and the possibility of rotating packaging about the vertical axis units is overlooked[1], [2], [4], [8], [15], [16]. With this in mind, the mathematical formulation of the optimization problem of forming loading units was developed by considering the mass, resistance and the possibility of rotating the packaging units about the vertical axis.
2. Review of Literature

In literature, the problem of unit load formation is quite extensively described and include formulation of models in one, two and three dimensions. One of the simplest optimization models of forming unit loads is proposed by T. Tlili, S. Faiz and S. Krichen [15]. In this model, the number of containers used to store objects is minimized. Additionally, in the mathematical formulation of the problem it is assumed that each object is characterized by volume and mass, each container has a predetermined maximum capacity, each object can be placed only in one container and must fit in it, container capacity cannot be exceeded by the located cargo, and the container may be stored in one container slot, whose loading area volume may not be exceeded. Considering an object’s location in a container is based on the criterion of capacity (not dimensions), it would be questionable if an object could fit in a container under actual conditions. This model allows only an initial estimate of the number of bins needed to arrange the cargo. Applying this tool in actual conditions carries an inevitable risk of acquiring an insufficient number of containers used to form unit loads.

A similar problem was formulated by M. Mongeau and Ch. Bes [8]. In the case of a two-dimensional loading unit formation problem, an interesting option is a mathematical model described by A. Savić, T. Sukilović and V. Filipović [14]. They modeled the formation of a loading unit as a arrangement problem of smaller rectangles in one larger rectangle. In this model, the possibility of rotating the rectangles was not considered. In the constraints of the model, it was provided that small rectangles are placed parallel to the walls and do not overlap or go beyond the outline of the large rectangle. Lacking the possibility of rotation and rectangle stacking, along with the relatively long optimization problem solving time for just six rectangles makes this an inefficient tool for the process of forming loading units.


Examining the problematic aspects of forming three-dimensional loading units, it should be mentioned that one of the first formulations of an optimization problem was suggested by S.C. Chen, S.M. Lee and Q. Shen [3]. The authors tackled the container loading problem, which is based on determining the positions of packaging units in a set number of containers. In this model, containers and packaging units were taken into account, which are characterized by various sizes, where the longest dimension is the length of packaging unit, the middle dimension is its width and the shortest dimension is height. In formulating the optimization problem, the continuous and binary variables are taken into account. The coordinate location of each packaging unit in a given container was described by discrete variables yielding a real and positive value. However, the locations of individual packaging units were mapped with the relative position of packaging unit pairs. To summarize, in the optimization problem described in [3] it was provided that there will be no overlap of packaging units, as well as the possibility of rotating them on the three axis. Still, the lack of stacking requirements cause the obtained solutions to not always have bearing surfaces of packaging units supported.

M. Hifi, I. Kacem, S. Negre and L. Wu [5] proposed an analogous model. In this model, the authors developed cuboidal packaging units which are placed in cubic containers which have an identical shape, where coordinates of the lower left-hand corner provide a description of its location in the space of the container. A similar mathematical model of forming loading units was presented by A. Ratkiewicz [13]. Each of the analyzed mathematical models of forming loading units is characterized by certain simplifications, which are lack of rotating packaging units, lack of contact requirement between bearing surfaces, and finally, omitting mass and resistance of packaging units. Taking this into consideration, the need for formulating an optimization problem which contains all of these elements was identified.
3. Proposal of mathematical model of three-dimensional loading unit formation

3.1. General Assumption of the Model

The model proposed in this article is based on the model[12], which considers the requirement of contact between bearing surfaces as well as rotation of packaging units about the vertical axis. This model also has additional constraints with regard to packaging unit resistance and loading of auxiliary loading equipment to assist with the formation of loading units.

The goal of formulating a three-dimensional optimization problem of loading units consists of the following assumptions:

- Packaging units have a cuboidal shape,
- Each of the packaging units is able to be rotated 90° about a vertical axis (Fig. 1), however, all units must be in a vertical position at all times.
- The mass, resistance, and dimension of each packaging unit may vary.
- The mass of each packaging unit is in a geometrical center.

Fig. 1. Graphical illustration of packaging unit arrangement with the possibility of rotation about the vertical axis.

3.2. Formulation of Optimization Problem

For data including:

- set of packaging unit numbers $JO = \{1, \ldots, i, \ldots, k, \ldots, n\}$, where: $n$ is the amount of packaging units, but $i$ and $k$ are number of packaging units,
- length of packaging units $p_i$, $i \in JO$,
- width of packaging units $q_i$, $i \in JO$,
- height of packaging units $r_i$, $i \in JO$,
- mass of packaging units $m_i$, $i \in JO$,
- permissible pressure on the bearing surfaces of packaging units $dm_i$, $i \in JO$,
- length of unit load $L$,
- width of unit load $W$,
- maximum height of unit $H$,
- height of auxiliary loading equipment to assist with the formation of loading units $rp$,
- permissible loading of auxiliary loading equipment to assist with the formation of loading units $B$,
- the values of the following variables should be assigned:
  - $t_i \in \{0, 1\}$ for $i \in JO$, where the binary variable $t_i$ accepts the value of 1 when the $i$-th packaging unit is placed in a loading unit, otherwise $t_i = 0$,
  - $x_i \in \mathbb{R}^+$ for $i \in JO$ is the coordinate of the left wall of the $i$-th packaging unit on the X axis,
  - $y_i \in \mathbb{R}^+$ for $i \in JO$ is the coordinate of the front wall of the $i$-th packaging unit on the Y axis,
  - $z_i \in \mathbb{R}^+$ for $i \in JO$ is the coordinate of the bottom wall of the $i$-th packaging unit on the Z axis,
  - $s_i \in \{0, 1\}$ for $i \in JO$, where the binary variable $s_i$ accepts the value of 1, when the $i$-th packaging unit is placed in a loading unit lengthwise along the X axis, otherwise $s_i = 0$,
  - $lx_i \in \mathbb{R}^+$ for $i \in JO$ is the dimension of the $i$-th packaging unit along the X axis,
  - $ly_i \in \mathbb{R}^+$ for $i \in JO$ is the dimension of the $i$-th packaging unit along the Y axis,
  - $tt_{ik} \in \{0, 1\}$ for $i \in JO$, where the binary variable $tt_{ik}$ accepts the value of 1, when the $i$-th or $k$-th packaging unit is placed in a loading unit, otherwise $tt_{ik} = 0$,
  - $a_{lk}^{lp} \in \{0, 1\}$ for $i, k \in JO$: $i \neq k$, where the binary variable $a_{lk}^{lp}$ accepts the value of 1, where the $i$-th packaging unit is on the left side of the $k$-th packaging unit, otherwise $a_{lk}^{lp} = 0$,
  - $a_{lk}^{pz} \in \{0, 1\}$ for $i, k \in JO$: $i \neq k$, where the binary variable $a_{lk}^{pz}$ accepts the value of 1,
where $i$-th packaging unit is in front of the $k$-th packaging unit, otherwise $a_{ik}^p = 0$,

- $a_{ik}^w \in \{0,1\}$ for $i, k \in JO$: $i \neq k$, where the binary variable $a_{ik}^w$ accepts the value of 1, where the $i$-th packaging unit is below the $k$-th packaging unit, otherwise $a_{ik}^w = 0$,

- $a_{ik}^s \in \{0,1\}$ for $i, k \in JO$: $i \neq k$, where the binary variable $a_{ik}^s$ accepts the value of 1, when the $i$-th packaging unit is underneath the $k$-th packaging unit, otherwise $a_{ik}^s = 0$,

- $a_{ik}^{st} \in \{0,1\}$ for $k \in JO$, where the binary variable $a_{ik}^{st}$ accepts the value of 1, where $k$-th packaging unit is directly located on the palette, otherwise $a_{ik}^{st} = 0$,

- $a_{ik}^{sp} \in \{0,1\}$ for $i, k \in JO$: $i \neq k$, where the binary variable $a_{ik}^{sp}$ accepts the value of 1, where the $i$-th packaging unit is placed on the $k$-th packaging unit, otherwise $a_{ik}^{sp} = 0$,

so that all the constraints are met:

- on the dimension of packaging units along the X axis:
  \[
  \forall i \in JO \quad l_x = p_i \cdot s_i + q_i (1-s_i)
  \] (1)

- on the dimension of packaging units along the Y axis:
  \[
  \forall i \in JO \quad l_y = q_i + p_i - l_x
  \] (2)

- on relative placement of packaging units along the X axis:
  \[
  \forall i, k \in JO: i \neq k \quad x_k \geq (x_i + l_x) \cdot a_{ik}^{lp}
  \] (3)

- on relative placement of packaging units along the Y axis:
  \[
  \forall i, k \in JO: i \neq k \quad y_k \geq (y_i + l_y) \cdot a_{ik}^{np}
  \] (4)

- on relative placement of packaging units along the Z axis:
  \[
  \forall i, k \in JO: i \neq k \quad z_k \geq (z_i + r_i) \cdot a_{ik}^{wn}
  \] (5)

- in the presence of a pair of packaging units in a unit load:
  \[
  \forall i, k \in JO: i \neq k \quad t_{tk} = t_i \cdot t_k
  \] (6)

- on not overlapping packaging units in a loading unit:
  \[
  \forall i, k \in JO: i \neq k \quad a_{ik}^{lp} + a_{ik}^{p} + a_{ik}^{np} + a_{ik}^{pn} + a_{ik}^{wn} + a_{ik}^{wn} \geq t_{tk}
  \] (7)

- on the possibility of placing packaging units relative to each other:
  \[
  \forall i, k \in JO: i \neq k
  a_{ik}^{lp} + a_{ik}^{p} + a_{ik}^{np} + a_{ik}^{wn} + a_{ik}^{wn} \leq 4 \cdot t_{tk}
  \] (8)

- on the possibility of contact between bearing surfaces for each pair of packaging units:
  \[
  \forall i, k \in JO: i \neq k \quad a_{ik}^{st} \leq a_{ik}^{sp}
  \] (9)

- on the possibility of contact between the bearing surfaces of the packaging unit and auxiliary loading equipment:
  \[
  \forall k \in JO \quad a_{ik}^{sp} \leq t_k
  \] (10)

- on ensuring contact between bearing surfaces for each pair of packaging units:
  \[
  \forall i, k \in JO: i \neq k \quad a_{ik}^{st} \cdot (z_i + r_i - z_k) = 0
  \] (11)

- on ensuring contact between bearing surfaces of the packaging unit and auxiliary loading equipment:
  \[
  \forall k \in JO \quad a_{ik}^{sp} \cdot (rp - z_k) = 0
  \] (12)

- on ensuring placement of each packaging unit on top of other packaging units or directly on the auxiliary loading equipment:
  \[
  \forall k \in JO \quad a_{ik}^{sp} + \sum_{i \in JO: i \neq k} a_{ik}^{st} \geq t_k
  \] (13)

- on situating packaging units above others along the X axis (coefficient of 0.5 stands for the geometrical center of the packaging unit edge):
\[\forall i, k \in \mathbf{JO}: i \neq k \quad x_i \cdot a_{ik}^{x} \leq x_k + 0.5 \cdot l x_k \quad (14)\]
\[\forall i, k \in \mathbf{JO}: i \neq k \quad (x_k + 0.5 \cdot l x_k) \cdot a_{ik}^{x} \leq x_i + l x_i \quad (15)\]

- on situating packaging units above others along the Y axis (coefficient of 0.5 stands for the geometrical center of the packaging unit edge):
\[\forall i, k \in \mathbf{JO}: i \neq k \quad y_i \cdot a_{ik}^{y} \leq y_k + 0.5 \cdot l y_k \quad (16)\]
\[\forall i, k \in \mathbf{JO}: i \neq k \quad (y_k + 0.5 \cdot l y_k) \cdot a_{ik}^{y} \leq y_i + l y_i \quad (17)\]

- on situating packaging units above others along the Z axis:
\[\forall i, k \in \mathbf{JO}: i \neq k \quad a_{ik}^{z} \cdot (z_i + r_i) \leq z_k \quad (18)\]

- on ensuring the packaging unit will not go beyond the dimensions of the auxiliary loading equipment along the X axis:
\[\forall i \in \mathbf{JO} \quad x_i + l x_i \leq L \quad (19)\]
\[\forall i \in \mathbf{JO} \quad x_i \geq 0 \quad (20)\]

- on ensuring the packaging unit will not go beyond the dimensions of the auxiliary loading equipment along the Y axis:
\[\forall i \in \mathbf{JO} \quad y_i + l y_i \leq W \quad (21)\]
\[\forall i \in \mathbf{JO} \quad y_i \geq 0 \quad (22)\]

- on not exceeding the maximum height of the loading unit:
\[\forall i \in \mathbf{JO} \quad z_i + r_i \leq H \quad (23)\]

- on ensuring the situation of packaging units above the surface of auxiliary loading equipment:
\[\forall i \in \mathbf{JO} \quad z_i \geq r p \quad (24)\]

- on securing packaging units from damage:
\[\forall i \in \mathbf{JO} \quad \sum_{k \in \mathbf{JO}} a_{ik}^{z} \cdot m_k \leq d m_i \quad (25)\]

- on securing against damage from the auxiliary loading equipment:
\[\sum_{i \in \mathbf{JO}} l_i \cdot m_i \leq B \quad (26)\]

and so that the criterion function with the interpretation of filling the loading unit:
\[\sum_{i \in \mathbf{JO}} p_i \cdot q_i \cdot r_i \cdot l_i \longrightarrow \max \quad (27)\]
accepts a maximum value.

### 3.3. Sample Calculation

The model of three-dimensional loading unit formation described in the previous section was implemented in LINGO. Next, the goal was to assess its usefulness, verify correctness and solve many sample calculations. The results obtained referred to solutions of analytical problem. In the article, one of these samples was described. The problem of placing 8 packaging units on a euro pallet with the dimensions of \( L = 1200 \text{ mm} \), \( W = 800 \text{ mm} \), \( r p = 144 \text{ mm} \) was solved. The maximum height of the pallet loading unit \( H \) was set at an even 1344 mm and the permissible pallet loading capacity \( B \) was set at 1000 kg. The parameters of packaging units are listed in Tab. 1.

The values of variables obtained as a result of the solution of the described problem with the help of implemented optimization formulation of loading units are show in Tab.2 (Some variables which results were equal to zero were omitted).

The criterion function for the obtained solution has the following result:
\[
\begin{align*}
900 \cdot 400 & = 360000
+ 900 \cdot 400 & = 360000
+ 900 \cdot 400 & = 360000
+ 500 \cdot 500 & = 250000
+ 900 \cdot 400 & = 360000
+ 500 \cdot 500 & = 250000
+ 400 \cdot 400 & = 160000
+ 450 \cdot 450 & = 202500
+ 200 \cdot 200 & = 40000
= 0.7416 \cdot 10^9 \text{ mm}^3
\end{align*}
\]

In the examined sample calculation, 4 out of 8 packaging units were placed on the pallet (Fig. 2). Additionally, the mass of the placed packaging units did not exceed the permissible pressure of the pallet and the packaging units would not be damaged due to stacking.
Optimization of unit load formation taking into account the mass of packaging units

On the basis of the obtained solution, it is possible to reflect the visualization of an optimal pallet loading unit. It is proof that for solutions obtained with the help of the developed tool, the pallet along with placed packaging units creates an integral whole. Solutions to problems are possible thanks to a logical physical interpretation.

Tab. 1. Characteristics of Packaging Units

<table>
<thead>
<tr>
<th>Number of Packaging Units ((i))</th>
<th>Length of Packaging Unit in mm ((p_i))</th>
<th>Width of Packaging Unit in mm ((q_i))</th>
<th>Height of Packaging Unit in mm ((r_i))</th>
<th>Mass of Packaging Unit in kg ((m_i))</th>
<th>Permissible Pressure on the Bearing Surfaces of Packaging Units in kg ((d m_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
<td>400</td>
<td>400</td>
<td>25</td>
<td>1200</td>
</tr>
<tr>
<td>2</td>
<td>900</td>
<td>400</td>
<td>600</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>900</td>
<td>400</td>
<td>560</td>
<td>150</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>900</td>
<td>400</td>
<td>500</td>
<td>390</td>
<td>400</td>
</tr>
<tr>
<td>6</td>
<td>1100</td>
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<td>7</td>
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<td>100</td>
</tr>
<tr>
<td>8</td>
<td>450</td>
<td>450</td>
<td>200</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Tab. 2. Results of Solving the Optimization Problem of Palet Loading Unit Formation

<table>
<thead>
<tr>
<th>Variable ((x))</th>
<th>Value</th>
<th>Variable ((t))</th>
<th>Value</th>
<th>Variable ((a))</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
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<td>0 mm</td>
<td>(t_5)</td>
<td>1</td>
<td>(p_z) ((a_{12}^p))</td>
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</tr>
<tr>
<td>(x_2)</td>
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<td>(t_6)</td>
<td>0</td>
<td>(p_z) ((a_{13}^p))</td>
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<tr>
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<td>0 mm</td>
<td>(t_7)</td>
<td>0</td>
<td>(p_z) ((a_{23}^p))</td>
<td>1</td>
</tr>
<tr>
<td>(x_4)</td>
<td>0 mm</td>
<td>(t_8)</td>
<td>0</td>
<td>(p_z) ((a_{23}^p))</td>
<td>1</td>
</tr>
<tr>
<td>(x_5)</td>
<td>0 mm</td>
<td>(l x_1)</td>
<td>900 mm</td>
<td>(p_x) ((a_{23}^p))</td>
<td>1</td>
</tr>
<tr>
<td>(x_6)</td>
<td>1200 mm</td>
<td>(l x_2)</td>
<td>900 mm</td>
<td>(w_n) ((a_{23}^w))</td>
<td>1</td>
</tr>
<tr>
<td>(x_7)</td>
<td>0 mm</td>
<td>(l x_3)</td>
<td>900 mm</td>
<td>(w_n) ((a_{11}^w))</td>
<td>1</td>
</tr>
<tr>
<td>(y_1)</td>
<td>0 mm</td>
<td>(l x_5)</td>
<td>900 mm</td>
<td>(l r) ((a_{12}^l))</td>
<td>1</td>
</tr>
<tr>
<td>(y_2)</td>
<td>400 mm</td>
<td>(l x_6)</td>
<td>1100 mm</td>
<td>(s_{11}^{sp})</td>
<td>1</td>
</tr>
<tr>
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<td>400 mm</td>
<td>(l x_7)</td>
<td>400 mm</td>
<td>(s_{12}^{sp})</td>
<td>1</td>
</tr>
<tr>
<td>(y_4)</td>
<td>0 mm</td>
<td>(l x_8)</td>
<td>450 mm</td>
<td>(s_{23}^{sp})</td>
<td>1</td>
</tr>
<tr>
<td>(y_5)</td>
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<td>(l y_1)</td>
<td>400 mm</td>
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<tr>
<td>(y_6)</td>
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<td>(l y_2)</td>
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<table>
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<td>$t l_{31}$</td>
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</tr>
<tr>
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<td>144 mm</td>
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<td>0</td>
<td>$t l_{32}$</td>
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</tr>
<tr>
<td>$z_6$</td>
<td>0 mm</td>
<td>$s_5$</td>
<td>1</td>
<td>$t l_{35}$</td>
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</tr>
<tr>
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<td>$s_6$</td>
<td>1</td>
<td>$t l_{51}$</td>
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</tr>
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<td>0</td>
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<td>$t 1$</td>
<td>1</td>
<td>$t 4$</td>
<td>0</td>
</tr>
<tr>
<td>$s_3$</td>
<td>1</td>
<td>$t 2$</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4. Conclusion
The mathematical models of forming loading units described in literature are characterized by certain simplifications. These simplifications include the inability to rotate packaging units about a vertical axis, lack of constraints regarding contact between the bearing surfaces, lack of integrating mass as well as resistance of the packaging loads. Consequently, applying these kinds of models to planning actual loading units does not always allow us to obtain an optimal solution which has the correct physical interpretation.

The optimization problem presented in this article is free from flaws present in the optimization models in the literature. The obtained solutions to the optimization problem of forming loading units allows for detailed projection of the optimal placement of packaging units on a pallet or in a container. Most importantly, resistance and mass of individual packaging units are taken into account. The quality of the solution and tests performed for other sample calculations allow us to assess the developed formulation concerning the mathematical problem of forming unit loads and its numerical implementation as valid. It should, however, be noticed that the accepted assumption which simplify the center of mass of packaging units in their geometrical center in the case of large packaging units placed on top of many smaller packaging units may introduce a certain error. In order to eliminate this described imperfection, the optimization problem should be modified, accepting the assumption that the mass of each packaging unit is distributed equally on the entire base surface of the packaging unit.
References


