Modeling of security and risk of bankruptcy of enterprise

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Abstract
The management problem is especially important due to the risk of enterprise bankruptcy under a market economy. Management by a business risk and to economic stability and safety of enterprise. Now, different methods are used in order that to prevent enterprises’ bankruptcy on the basis of prognoses and safety of risk methods, among from that, maybe, would be marked the following to those: method of DuPont, model of Altman, model of Olson, statistical methods well-known as the “Winnings” methods. The practical use of these models gave us a high result in case of small and middle enterprise. In every model the used discoveries are undertaken from the documents of the official financial reporting.

The article analyzes mathematical methods and algorithms used to evaluate financial stability of an enterprise, such as the Altman Z-Score model. The models are designed to evaluate enterprise bankruptcy risks. The central concept is evaluation of the enterprise bankruptcy risk and presentation of an automated risk calculation system.

Introduction

In order to evaluate the enterprise bankruptcy risk it is needed to use logical probability modeling to check validity of the hypothesis regarding the structure and nature of the integrated enterprise financial status indicator, i.e. to select relevant economic indicators and prove the possibility to evaluate financial status of an enterprise using these:

1. Calculate reliability / unreliability of each indicator and combination thereof in case of possible financial statuses of an enterprise based on the logical relations between the selected economic indicators.
2. Show the impact of the weighted share of each indicator and a combination thereof has on the anticipated financial status.
3. Calculate the risk of financial instability and develop measures for its elimination.
4. Use the modeling approach to select the optimal scenario for financial development of the enterprise and use such scenario to monitor financial stability of the organization.
5. Use the actual data to evaluate financial stability and forecast the bankruptcy risk.

Now, different methods are used in order that to prevent enterprises’ bankruptcy on the basis of prognoses and safety of risk methods, among from that, maybe, would be marked the following to those: method of DuPont, model of Altman, model of Olson, statistical methods well-known as the “Winnings” methods. The practical use of these models gave us a high result in case of small and middle enterprise. In every model the used discoveries are undertaken from the documents of the official financial reporting.

Contents

Enterprise bankruptcy risk is evaluated using Altman 5-factor model. The indicator calculated using the Altman model above is:

\[ Z = 1.2K_1 + 1.4K_2 + 3.3K_3 + 0.6K_4 + K_5 \]  \hspace{1cm} (1)

where:
\[ K_1 = \text{Working Capital / Total Assets}; \]
\[ K_2 = \text{Retained Earnings / Total Assets}; \]
Altsman model defines probability of bankruptcy using the following scale (Table 1).

Table 1. Probability of bankruptcy

<table>
<thead>
<tr>
<th>Z</th>
<th>Probability of bankruptcy</th>
<th>Z</th>
<th>Probability of bankruptcy</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1.8</td>
<td>Very high</td>
<td>2.71 &lt; Z &lt; 2.99</td>
<td>Medium</td>
</tr>
<tr>
<td>1.8 &lt; Z &lt; 2.7</td>
<td>high</td>
<td>Z &gt; 3</td>
<td>Low</td>
</tr>
</tbody>
</table>

In authors’ opinion at prognostication bankruptcy effectively calculation of Z - score by means of logic – probability model should be used.

As already mentioned above, a logical probability model is developed using 4 stages:
1. Definition of the question, development of a structural design of the system.
2. Definition of the logical function.
3. Definition of the polynomial of the probability functional.
4. Calculation of probabilities for the system indicators.

1. Development of the structural design

The Altman model presents evaluation of probability of bankruptcy in the form of a system comprising 5 indicators. Relations between these indicators can be schematically represented as shown in figure 1.

2. Definition of the logical function

Let us assume the logical function for an operational status of the system that is represented in the shortest manner:

\[ Z(5) = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_4 & x_5 \\ x_1 & x_4 & x_5 \end{vmatrix} \]  

(2)

Define Bull difference by \( X_i \) argument. As a result of transformations we will receive the value of:

\[ X_i \text{ is higher than its allowed value } - R_i \]

\[ X_i \text{ is lower than its allowed value } - Q_i \]

\[ \Delta \]  

\[ \Delta_{x_i} Z(5) = \begin{vmatrix} K_1 & K_2 & K_3 \\ \bar{K}_1 & \bar{K}_2 & \bar{K}_3 \end{vmatrix} \]  

(4)

Where:

\[ \bar{K}_i = \begin{vmatrix} x_2 & \bar{x}_3 \\ x_3 & \bar{x}_4 \end{vmatrix} \]  

(5)

\[ \bar{K}_2 = \begin{vmatrix} x_2 & \bar{x}_4 \\ x_3 & \bar{x}_5 \end{vmatrix} \]  

(6)

\[ \bar{K}_3 = x_2 \bar{x}_3 \bar{x}_4 \bar{x}_5 \]  

(7)

\[ \Delta_{x_i} Z(5) = \begin{vmatrix} K_1 & K_2 & K_3 \\ \bar{K}_1 & \bar{K}_2 & \bar{K}_3 \end{vmatrix} = \begin{vmatrix} \bar{x}_2 & \bar{x}_3 & \bar{x}_4 \bar{x}_5 \\ x_2 & x_3 & x_4 \end{vmatrix} \]  

(8)

\[ P(\Delta_{x_i} Z(5) = 1) = Q_1, R_i + Q_2, R_i, R_i + R_i, Q_2, Q_5 \]  

(9)

In accordance with the logical probability method it is a necessary and sufficient condition that: \( R_i = Q_i, 0.5 \), \( i = 1, 5 \).

According the calculation, the weight of \( \forall X_i \) will be \( g_{x_i} = 0.375 \). This results from the Altman Model. According to Altman model any economic indicator in the model is important for:

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\[ Z(XS) = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \\ x_1 & x_4 & x_3 \\ x_2 & x_3 & x_4 \end{bmatrix} \] (10)

Let’s transfer the equation into the Orthogonal-normal dysfunctional function:

\[ Z(XS) = \begin{bmatrix} x_1 & x_3 \\ x_2 & \bar{x}_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \\ \bar{x}_1 & x_2 & \bar{x}_3 & x_4 & x_5 \end{bmatrix} \] (11)

\[ R_c = R_1X_5 + R_2(1 – R_1)X_4 + (1 – R_1)R_2X_2 + R_3 = R_1X_5 + R_2(1 – R_1)X_4 + (1 – R_1)R_2X_2 + R_3 \] (12)

3. Definition of the probability polynomial

The transformations will yield the polynomial:

\[ R = 2R^2 + 2R^3 - 5R^4 + 2R^5 \] (13)

4. Calculation of the probabilities for system indicators

It presents: 1) share of each economic indicator \( g_i \) in the financial stability of an enterprise; 2) value of the economic indicator in defining reliability of the system.

Table 2. Calculation of the probabilities for system indicators

<table>
<thead>
<tr>
<th>( X_i )</th>
<th>( R_i )</th>
<th>( Q_i )</th>
<th>( g_{i0} )</th>
<th>( \frac{\partial R_i}{\partial R} )</th>
<th>( B_{x_i} )</th>
<th>( Y_{x_i} )</th>
<th>( Range )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>0.90</td>
<td>0.10</td>
<td>0.375</td>
<td>0.1967</td>
<td>0.2500</td>
<td>0.7500</td>
<td>0.0800</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.80</td>
<td>0.20</td>
<td>0.375</td>
<td>0.2030</td>
<td>0.3000</td>
<td>0.7000</td>
<td>0.1000</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.70</td>
<td>0.30</td>
<td>0.375</td>
<td>0.35469</td>
<td>0.0500</td>
<td>0.9500</td>
<td>0.1500</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>0.95</td>
<td>0.05</td>
<td>0.375</td>
<td>0.2940</td>
<td>0.3400</td>
<td>0.6600</td>
<td>0.0400</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>0.99</td>
<td>0.01</td>
<td>0.125</td>
<td>0.0541</td>
<td>0.0559</td>
<td>0.0054</td>
<td>0.0554</td>
</tr>
</tbody>
</table>

Where: \( R_i \) – probability of reliability for \( X_i \) indicator; \( Q_i \) – probability of unreliability for \( X_i \) indicator; \( B_{x_i} \) – role of \( X_i \) indicator in reliability of the system (the weighted share of the indicator in the improvement of system reliability); \( Y_{x_i} \) – role of \( X_i \) indicator in the worsening of system reliability.

The \( x_i \) share calculated above (Table 2) enables us to study reliability of the system when initial probabilities regarding reliability / unreliability of the system are not available.

The weight of an element is calculated as:

\[ g_{x_i} = n^{(i)}_{1} - n^{(i)}_{0} \] (14)

The bigger the weighted share of the \( x_i \) element, the higher its role in the probability of system reliability.

It can also calculate the weighted share for pairs of elements when \( R_i = Q_i = 0.5, \ i = 1, 5 \).

The highest uncertainty of the element status, i.e. dispersion is equal to 0.25. Table 3 shows that the pair \( x_2X_3 \) is the most uncertain.

It follows from our calculations that the value of \( x_4 \) has a higher share compared to other indicators in the evaluation of enterprise bankruptcy risk. In the Altman model there is:

\[ X_4 = \text{Chartered capital + additional capital assets} \]

Meanwhile, we are dealing with a paradox with regard to the pairs that the worst and most unreliable pair is the combination of \( x_2X_3 \), which, in our opinion, requires additional research.

Table 3. Weight of pairs

<table>
<thead>
<tr>
<th>Pairs</th>
<th>( R^{(i)}_{ij} )</th>
<th>( B_{x_iX_j} )</th>
<th>( R^{(ii)}_{ij} )</th>
<th>( Y_{x_iX_j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1, x_2 )</td>
<td>0.00000001</td>
<td>0.9650000</td>
<td>0.00000001</td>
<td>0.202041</td>
</tr>
<tr>
<td>( x_3, x_4 )</td>
<td>0.76000000</td>
<td>0.2050000</td>
<td>1.00000000</td>
<td>0.015241</td>
</tr>
<tr>
<td>( x_5, x_4 )</td>
<td>0.55440000</td>
<td>0.41040000</td>
<td>0.99940000</td>
<td>0.034641</td>
</tr>
<tr>
<td>( x_1, x_5 )</td>
<td>0.76000000</td>
<td>0.20480000</td>
<td>0.98500000</td>
<td>0.020241</td>
</tr>
<tr>
<td>( x_2, x_5 )</td>
<td>0.84600000</td>
<td>0.84600000</td>
<td>0.99995000</td>
<td>0.352410</td>
</tr>
<tr>
<td>( x_3, x_5 )</td>
<td>0.63000000</td>
<td>0.33480000</td>
<td>1.00000000</td>
<td>0.015241</td>
</tr>
<tr>
<td>( x_2, x_3 )</td>
<td>0.63000000</td>
<td>0.33480000</td>
<td>0.98000000</td>
<td>0.020241</td>
</tr>
<tr>
<td>( x_3, x_5 )</td>
<td>0.76000000</td>
<td>0.20480000</td>
<td>0.98000000</td>
<td>0.035241</td>
</tr>
<tr>
<td>( x_4, x_5 )</td>
<td>0.63000000</td>
<td>0.33480000</td>
<td>0.98000000</td>
<td>0.035190</td>
</tr>
</tbody>
</table>

Field the receipt of such results is needed calculations a few different scenarios. On it we created the mini automated program for the calculation of risk of bankruptcy, in the algorithm of that there is a logical probability method. The enterprise bankruptcy risk evaluation package is a part of a program designed to financially analyze an enterprise. We have not aimed at analyzing algorithms used to calculate indicators of financial analysis. They are defined by the ratio method, using the available algorithms of direct calculation. Therefore, our attention has been focused on the structure, operational modes and specificities of the financial evaluation and forecast package that uses the logical probability method. The program is divided into below described stages.

Tasks 1–4 are realized at the financial planning stage, and the last two tasks are solved based on the actual data and therefore, they are realized in the real time. The first three are accessory, and yet, necessary tasks. Their solutions are used to realize
Task 4 (Modeling Financial Status of an Enterprise) of the primary research objective. Results of the task allow us to develop possible development strategies for the enterprise and evaluate them using definite criteria, as well as plan measures to financially strengthen the enterprise. They also make it possible to check validity of various hypotheses regarding financial development of the enterprise, i.e. show its possible financial statuses under various financial and economic scenarios. Outcomes of tasks 5 and 6 will help us evaluate and forecast financial status of the enterprise using the actual values; while task 7 aims at combining the estimated values using actual values with the values we selected at the modeling stage. This will enable us to develop a program of measures to approximate financial situation with the scenario selected at the modeling stage (Fig. 2).

**Fig. 2. The general structure of the program package**

In order to solve the task “Calculation of probabilities of events for allowed values of each economic indicator covered by the complex financial stability evaluation indicator” it is necessary to solve the following tasks (Fig. 3).

Probability of indicator reliability (acceptance of allowed values) and unreliable (values less than the allowed values) values is calculated. These data are used to calculated the weighted impact of an indicator on system stability. According to the algorithm, possible indicator combinations are calculated for system indicators. These are further used to select combinations that are logically allowed (Fig. 4).

**Fig. 3. Program package for calculation of probabilities of events for allowed values of each economic indicator covered by the complex financial stability evaluation indicator**

**Fig. 4. Calculation of scales of influence of factors**

Structure of the program package calculating impact of allowed value probabilities for each economic indicator.

Results of the calculations allows us to define conditions needed for a system to operate in a stable manner. To this end, shortest ways for a financially stable operation of the enterprise are calculated and orthogonalization is used to choose one shortest way from the multitude of the selected ways (Fig. 5).

**Fig. 5. Program package evaluating financially stable operation mode for an enterprise**
Detailed structure of the program package evaluating financial stability of an enterprise based on logical probability model may be represented in the following manner (Fig. 6).

After defining the conditions necessary and sufficient for an enterprise to operate in a financially stable way it is possible to model financial status of an enterprise under various assumptions (hypotheses), evaluate results of each hypothesis, select the acceptable versions (including taking into account an expert’s conclusion) and develop a package of organizational and technical measures needed to achieve the objectives. Modeling is realized using the scenario method.

Universal scenario-modeling program package is integrated in Excel (Scenario). It is needed to adapt it taking into account specific conditions of a given task and make it possible to launch it when developing the software.

Modeling of planned decisions based on the scenario principle is a representation of centralized planning and management system. The mechanism used to formulate a realized plan in Excel is effectively used in the formation of strategic and current plans of a company.

The essence of a scenario-modeling of financial status of an enterprise is the following. A task is recommended to be solved by the software. However, its solution without a program drawn in advance is also possible. In such case the task will be solved in a semi-automated manner. Let us look at both modes. In both modes various scenarios of financial stability of an enterprise is calculated our polynomial formula. In the first scenario, in order to carry out multivariate calculations using the Scenario software, we input initial values into the cells selected in advance. The formula is written in the cell that is to produce the result. In our case, on the page called “sheet” we select the range with the initial data and select Tools / Scenarios. A dialogue window will appear (Fig. 7).

Clicking on the OK in the dialogue window will produce another dialogue window (Fig. 8). In the Scenario Name field we write the title of the plan scenario. In the Changing Cells field the system has already written the address of the range we selected, including values of the variable indicators.

After clicking the OK button another dialogue window will appear (Fig. 8) where we will write estimated values of the variables. After clicking the Add button, we will again see the dialogue window entitled Scenario Manager. Input the plan scenario title and click the OK button. Write new values of the variables in the Scenario Values window, etc. After plan scenarios have formed it has to click the Summary button. A new sheet under the name Scenario Summary will be created showing results of multivariate calculations.
cells are given names. In order to give a name to a cell or range is has to select Insert / Name / De-
fine and write a name in the dialogue window that appears.
Below it can find a program developed by authors, which carries out multivariable calculations using the Scenario software. The software focus in this case is on the calculation of a multi-
variant plan.
Let us make a 5-column, 2-row table on the Sheet page.
Range 2:2 shows initial values of the variables, while Cell E2 contains the calculation formula. Let us design a form that will write the initial values. In order to write a change in the value with a definite step size we will use the Spinbutton element (Table 4).

Table 4. The management elements on the form and their functions

<table>
<thead>
<tr>
<th>Element</th>
<th>Program name</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpinButton</td>
<td>SpinButton1</td>
<td>modify $X_1$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton2</td>
<td>modify $X_2$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton3</td>
<td>modify $X_3$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton4</td>
<td>modify $X_4$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton5</td>
<td>modify $X_5$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton6</td>
<td>modify $X_6$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton7</td>
<td>modify $X_7$ probability values</td>
</tr>
<tr>
<td>SpinButton</td>
<td>SpinButton8</td>
<td>modify $X_8$ probability values</td>
</tr>
<tr>
<td>CommandButton</td>
<td>CommandButton1</td>
<td>Launch the program</td>
</tr>
<tr>
<td>Label</td>
<td></td>
<td>Labeling</td>
</tr>
</tbody>
</table>

Conclusions

The programs are written in the form program module. The declaratory part thereof shows the changes used by various programs in the module.
To the article a fragment is driven by us to the created CAS of management of process of estimation of risk of safety of bankruptcy of enterprise.

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