Two-dimensional coordinate estimation for missing automatic identification system (AIS) signals based on the discrete Kalman filter algorithm and universal transverse mercator (UTM) projection

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Abstract
Due to safety reasons, the movement of a ship in coastal areas should be monitored, tracked, recorded, and stored. The Automatic Identification System (AIS) is a suitable tool to use in performing these functions. The probability limit for the AIS dynamic data availability can be limited by the lack of a Global Position System (GPS) signal, heading (HDG), and rate of turn (ROT) data in the position report. The unavailability of a data link is an additional limitation. To fill this gap, it is possible to attach the discrete Kalman filter (KF) for the position and course estimation. Coordinate estimation in the absence of a transmission link can improve the quality of the AIS service at Vessel Traffic Service (VTS) stations. This paper has presented the Kalman filtering algorithm to improve the possibilities for ship motion tracking and monitoring in the TSS (Traffic Separation Scheme) and fairways area. More than 570 iterations were calculated and the results have been presented in figures to familiarize the reader with the operating principle of the Kalman filter algorithm.

Introduction
Currently, due to increased maritime transport, particular attention should be paid to the safety aspect of shipping. The downsizing of ship crews has forced the introduction of new technological solutions to ensure safe navigation. Using onboard navigation systems, it is possible to define the coordinates of a ship. But to find a ship’s position in relation to other ships one must rely on navigation systems, e.g. the Automatic Identification System (AIS). Unfortunately, AIS developers have not met the integrity, availability, and reliability requirements (ITU-R M.1371, 2014). Therefore, appropriate steps must be taken to minimize the risk of unreliable information.

Over the years, a number of papers have been published on AIS data integrity and availability (Hori et al., 2009; Banyś, Noack & Gewies, 2012; Felski, Jaskólski & Banyś, 2015). In (Konatowski & Sipa, 2004; Kaniewski, 2010), a solution to the reliability problem of navigation systems was presented, suggesting the use of the Kalman Filter (KF) algorithm to estimate the coordinates for the navigation system. VTS operators have repeatedly encountered a lack of data reception from the onboard AIS. This phenomenon is a result of the limitations of the VHF data link and was presented by (Jaskólski, 2017).

In this paper, the Kalman filtering algorithm has been applied to extend the possibilities of ship motion tracking and monitoring in the TSS (Traffic Separation Scheme) and fairways area. The discrete Kalman Filter algorithm has been proposed for ship movement prediction in case of unavailability of the AIS data.
Background

According to technical specification (ITU-R M.1371, 2014) every vessel equipped with an AIS receiver transmits position reports based on its movement. These selected data have been presented in Table 1.

Table 1. Selected data of AIS position report (ITU-R M.1371, 2014)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Message ID</td>
<td>Identifier for position reports</td>
</tr>
<tr>
<td>User ID</td>
<td>Unique identifier such as maritime mobile service identity (MMSI) number</td>
</tr>
<tr>
<td>Rate of turn</td>
<td>0 to +126 = turning right at up to 708° per min or higher; 0 to -126 = turning left at up to 708° per min or higher</td>
</tr>
<tr>
<td>Speed over ground</td>
<td>Speed over ground in 1/10 knot steps (0–102.2 knots)</td>
</tr>
<tr>
<td>Longitude</td>
<td>Longitude in 1/10 000 min (±180°)</td>
</tr>
<tr>
<td>Latitude</td>
<td>Latitude in 1/10 000 min (±90°)</td>
</tr>
<tr>
<td>Course over ground</td>
<td>Course over ground in 1/10 (0–3599)</td>
</tr>
</tbody>
</table>

The Reporting Intervals between two consecutive AIS position reports received from the same vessel equipped with an AIS Class A receiver have been presented in Table 1.

Table 2. Class A shipborne mobile equipment reporting intervals (ITU-R M.1371, 2014)

<table>
<thead>
<tr>
<th>Ship’s dynamic conditions</th>
<th>Nominal reporting interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship at anchor or moored and not moving faster than 3 knots</td>
<td>3 min</td>
</tr>
<tr>
<td>Ship at anchor or moored and moving faster than 3 knots</td>
<td>10 s</td>
</tr>
<tr>
<td>Ship 0–14 knots</td>
<td>10 s</td>
</tr>
<tr>
<td>Ship 0–14 knots and changing course</td>
<td>3.33 s</td>
</tr>
<tr>
<td>Ship 14–23 knots</td>
<td>6 s</td>
</tr>
<tr>
<td>Ship 14–23 knots and changing course</td>
<td>2 s</td>
</tr>
<tr>
<td>Ship &gt; 23 knots</td>
<td>2 s</td>
</tr>
<tr>
<td>Ship &gt; 23 knots and changing course</td>
<td>2 s</td>
</tr>
</tbody>
</table>

If the vessel is underway, the officers keeping watch know its speed over the ground \(V\) in knots, the course over ground \(\psi\), and the geographic position \((\phi, \lambda)\) (Czapiewska & Sadowski, 2015). For research purposes \(V'\) was converted to m/s according to equation (11), and the geographic position was converted to Cartesian coordinates \((x, y)\) according to equations (1)–(5). Finally, the coordinates will be presented with the use of a 2-dimensional Cartesian coordinate system – The Universal Transverse Mercator (UTM).

With the use of ellipsoid WGS-84 parameters, the square of the first eccentric \(e^2\) was estimated as (Banachowicz & Urbaniński, 1988):

\[
e^2 = \frac{a^2 - b^2}{a^2}
\]

where: \(a\) – semi-major axis, \(b\) – semi-minor axis, and: \(a = 6,378,137.0 \text{ m}, b = 6,356,752.3\text{ m}\) determine the radius of curvature for the first vertical circle \(N\). The latter is calculated as (Banachowicz & Urbaniński, 1988):

\[
N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}
\]

where: \(\phi\) – latitude.

Then, the Cartesian coordinates take the following form (Banachowicz & Urbaniński, 1988):

\[
X = (N + H) \cos \phi \cdot \cos \lambda
\]

\[
Y = (N + H) \cos \phi \cdot \sin \lambda
\]

\[
Z = [N (1 - e^2) + H] \cdot \sin \phi
\]

where: \(H\) – height of the point “P”, \(\lambda\) – longitude.

According to equation (6), speed over ground given in [kt] should be converted to [m/s]:

\[
V' = 0.514(4) \cdot V
\]

where: \(V', V\) – speed over ground in [m/s] and [kt], respectively and the unit conversion for the rate of turn should be as follows:

\[
\omega' = \frac{\omega}{60}
\]

where: \(\omega', \omega\) – rate of turn in [deg/s] and [deg/min], respectively.

Recording signals, operating database

The post-processing method was used in order to conduct analyses of the AIS messages recorded. The station for recording the AIS signals was prepared in the Institute of Navigation and Marine Hydrography, at the Gdynia Naval Academy. The data was recorded on the data carrier of a signal recorder with sentence VDM. Position Reports (message No. 1) were used to analyze the data. The data recorded date was for the 1 April 2017. They were recorded in text files. Figure 1 shows the onshore setup of the AIS traffic data acquisition.
With the above configuration it is possible to conduct a measurement campaign and have a synchronised collection of the data streams, which will serve as groundwork for analysis of the signals latency in a multi-sensor NMEA environment.

A VDM sentence contains navigational datasets of vessels, which are received from other vessels equipped with an AIS transponder in the area of the VHF operation zone. These data are encapsulated according to (ITU-R M.1371, 2014) specifications. Afterwards, the comparison of the AIS timestamps and GNSS time is carried out. The AIS database was compiled by an IB Expert Database Client type.

The AIS data from one vessel in the area of the Gulf of Gdańsk, which was recorded for a time period of 571 s have been presented in Table 3 with the use a 2-dimensional Cartesian coordinate system – UTM.

Taking into consideration the latency of the data in Table 3 – lines with gray-scale color, the limitations of the VHF data link availability and the latency of the AIS position reports can be observed. The latency of the position report data exceeds 42 s. For this purpose, a discrete Kalman Filter can be used to reduce the unavailability of the AIS data and to complete the missing coordinates for the defined algorithm interval of 1 s.

**The discrete Kalman filter algorithm methodology**

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at a given time and then obtains feedback in the form of (noisy) measurements. The equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting the current state forward (in time) and error covariance estimates to obtain a priori estimates for the next time step. The measurement update equations are responsible for the feedback – i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate (Welch & Bishop, 2006).

The time update equations can also be perceived as predictor equations, while the measurement update equations can be considered as corrector equations. In fact, the final estimation algorithm resembles that of a predictor-corrector algorithm for

![Figure 1. The onshore station setup of the AIS traffic data acquisition](image)

**Table 3. UTM input data**

<table>
<thead>
<tr>
<th>(t) [s]</th>
<th>(X) [m]</th>
<th>(Y) [m]</th>
<th>(\psi) [deg]</th>
<th>(V') [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6032926</td>
<td>4347635</td>
<td>148.0</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>6032858</td>
<td>4347678</td>
<td>147.7</td>
<td>3.8</td>
</tr>
<tr>
<td>20</td>
<td>6032824</td>
<td>4347696</td>
<td>146.8</td>
<td>3.8</td>
</tr>
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<td>30</td>
<td>6032790</td>
<td>4347715</td>
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<td>3.8</td>
</tr>
<tr>
<td>40</td>
<td>6032755</td>
<td>4347740</td>
<td>147.2</td>
<td>3.9</td>
</tr>
<tr>
<td>50</td>
<td>6032721</td>
<td>4347758</td>
<td>148.3</td>
<td>3.9</td>
</tr>
<tr>
<td>82</td>
<td>6032619</td>
<td>4347813</td>
<td>152.2</td>
<td>3.8</td>
</tr>
<tr>
<td>102</td>
<td>6032551</td>
<td>4347843</td>
<td>155.5</td>
<td>3.8</td>
</tr>
<tr>
<td>142</td>
<td>6032405</td>
<td>4347897</td>
<td>156.6</td>
<td>3.8</td>
</tr>
<tr>
<td>162</td>
<td>6032326</td>
<td>4347927</td>
<td>153.7</td>
<td>3.8</td>
</tr>
<tr>
<td>173</td>
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<td>4347945</td>
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<td>4347994</td>
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<tr>
<td>216</td>
<td>6032145</td>
<td>4348005</td>
<td>156.4</td>
<td>3.8</td>
</tr>
<tr>
<td>258</td>
<td>6032010</td>
<td>4348059</td>
<td>161.0</td>
<td>3.8</td>
</tr>
<tr>
<td>268</td>
<td>6031965</td>
<td>4348070</td>
<td>162.0</td>
<td>3.8</td>
</tr>
<tr>
<td>288</td>
<td>6031885</td>
<td>4348120</td>
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<td>3.8</td>
</tr>
<tr>
<td>319</td>
<td>6031673</td>
<td>4348152</td>
<td>163.7</td>
<td>3.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(t) [s]</th>
<th>(X) [m]</th>
<th>(Y) [m]</th>
<th>(\psi) [deg]</th>
<th>(V') [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>329</td>
<td>6031525</td>
<td>4348231</td>
<td>162.4</td>
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<td>154.5</td>
<td>3.7</td>
</tr>
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<td>4348364</td>
<td>153.0</td>
<td>3.7</td>
</tr>
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<td>6031230</td>
<td>4348397</td>
<td>142.3</td>
<td>3.7</td>
</tr>
<tr>
<td>470</td>
<td>6031208</td>
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<td>138.0</td>
<td>3.7</td>
</tr>
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<td>480</td>
<td>6031162</td>
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<td>133.0</td>
<td>3.6</td>
</tr>
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<td>3.6</td>
</tr>
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<td>4348517</td>
<td>121.7</td>
<td>3.6</td>
</tr>
<tr>
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<td>6031114</td>
<td>4348549</td>
<td>118.5</td>
<td>3.6</td>
</tr>
<tr>
<td>521</td>
<td>6031090</td>
<td>4348587</td>
<td>114.0</td>
<td>3.7</td>
</tr>
<tr>
<td>531</td>
<td>6031078</td>
<td>4348613</td>
<td>112.4</td>
<td>3.7</td>
</tr>
<tr>
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<td>6031066</td>
<td>4348651</td>
<td>111.5</td>
<td>3.7</td>
</tr>
<tr>
<td>551</td>
<td>6031054</td>
<td>4348683</td>
<td>110.9</td>
<td>3.7</td>
</tr>
<tr>
<td>561</td>
<td>6031030</td>
<td>4348728</td>
<td>110.3</td>
<td>3.7</td>
</tr>
<tr>
<td>571</td>
<td>6031029</td>
<td>4348754</td>
<td>109.1</td>
<td>3.7</td>
</tr>
</tbody>
</table>

\(X, Y\) – UTM coordinates, \(\psi\) – course over ground, \(V\) – speed over ground.

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The equations for the time and measurement updates are presented below:

According to equations (8), (9), the Kalman filter time update equations are:

\[
\hat{x}_k = A \cdot \hat{x}_{k-1} + B \cdot u_k + w_{k-1}
\] (8)

where:
- \(\hat{x}_k\) – estimated state vector in time step \(k\),
- \(A\) – transition matrix,
- \(\hat{x}_{k-1}\) – estimated state vector in the preceding time step \((k-1)\),
- \(B\) – output matrix,
- \(u_k\) – control variable matrix,
- \(w_{k-1}\) – previous state noise matrix, and:

\[
P_k = A \cdot P_{k-1} \cdot A^T + Q
\] (9)

where:
- \(P_k\) – process error covariance,
- \(P_{k-1}\) – previous state process covariance,
- \(A^T\) – transpose of a transition matrix,
- \(Q\) – process noise covariance.

The state and covariance matrix estimates forward from time step \(k-1\) to step \(k\).

According to formulas (10), (11), (12), the discrete Kalman filter measurement update equations are as presented below:

\[
K_k = P_k \cdot H^T \cdot (H \cdot P_k \cdot H^T + R)^{-1}
\] (10)

where:
- \(K_k\) – Kalman gain at time step \(k\),
- \(H^T\) – transpose of simple transformation matrix,
- \(H\) – simple transformation matrix, a design matrix consisting of partial derivations of the measurements,
- \(R\) – sensor noise covariance and:

\[
\hat{x}_k = \hat{x}_k + K_k \cdot (z_k - H \cdot \hat{x}_k)
\] (11)

where:
- \(\hat{x}_k\) – a posteriori estimate of the state at step \(k\),
- \(\hat{x}_{k-1}\) – a priori estimated state,
- \(z_k\) – actual measurement vector.

\[
P_k = (I - K_k \cdot H) \cdot P_k
\] (12)

where:
- \(P_k\) – process error covariance matrix,
- \(I\) – identity matrix.

The operating principle of the KF algorithm is as follows (Welch & Bishop, 2006):
1. The first task during the measurement update is to compute the Kalman gain \(K_k\).
2. The next step is to actually measure the process to obtain \(z_k\) and then generate an a posteriori state estimate by incorporating the measurement as in equation (11).
3. Again, using equation (11), an a posteriori state estimate \(\hat{x}_k\) is obtained as a linear combination of an a priori estimate \(\hat{x}_{k-1}\) and a weighted difference between an actual measurement \(z_k\) and a measurement prediction \(H \cdot \hat{x}_{k-1}\).
4. The final step is to obtain an a posteriori error covariance estimate via equation (12).

Figure 3 depicts the operation principle of the KF filter.

The Kalman filter (KF) has been the subject of extensive research and application, particularly in the areas of autonomous or assisted navigation.
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(2010; Bezrucka, 2012; Naus & Nowak, 2016). This is a recursive algorithm employed for the discrete linear dynamic process estimation. The algorithm is responsible for the minimization of the mean squared error. Due to this fact, KF can be applied for ship movement estimation (Welch & Bishop, 2006).

For \( k \) iterations, 1 s interval and for a two-dimensional model the state vector \( X_i \) is:

\[
X_i = \begin{bmatrix}
  x_k \\
  y_k \\
  V'_k \cos (\psi_k) \\
  V'_k \sin (\psi_k)
\end{bmatrix}
\]

where: \( x_k, y_k \) – UTM coordinates; \( V'_k \cos (\psi_k), V'_k \sin (\psi_k) \) – linear speed in the \( x \) and \( y \) axis, respectively.

For the initial state:

\[
X_0 = \begin{bmatrix}
  0 m \\
  0 m \\
  0 m/s \\
  0 m/s
\end{bmatrix}
\]

For the first iteration:

\[
X_1 = \begin{bmatrix}
  6032926 m \\
  4347635 m \\
  -3.2 m/s \\
  2 m/s
\end{bmatrix}
\]

The transition matrix for a two-dimensional model is (Jaskólski, 2017):

\[
A = \begin{bmatrix}
  1 & 0 & \Delta t & 0 \\
  0 & 1 & 0 & \Delta t \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

where: \( \Delta t = 1 \) s – interval between the current and the previous measurements for the calculated data.

Every 10 s, nine out of ten coordinates were calculated according to formulas (15)–(20) using \( \Delta t = 1 \) s.

If the coordinates are estimated as follows:

\[
X_k = X_{k-1} + \Delta X \quad (15)
\]

\[
Y_k = Y_{k-1} + \Delta Y \quad (16)
\]

where: \( X_{k-1}, Y_{k-1} \) – coordinates for the previous moment – AIS data; \( \Delta X, \Delta Y \) – shift coordinates in the \( x \) and \( y \) axis, respectively

and (Jaskólski, 2017):

\[
\Delta X = \frac{a_x \cdot \Delta t^2}{2} \quad (17)
\]

\[
\Delta Y = \frac{a_y \cdot \Delta t^2}{2} \quad (18)
\]

and (Richert, 2017):

\[
a_x = \frac{V'_k \cdot \cos (\psi_k + \omega_k \cdot \Delta t) - \cos (\psi_{k-1})}{\Delta t} \quad (19)
\]

\[
a_y = \frac{V'_k \cdot \sin (\psi_k + \omega_k \cdot \Delta t) - \sin (\psi_{k-1})}{\Delta t} \quad (20)
\]

where: \( a_x, a_y \) – acceleration in the \( x \) and \( y \) axis, respectively; \( \omega_k \) – rate of turn in time step \( k \); \( V' \) – speed over ground; \( \psi \) – course over ground.

Then, the predicted state vector \( \hat{x}_{k-1} \), the product of output matrix \( B \) and the control variable matrix \( u_k \) for a two-dimensional model is presented as follows:

\[
B \cdot u_k = \begin{bmatrix}
  a_x \cdot \Delta t \cdot [m] \\
  \frac{a_x \cdot \Delta t^2}{2} [m] \\
  a_y \cdot \Delta t \cdot [m/s] \\
  \frac{a_y \cdot \Delta t^2}{2} [m/s]
\end{bmatrix}
\]

If there is no information about the imperfection of the measuring sensors for an observed vessel via AIS, the noise in the process \( w_{k-1} \) for the previous iteration is:

\[
w_{k-1} = \begin{bmatrix}
  0 m \\
  0 m \\
  0 m/s \\
  0 m/s
\end{bmatrix}
\]

then, the predicted state matrix \( \hat{x}_{k-1} \) is:

\[
\hat{x}_{k-1} = A \cdot \hat{x}_{k-1} + B \cdot u_k + w_{k-1} \quad (22)
\]

where: \( \hat{x}_{k-1} \) – previous state vector (in time step \( k-1 \)).

To estimate the previous state process covariance matrix \( P_{k-1} \), the following assumptions were adopted for the first three iterations:

According to IMO performance standards for Marine Speed Logs (MSC.96(72), 2000) and GPS: (MSC.115(73), 2000):

\[
\sigma_x = 10 m, \quad \sigma_y = 10 m, \quad \sigma_x' = 0.2 m/s, \quad \sigma_y' = 0.2 m/s
\]
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and

\[
P_{k-1} = \begin{bmatrix} \sigma_x^2 & \text{cov}(x, y) & \text{cov}(x, V_x) & \text{cov}(x, V_y) \\ \text{cov}(y, x) & \sigma_y^2 & \text{cov}(y, V_x) & \text{cov}(y, V_y) \\ \text{cov}(V_x, x) & \text{cov}(V_x, y) & \sigma_{V_x}^2 & \text{cov}(V_x, V_y) \\ \text{cov}(V_y, x) & \text{cov}(V_y, y) & \text{cov}(V_y, V_x) & \sigma_{V_y}^2 \end{bmatrix}
\]  

(23)

then:

\[
P_{k-1} = \begin{bmatrix} 100 \text{ m}^2 & 100 \text{ m}^2 & 2 \text{ m}^2/\text{s} & 2 \text{ m}^2/\text{s} \\ 100 \text{ m}^2 & 100 \text{ m}^2 & 2 \text{ m}^2/\text{s} & 2 \text{ m}^2/\text{s} \\ 2 \text{ m}^2/\text{s} & 2 \text{ m}^2/\text{s} & 0.04 \text{ m}^2/\text{s}^2 & 0.04 \text{ m}^2/\text{s}^2 \\ 2 \text{ m}^2/\text{s} & 2 \text{ m}^2/\text{s} & 0.04 \text{ m}^2/\text{s}^2 & 0.04 \text{ m}^2/\text{s}^2 \end{bmatrix}
\]

To estimate the previous state process covariance matrix \( P_{k-1} \) according to equation (23) for \( k = 3 + i \) iteration, elements of the \( P_{k-1} \) matrix have been presented in equations (24)–(33) (Kantak, Stateczny & Urbański, 1988):

\[
\sigma_i^2 = \Delta t^2 \left[ (\sigma_i \cos(\psi_{i-1}))^2 + \Delta t^2 \sigma_i^2 V_i \cos(\psi_i) \right] 
\]

(24)

\[
\sigma_j^2 = \Delta t^2 \left[ (\sigma_j \sin(\psi_{i-1}))^2 + \Delta t^2 \sigma_j^2 V_j \cos(\psi_j) \right] 
\]

(25)

\[
\sigma_{V_i}^2 = \left[ (\sigma_i \cos(\psi_j))^2 + \sigma_j^2 V_j \sin(\psi_j) \right] 
\]

(26)

\[
\sigma_{V_j}^2 = \left[ (\sigma_j \sin(\psi_j))^2 + \sigma_j^2 V_j \cos(\psi_j) \right] 
\]

(27)

\[
\text{cov}(x, y) = \text{cov}(y, x) = \frac{1}{2} \Delta t^2 \sin(2\psi_k) \left[ \sigma_b^2 - \left( \sigma_y V_y \right)^2 \right] 
\]

(28)

\[
\text{cov}(x, V_x) = \text{cov}(V_x, x) = \Delta t \cos(\psi_k) \left[ \sigma_b^2 - \left( \sigma_y V_y \right)^2 \right] 
\]

(29)

\[
\text{cov}(y, V_y) = \text{cov}(V_y, y) = \Delta t \sin(\psi_k) \left[ \sigma_b^2 + \left( \sigma_y V_y \right)^2 \right] 
\]

(30)

\[
\text{cov}(V_x, V_y) = \text{cov}(V_y, V_x) = \frac{1}{2} \sin(2\psi_k) \left[ \sigma_b^2 + \left( \sigma_y V_y \right)^2 \right] 
\]

(31)

\[
\text{cov}(y, V_x) = \text{cov}(V_x, y) = \frac{1}{2} \Delta t \sin(2\psi_k) \left[ \sigma_b^2 - \left( \sigma_y V_y \right)^2 \right] 
\]

(32)

\[
\text{cov}(V_j, x) = \text{cov}(x, V_j) = \frac{1}{2} \Delta \sin(2\psi_k) \left[ \sigma_b^2 - \left( \sigma_y V_y \right)^2 \right] 
\]

(33)

If the process noise covariance \( Q \) is:

\[
Q \equiv P_{k-1} 
\]

(34)

then the process error covariance \( P_k \) for the two-dimensional model is calculated according to equation (9).

If the simple transformation matrix \( H \) is as follows:

\[
H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} 
\]

(35)

and according to the model’s assumptions, the sensor noise covariance matrix \( R \) is calculated in every iteration, where the diagonal values are variances of the last three measurements of the coordinates and velocity, namely:

\[
R = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_{V_x}^2 & 0 \\ 0 & 0 & 0 & \sigma_{V_y}^2 \end{bmatrix} 
\]

(36)

then, the Kalman gain \( K_k \) is calculated in every iteration according to formula (10).

If the actual measurement vector \( z_k \) is presented as follow:

\[
z_k = \begin{bmatrix} x_k [\text{m}] \\ y_k [\text{m}] \\ V_x \cos(y_k) [\text{m/s}] \\ V_y \sin(y_k) [\text{m/s}] \end{bmatrix} 
\]

(37)

then an a posteriori estimate of the state at step \( k \hat{x}_k \) is estimated according to formula (11).

Finally, if the identity matrix takes the following form:

\[
I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} 
\]

(38)

then the process error covariance matrix \( P_k \) is calculated according to formula (12).

If the coordinates are unavailable, then the course over ground for iteration \( k \) is calculated according to the formula (Kantak, Stateczny & Urbański, 1988):
\[ \psi_k = a \tan \frac{y_{k} - y_{k-1}}{x_{k} - x_{k-1}} \]  

and the linear speed over ground in the \( x \) and \( y \) axis is calculated according to the following equations, respectively:

\[ V_k^x = \cos(\psi_k) \cdot V_k \quad [\text{m/s}] \]  
\[ V_k^y = \sin(\psi_k) \cdot V_k \quad [\text{m/s}] \]

**Research outcomes**

More than 570 iterations using the Kalman Filtering Algorithm were conducted to reduce VHF AIS data link unavailability and to complete missing AIS data.

The use of the Kalman filter was intended to improve the availability of AIS dynamic information displayed on the Vessel Traffic Service (VTS) stations. The research outcomes for the discrete KF estimation for UTM coordinates have been presented in Figure 4.

![Figure 4. UTM coordinates for the AIS data and estimated UTM coordinates with the use of the Kalman filtering algorithm](image)

Figure 4. UTM coordinates for the AIS data and estimated UTM coordinates with the use of the Kalman filtering algorithm

The coordinate differences for the \( x \) and \( y \) direction with a maximum 38 m difference have been presented in Figure 5. At this time the ship has covered a distance of 148 meters. The AIS and KF estimated coordinates were compared to show the differences between the coordinates. According to the research results, the largest differences were typically in the first few iterations, where the KF algorithm did not work correctly.

Taking into account the data from Table 3 and from Figure 4, it can be seen that for 570 seconds, only 38 position reports were received. In addition, the ship significantly diverted course, as evidenced by the value of the rate of turn parameter. In accordance with the assumptions contained in (ITU-R M.1371, 2014), the vessels alternating heading, with \( V \leq 14 \) knots, equipped with class A AIS should provide the dynamic data at an interval of 3.33 seconds. According to the formulas (19) and (20) acceleration in the \( x \) and \( y \) direction can be determined. This ship was deliberately selected to analyze the filter’s performance with limited availability of AIS position reports. This is easily noticeable in the coordinate

![Figure 5. Differences between the AIS and KF estimated \((x, y)\) coordinates; \(\Delta POS [m]\) – distance between the AIS and KF estimated coordinates](image)

Figure 5. Differences between the AIS and KF estimated \((x, y)\) coordinates; \(\Delta POS [m]\) – distance between the AIS and KF estimated coordinates
Two-dimensional coordinate estimation for missing automatic identification system (AIS) signals...

...differences, for individual iterations. Despite the AIS position reports being characterized by 42 seconds of latency (Richert, 2017), the KF correctly estimated the coordinates. After 420 s of registration data and after 42 seconds of unavailability of the AIS data, an increase in the difference between the AIS and Kalman filtered data was observed. The essential factor affecting the correct operation of the Kalman filter is the appropriate estimation of the sensor noise covariance matrix $R$ and the process noise covariance $Q$. At least 85% of the estimated coordinates were located not more than 10 meters from the AIS positions. The AIS data has been deliberately selected to analyze the performance of the Kalman Filter, given a limited amount of measurement data, and to carry out the state correction.

Conclusions

In this article, the discrete Kalman algorithms have been used to estimate the coordinates and improve the availability of the AIS data. At least 570 iterations were presented to demonstrate the principle of the KF algorithm. An incomplete position report was selected for the presentation of the KF algorithm. The KF algorithm did not work properly for the first few iterations. This algorithm produces an estimation of the positions of the missing AIS signals, and is useful in patching the AIS data in cases when high resolution data would be needed but is not available.

References

13. MSC.96(72) (2000) Adoption of amendments to performance standards for devices to measure and indicate speed and distance (Resolution A.824(19)). London: IMO.