UNCERTAINTY ANALYSIS OF THE DETERMINATION NOISE SOURCE DIRECTION USING THE TETRAHEDRON HYDROPHONE ARRAY

IGNACY GLOZA¹, RAFAŁ JÓZWIAK¹, PAWEŁ PAWLIK², WOJCIECH BATKO²

¹Polish Naval Academy
Śmidowicza 69 81-103 Gdynia, Poland
²AGH University of Science and Technology, Department of Mechanics and Vibroacoustics
Al. Mickiewicza 30, 30-059 Cracow, Poland
i.gloza@amw.gdynia.pl

The analysis of the uncertainty of the determination of the noise source direction in the aquatic environment is presented in this paper. The type A uncertainty of the bearing algorithm was determined on the bases of multiple measurements of the hydroacoustic pressure by means of the TC4032 type hydrophones and the GPS device data.

The influence of the non-linearity of the amplitude-frequency characteristic on the uncertainty of the determination of the noise source direction was also described. The influence of water density, electromagnetic disturbances and irregular noises on the final uncertainty was determined.

On the bases of the determined uncertainty components, the uncertainty budget of the determination of the angle showing the direction from which hydroacoustic waves are coming, was prepared.

INTRODUCTION

The detection, classification and tracing of ships is important to improve the safety of harbours and seaways. Hydroacoustic converters and hydrophones can be applied for detections of noises and disturbances generated by ships. The method of tracing noise sources by means of an antenna consisting of four hydrophones is presented in this paper. Such a solution constitutes the passive measuring system which - when localising the traced object - does not emit acoustic energy to the sea environment. This is undoubtedly an advantage of the passive measuring system, since such system does not require a high power supply and can
operate at the accumulator supply. In addition, such system does not introduce any disturbances during measurements and the traced object is unable to detect that it is being traced.

However, the obtained result is exposed to several disturbing factors and, on account of this fact, the components of the type A and B uncertainties were determined and their fractions in the uncertainty budget were presented in this study.

1. DESCRIPTION OF THE MEASURING SYSTEM

Four hydrophones of the TC4032 type were arranged on the frame, forming the regular tetrahedron – hence, a tetrahedral antenna. It is well known that lengths of all tetrahedron edges are the same. A similar situation applies with lengths between hydrophones acoustic centres, which is equal to 250mm. Due to such arrangement of hydrophones, it is possible to determine six virtual hydrophones, which are situated in the middle of the segment joining two hydrophones (tetrahedron edges). Virtual hydrophones situated on opposite edges form pairs responsible for axes: V2 – V4 is OX, V5 – V6 is OY and V1 – V3 is OZ in the Cartesian co-ordinate system. The origin of the Cartesian co-ordinate system determines the point of intersection of lines joining individual pairs of virtual hydrophones, it means: V1 – V3, V2 – V4, V5 – V6, as seen in the Figure below (Fig. 1).

Fig. 1. Arrangement of hydrophones in the tetrahedral antenna

Distances $D = 2x$ - in between virtual hydrophones - can be determined from the dependence of the length radius of the sphere circumscribed on the regular tetrahedron and the length of the right-angle triangle.

The radius of the sphere circumscribed on the regular tetrahedron, $R$ is
\[ R = \sqrt{\frac{6}{4}} d, \]  

where \( d \) is a distance between hydrophones, \( x \) is a half length between virtual hydrophones and \( R \) is the radius of the sphere circumscribed on the regular tetrahedron.

The distance between virtual hydrophones \( D \), in dependence of the distance between hydrophones, describes the dependence:

\[ D = \frac{d}{\sqrt{2}} \]  

The distance between virtual hydrophones: \( \frac{250\text{mm}}{\sqrt{2}} = 176.8\text{mm} \).

Phase displacements and amplitudes of signals received by hydrophones depend on the noise source placement. Signals received by four hydrophones - for one passage of the object above the measuring module - are seen in Figure 2.

\[
V_1(t) = \frac{H_1(t) + H_2(t)}{2}
\]  

In order to determine \( V_1 \) signal for the first virtual hydrophone, the resultant signal between the hydrophone pairs \( H_1 \) and \( H_2 \) should be determined:

Comparison of signals from hydrophones \( H_1 \) and \( H_2 \) and the virtual signal \( V_1 \) is presented in Figure 3.
Fig. 3. Comparison of signals from hydrophones H1 and H2 and the virtual signal V1.

After determining all six virtual signals, the sound intensity components $I_x$, $I_y$ should be determined. The cross spectrum function was applied [6-10]:

$$I_x = -\frac{\text{Im}\{G_{V_4V_2}\}}{\rho \omega},$$

(4)

where $\rho$ is water density, $\omega = 2\pi f$ is pulsation of the analysed signal ($f$ is frequency), $D$ is distance between virtual hydrophones, $\text{Im}\{G_{V_4V_2}\}$ is the imaginary part of the cross spectrum. Knowing the sound intensity components $I_x$, $I_y$, the angle $\phi_i$ was determined. This angle determines the bearing of the noise source, according to (5):

$$\phi_i = \arctan \left( \frac{I_y}{I_x} \right)$$

(5)

2. UNCERTAINTY OF TYPE A

Using the algorithm described in Chapter 2 the angle $\phi_i$ was determined on the basis of measurements from four hydrophones of TC4032 type. The algorithm results were compared with the measurements performed by means of the geodesic GPS device. In order to perform...
the comparative analysis, the angle $\phi$ was determined by transforming the co-ordinates of the monitored object from the Cartesian system into the polar system. Figure 4 presents the angle $\phi$ changing as a function of time, with the blue curve indicating the angle $\phi$ determined by means of the bearing algorithm, and the red curve indicating the angle $\phi$ determined from the GPS data.

![Graph showing angle $\phi$ as a function of time with blue and red curves representing data from bearing algorithm and GPS respectively.]

Fig. 4. Angle $\phi$ changes as the time function – measurement No. 1

Then the difference between the data obtained by means of the bearing algorithm and by means of the GPS device was determined. The data from the GPS device can be treated as the reference standard since they are characterised by a small uncertainty of measurements (of the order of 0.1m). This difference can be called the error, and this error is a function of time as shown in Figure 5.
The histogram of the error of the angle $\phi$ determination, by means of which the standard uncertainty was determined [5], is presented in Figure 6. The standard uncertainty of the angle $\phi$ determination for each measuring file is given in Table 1.

Table 1. Standard uncertainty for the data from each measuring file.

<table>
<thead>
<tr>
<th>Measurement name</th>
<th>Standard uncertainty of the angle $\phi$ determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement No. 1</td>
<td>16.56</td>
</tr>
<tr>
<td>Measurement No. 2</td>
<td>13.56</td>
</tr>
<tr>
<td>Measurement No. 3</td>
<td>22.32</td>
</tr>
<tr>
<td>Measurement No. 4</td>
<td>12.87</td>
</tr>
</tbody>
</table>
Then the arithmetic mean of the standard uncertainty from all measuring files was determined. This uncertainty will be utilised in the determination of the complex uncertainty.

3. UNCERTAINTY OF TYPE B

On the bases of the amplitude-frequency characteristic of the TC4032 hydrophone (Fig. 7), the expanded uncertainty of non-linear characteristic was determined. This uncertainty equals ±2dB, for the accuracy given by the producer.

![Characteristics of the TC4032 hydrophone](image)

Fig. 7. Characteristic of the TC4032 hydrophone

Then the influence of this uncertainty on the result of the angle determination by means of the algorithm was determined. Since the measuring data from four hydrophones of the same type are used in the algorithm, the error propagation was investigated in such way as to take into account the influence of the uncertainty of each hydrophone occurring independently as well as simultaneously. The formalism of the interval arithmetic was applied for this aim [1,2,4]. The lower and upper limit of the uncertainty interval of the angle was determined, as a function of time (Fig. 8).
Half of the interval width is equal to the limiting uncertainty $\Delta f_i$ (Fig. 9.).

Then the standard uncertainty was estimated, according to the dependence (6) [5]

$$u(f_i) = \frac{\Delta f_i}{\sqrt{3}},$$  \hspace{1cm} (6)

where

$u(f_i)$ – standard uncertainty,

$\Delta f_i$ – limiting uncertainty of the angle $f_i$,

$\sqrt{3}$ – expansion factor at the assumption of the uniform distribution.
The standard uncertainty caused by non-linearity of the amplitude-frequency characteristic is equal to:

\[ u(\phi) = 3.50^\circ \]  

(7)

4. UNCERTAINTY BUDGET

On the bases of the estimation of the type A and B uncertainties, presented in previous chapters, the uncertainty budget was developed and shown in Table 2. The uncertainty sources due to a water density, electromagnetic disturbances and to irregular noise influences were also taken into account in the budget. These data were taken from references, where item [3] contains the experiment by means of which the data were determined.

Table 2. Uncertainty budget of the angle \( \phi \) determination.

<table>
<thead>
<tr>
<th>Uncertainty source</th>
<th>Uncertainty of the input parameter</th>
<th>Fraction in the complex standard uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-linearity of the amplitude-frequency characteristic of hydrophones</td>
<td>2.00 dB</td>
<td>3.50 °</td>
</tr>
<tr>
<td>Water density (data from references [3])</td>
<td>0.03 dB</td>
<td>0.03 °</td>
</tr>
<tr>
<td>Electromagnetic disturbances (data from references [3])</td>
<td>0.05 dB</td>
<td>0.05 °</td>
</tr>
<tr>
<td>Influence of irregular noises (data from references [3])</td>
<td>0.06 dB</td>
<td>0.07 °</td>
</tr>
<tr>
<td>Uncertainty of the reference data from the GPS device</td>
<td>0.1 m</td>
<td>0.05 °</td>
</tr>
<tr>
<td>Uncertainty of the transforming algorithm</td>
<td>17.88 °</td>
<td></td>
</tr>
</tbody>
</table>

|                                  |                                  |                                             |
|                                  | Complex standard uncertainty      | 18.22 °                                    |
|                                  | Expanded uncertainty (k=2)        | 36.44 °                                    |
|                                  | Percentage expanded uncertainty (k=2) (in relation to 180 ° range) | 20.2 %                                    |

5. CONCLUSIONS

The assessment of the uncertainty in the determination of the angle \( \phi \), based on measurements obtained by means of four hydrophones, is presented in the instant paper. The reference data were obtained on the basis of measuring the position of the floating object by means of the geodesic GPS device. The influence of non-linearity of the amplitude-frequency
characteristic of hydrophones on the uncertainty of the determination of the angle $\phi_i$ was also investigated in the study.

The assessment of the type A uncertainty was performed utilising the analysis of the standard deviation of the difference between the algorithm and GPS data. In the type B assessment, the non-linearity of the hydrophones characteristic and the uncertainty of the GPS data were taken into account. The influence of such factors as water density, electromagnetic disturbances and irregular noises was also determined; however these factors have a negligible weight in the complex uncertainty. The type B uncertainty propagation was investigated using the interval arithmetic.

The analysis performed indicates that the error of the algorithm determining the angle $\phi_i$ is characterised by the normal distribution, which allows its elimination by the application of the proper methods of signal analysis.

ACKNOWLEDGEMENT

The investigation was supported by The National Centre for Research and Development (Grant No DOBR/0067/R/ID2/2012/03).

REFERENCES


[3] Chen Yi, A.E. Isaev (and others), Comparison of hydrophone calibrations in the frequency range 250 Hz to 200 kHz, Metrologia 2011, 48Tech. Suppl.09004, 2011.


