PREDICTION OF ESTIMATES OF TECHNICAL OBJECT’S RELIABILITY ON THE BASIS OF DAMAGE DETERMINED FROM LINDBERG LEVY’S CLAIM AND MULTIPLICITY OF THE SET SPECIFIED FROM ERGODICITY STREAM DAMAGE

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Abstract

During the exploitation one problem still unsolved is that of determining reliability characteristics for single (individual or small set) technical object. This particularly concerns identification of damage set (parametrical and momentary) as well as determination of substitute cardinality of objects set, where in fact this equals 1, 2, 3 at the most. The article presents a method of determining parametrical and momentary damages from iteration equations:

\[ D_K = a_{Rb} D_K + b_{Rb} U, \]  \( (1) \)

\[ \dot{U}_W = a_{Rc} U + b_{Rc} D_K, \]  \( (2) \)

where: \( D_K \) – complex diagnostic signal, \( \dot{U}_W \) – net environment signal (manner of use), \( a_{Rb} \) – technical condition parameter, \( a_{Rc} \) – exploitation condition parameter (exploitation potential), \( b_{Rb} \) – parameter of environment (manner of use) influence intensity on technical condition, \( b_{Rc} \) – intensity parameter of technical condition (physical wear) influence on technical object operation quality.

Assumption is made that if current value of \( a_{Rb} \) technical condition parameter is higher than \( a_{Rb, dop} \) (permissible) stemming from central limit theorem, parametrical damage occurs and analogically if value of exploitation condition parameter \( a_{Rc} \) is higher than \( a_{Rc, dop} \), momentary damage occurs. Additional assumption is made that substitute cardinality of objects set might be deputized by properly long observation of single object, as resulting from assumption of damage flux ergodicity. Therefore, having damages number (parametrical and momentary) and large enough substitute cardinality for a single object, unreliability estimators might be calculated and thus reliability characteristics of technical object. Presented method of determining reliability characteristics is based on virtually not dangerous parametrical and momentary damages that occur before extremely dangerous catastrophic damage.

Method has been verified in practices on bearing systems of two pump complexes of second grade pump station in Jurowce Water Purification Plant of Białystok Water Department.

Keywords: damage identification, damage flux ergodicity, reliability estimators
1. Introduction

In the exploitation process of complex technical object, its destruction occurs, i.e. maladjustment, wear and unreliability increase. Hence necessary is to observe adjustment condition, technical condition and reliability during standard utilization. Results of these observations are the base for synthetic (comprehensive) assessment of object aptitude, i.e. ability for proper realization of tasks according to its destination.

Object adjustment control consists in constant observation of ability to sustain proper relation between utility signals (output) and input signals (environment) [4, 5].

Diagnostics consists in constant observation of technical condition parameter changes based on relation between diagnostic signals connected to elements wear and environment, utility and accompanying signals [4].

Analysis and assessment of object reliability consists in constant gathering of knowledge on object damage that are border levels of maladjustment and wear and, subsequently (according to PN-77/N-04005) calculating the identified number of damages to reliability characteristics.

Assessment of adjustment, diagnostics and reliability conditions are basic, strictly interconnected elements that are base for comprehensive object aptitude assessment within cybernetic exploitation system [1, 6, 7].

Method described in the article consists in assessment of technical condition of exploited object basing on changes of technical condition parameter $a_{Rb}$ and adjustment parameter $a_{Rc}$ and thus number of parametrical $m_b$ and momentary damages $m_c$ that are basis for assessment of object reliability. Such assessment will be possible after determining threshold and permissible values for parameters $a_{Rb}$ and $a_{Rc}$ as well as object elements wear in time shorter than object service time set by manufacturer.

Therefore threshold values of technical condition $d_{pr.dop}(\theta)_{aRb}$ and adjustment $d_{pr.dop}(\theta)_{aRc}$ parameters are determined. Relations between current threshold value and threshold value permissible in certain moment allow for identifying parametrical (from course of parameter $a_{Rb}$) and momentary (from course of parameter $a_{Rc}$) damage.

2. Damage identification basics

The idea of aptitude assessment method is assumption that every technical object is described by two correlated equations of state: [2, 3]

$$\frac{dD_K}{d\theta} = a_{Rb} D_K + b_{Rb} U, \quad \frac{dU}{d\theta} = a_{Rc} U + b_{Rc} D_K,$$

(1a, 1b)

where:
- $U$ – vector of signal automatics resulting from technical object operation,
- $D_K$ – vector of diagnostic signals connected to object technical condition,
- $a_{Rb}$ – object technical condition parameter,
- $b_{Rb}$ – parameter of influence of technical condition on adjustment capabilities,
- $a_{Rc}$ – adjustment (operation) parameter,
- $b_{Rc}$ – parameter of influence of utility condition on technical condition.

Equations are based on obvious notion that utility value of an object depends on its technical condition (physical wear) and technical condition depends on use intensity and adjustment quality, i.e. technical condition $D_K$ is an environment to utility condition $U$ and utility condition $U$ is an environment to technical condition $D_K$.

From state equations (1a, 1b) technical condition parameters “$a_{Rb}$” and “$a_{Rc}$” are calculated:

$$a_{Rb} = \frac{\Delta D_K}{\Delta \theta (D_K + \tilde{a}_{Rb} U)}, \quad a_{Rc} = \frac{\Delta U}{\Delta \theta (U + \tilde{a}_{Rc} D_K)},$$

(2a, 2b)
where:

\[
\hat{a}_{Rb} = -\frac{\sum_{i=1}^{n} \Delta D_{ki} \Delta U_i}{\sum_{i=1}^{n} \Delta U_i^2}, \quad \hat{a}_{Re} = -\frac{\sum_{i=1}^{n} \Delta D_{ki} \Delta U_i}{\sum_{i=1}^{n} \Delta U_i^2}.
\] (3a, 3b)

Dependencies (2a, 2b) show, that in any moment \( \theta + \Delta \theta \) (\( \Delta \theta \) – sufficiently high) momentary values \( aRb_i \) and \( aRe_i \) might be calculated that are base to quantitative assessment of adjustment and technical condition. Moreover courses of series of values \( aRb(\theta) \) and \( aRe(\theta) \) might be base for identifying momentary and parametric damages and thus reliability characteristics (before the occurrence of catastrophic damage).

While analyzing momentary values of \( aRb \) and \( aRe \) (determined from 2a and 2b formulas), parametrical and momentary damages were noticed to be identifiable also basing on quantitative relations between momentary threshold value and respective momentary permissible value [1]:

\[
d_{pr}(\theta) = i \cdot \mu + a \cdot \sigma \cdot \sqrt{i},
\] (4)

\[
d_{dop} = m \cdot \mu + a \cdot \sigma \cdot \sqrt{m},
\] (5)

\[
d_{pr,dop} = \frac{\theta}{\theta_{max}} d_{dop},
\] (6)

where:

- \( d_{pr}(\theta) \) – \( aRb \) and \( aRe \) parameters threshold,
- \( d_{dop} \) – permissible value of \( aRb \) and \( aRe \) parameters,
- \( d_{pr,dop} \) – permissible value of \( aRb \) and \( aRe \) parameters for moment \( \theta \),
- \( \mu \) – average,
- \( \sigma \) – standard deviation,
- \( i \) – number of subsequent measurement,
- \( a < 3, \)
- \( m = \theta_{max}/\Delta \theta \).

Damage occurs when for moment \( i \) \( d_{pr,i} > d_{dop,i} \).

Having number of damages, reliability characteristics might be determined for each moment of technical object exploitation. Presented method might be applied to relatively low number of measurements and serve to verify reliability characteristics determined basing on large number of measurements.

3. Exploitation reliability characteristics of technical object

Currently, the basis for calculating reliability characteristics is catastrophic, parametrical and transitory damage occurring in exploitation process.

Catastrophic damage is sudden, total and cause immediate and utter loss of object aptitude to proper operation. Following elements are counted to this kind of damage: fracture, deformation, cut, incineration or liquefaction of elements. Number of catastrophic damages \( ma \) is determined for whole set of objects.

Parametrical damage (aging, gradual) leads to partial and removable damage of certain elements, causing in initial phase increasing deterioration of their operation. This damage might be conventional (e.g. surpassing permissible value of adjustment quality). Number of parametrical damages \( mb \) might be determined for single object as well as for set of objects.

Characteristically momentary damage may automatically appear and disappear with no intervention from staff after unknown cause that led to them disappeared. Examples of such damage are sporadic fluctuations of temperature, humidity, accelerations and vibrations. Number of momentary damages
might be determined for each single object. Basing on observed number of single damages, reliability estimators are $R_a^*$, $R_b^*$, $R_c^*$ determined:

$$
R_a^* = \frac{n - m_a}{n}, \quad R_b^* = \frac{n - m_b}{n}, \quad R_c^* = \frac{n - m_c}{n},
$$

(7a, 7b, 7c)

where $n$ is a set of possible damages, that may occur from the beginning to the end of service time.

Determined estimators are basis for determining reliability characteristics:

$$
R(t) = R_a(t) R_b(t) R_c(t) = (R_a^*, R_b^*, R_c^*).
$$

(7d)

As determining $R_b^*$ and $R_c^*$ is almost always very difficult, characteristics is usually based only on $R_a^*$, so only when number of catastrophic damages of numerous set of objects. Course of determining reliability characteristics based on $R_a^*$ ($R_b^*$ might also be partially included) is unequivocally described by standards: PN-77/N-04005, PN-79/N-04031, PN-83/N01052.07.

$$
R(t) = (R_a^*).
$$

(8)

A flaw of this method is being based on dangerous catastrophic damages and should be realized on numerous set objects $n$.

Therefore more and more attempts are made to determine individual reliability characteristics based only on parametric and momentary damages (without catastrophic) [4, 7]:

$$
R(t) = (R_b^*, R_c^*).
$$

(9)

This way requires however special method of identification of number of parametrical $m_b$ and momentary $m_c$ damages as well as set of substitute objects (set of possible damages) $n$. Number $n$ is determined using properties of damage flux ergodicity:

$$
n = \frac{\Delta m(t) + m(t)\lambda \Delta t}{\lambda \Delta t},
$$

(10)

where:

- $\Delta m(t)$ – increase in number of damages,
- $m(t)$ – sum of previous damages,
- $\lambda$ – damage intensity specified by manufacturer or value from service time $\lambda = 1/T$,
- $\Delta t$ – time between damage occurrences.

**4. Object description**

Exploited object is a pump complex presented in Fig. 1.

**Fig. 1. Pump complex: 1 – three-phased electric engine CELMA type Sg 280 M4, 2 – clutch, 3 – 20A40-type pumps with horizontal shaft with NU-type bearings (b)**

Critical point of complex is bearing system with NU-type bearings. Two bearings are located in engine chassis (1) on counter-driving and driving side and two bearings are located in pump chassis (3).
During exploitation of this complex and expensive technical object, characteristic utility, environment and accompanying signals are constantly (or periodically) observed. Their proper processing should grant an unequivocal answer whether object is used and serviced properly, i.e. properly identified in terms of adjustment, diagnostics and reliability.

6. Exploitation research of bearing system in pump complex

Values of parameters $a_{Rb}$ and $a_{Rc}$ as well as average $\mu_i$ and standard deviation $\sigma_i$ for each moment of pump exploitation are presented in Tab. 1.

**Tab. 1. Values of technical condition parameter $a_{Rb}$ and adjustment $a_{Rc}$ for pump complex bearing system [7]**

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$a_{Rb}$</th>
<th>$a_{Rc}$</th>
<th>$d_{pr aRb}$</th>
<th>$d_{pr dop aRb}$</th>
<th>$d_{pr aRc}$</th>
<th>$d_{pr dop aRc}$</th>
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Basing on data from Tab. 1 courses of parameters $a_{Rb}$, $a_{Rc}$, $d_{pr aRb}$, $d_{pr aRc}$, $d_{pr dop aRb}$, $d_{pr dop aRc}$ were determined.

![Fig. 2. Courses of technical condition parameters $a_{Rb}$ (square), adjustment $a_{Rc}$ (square), $d_{pr aRb}$ (asterisk), $d_{pr aRc}$ (asterisk), $d_{pr dop aRb}$ (dot), $d_{pr dop aRc}$ (dot)](image-url)
Subsequently reliability characteristics $R(t)$ is determined according to formula (9).

![Fig. 3. Courses of reliability estimators $R^*(t)$ – squares and courses of reliability characteristics $R(t)$ – black line](image)

Subsequently estimated object lifetime $E(t)$ was determined:

$$E(t) = \int_{0}^{80000} 1.1097 e^{-t^{1.5}} dt = 594\,[h]. \quad (11)$$

7. Conclusions

Method of damage identification based on central limit theorem is credible and efficient. Except for being data obtained from exploitation process (utility and diagnostic signals are measured), method is used without information from machine manufacturer (damage intensity $\lambda$ or service life $E(t)$). This gives good basis for identification of damage and cardinality.

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References