PARTIAL FACTORS IN MODELLING OF STEEL STRUCTURES RELIABILITY ACCORDING TO EUROCODES

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Abstract
Problems associated with the estimation of partial factors for structural systems, subsystems and buckling of individual elements are dealt with in this paper. Aspects related to resistance factors for the section resistance and member resistance are in particular referred to. The Eurocode’s approach of resistance partial factor calibration assisted by experimental data for subframe systems is also summarized.

Keywords: calibration of partial factors, resistance model, partial resistance factor, design resistance, buckling reduction factor

1. INTRODUCTION
Calibration exercise of resistance partial factors in the limit states method of design may be carried out on the basis of different input assumptions. Reliability model and general rules for independent evaluation of design values of actions and their combinations as well as design values of resistance with regard to the adopted target reliability index were presented in the Eurocode dealing with design basis, PN-EN 1990 [1]. Aspects of practical interpretation of Eurocode’s reliability model and calibration of partial factors were presented by Biegus [2] and Gwóźdź et al. [3].

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2. CALIBRATION OF PARTIAL FACTORS

The Eurocode’s global criterion of structural system ultimate limit state is formulated with use of the structure load multiplier $\alpha_{ult}$, evaluated on the basis of nonlinear equilibrium path of imperfect structural model and the level $F_d = \gamma F_k$ of the most unfavourable combination of design actions and nominal mechanical properties of steel and geometrical properties of member sections:

$$\alpha_{ult} = \frac{R_k}{\gamma_M F_d} = \frac{R_k}{\gamma_M \gamma_F F_k} \geq 1.0$$  \hspace{1cm} (2.1)

where: $\alpha_{ult}$ - structure load multiplier referred to the limit point on the nonlinear equilibrium path evaluated for imperfect structural model that, in elastic design, corresponds to the ultimate state of the most stressed span or support cross section from finite number of structural elements (members and joints),

$\gamma = \gamma_f \gamma_S$ - global partial factor for combination of actions $F_k$:

$$\gamma = \sum_j \left( \sum_i \left( \sum_{ji} \frac{F_{repG,j} \gamma_{FG} + K_{Fl} \sum_{ji} F_{repQ,j} \gamma_{FQ,j} \psi_{Q,ji}}{\sum_{ji} \xi_{G,j} G_k} \right) \right)$$  \hspace{1cm} (2.2)

in which the notation according to [1] is adopted,

$\gamma = \gamma_m \gamma_{rd}$ - partial factor for structural resistance in the limit point on the equilibrium path.

Partial coefficients are expressed in a multiplicative format in which the first factor component $\gamma$ includes the randomness of action model, $\gamma_m$ - the random character of structural material properties, while the second factor component $\gamma_{rd}$ refers to the accuracy of action model while $\gamma_{rd}$ to the resistance model.

In Fig. 1 the calibration methodology of partial factors is illustrated. Point A of coordinates (1.00; $\gamma_{F,EN}$) is directly related to the situation recommended in Eurocodes EN and adopted in the basic steel design Eurocode implemented in Poland PN-EN 1993-1-1 [4] as well as in the other parts of PN-EN 1993. Coefficient $\gamma_{F,EN}$ plays the role of substitute, global coefficient of safety. The term substitute in reference to the coefficient $\gamma_{F,EN}$ means that it is not the conventional global safety factor of a constant value independent from the $j$-th combination of ,,i” variable action components. Its value depends upon the permanent and variable load component coefficients $\gamma_i$, $\gamma_{qi}$ and simultaneousness action coefficients $\psi_{Q,ji}$ for the evaluation of representative
values of variable action components for the combination „j” that is considered in the ultimate limit state verification criterion according to (2.1).

Point C of coordinates \((\gamma_{M,\text{LSD}}, 1,00)\) is referred to such a calibration rule in which the most important factors related to uncertainties of both action model and models are accounted for in the substitute, global safety factor \(\gamma_{M,\text{LSD}}\) used for the evaluation of design resistance.

![Figure 1: Possible approaches for calibration of partial factors in semi-probabilistic method of limit states](image)

Calibration of substitute partial factor at point A in Fig. 1 was carried out in Eurocodes assuming that the nominal resistance \(R\) would be of 5% fractile of probability distribution (and equals to the characteristic value \(R_k\)), while the design resistance \(R_d\) would be of 0,1% fractile of the same distribution. National standardization committees implementing Eurocodes may verify the values of partial factors recommended in model codes EN in order to account for local technical and service conditions. In the simplest approach, statistical analysis of data for calibration of partial factor \(\gamma_{M,\text{loc}}\) may be carried out with the assumption that empirical statistics and model coefficients for actions remain at the same level as adopted in [1], and the calibration exercise is limited only to the resistance partial factor. Such an approach is referred to the point B in Fig. 1 and is an approximation the accuracy of which is difficult or even not possible to verify without complete probabilistic simulations that combines the effects of both actions and resistance. If the complete reliability simulations have been carried out taking into account the local conditions of production and service, it might appear that the point B in Fig. 1 is to be shifted to the location of point \(B^*\). Coordinates of the latter point \((\gamma_{M,\text{loc}}^*, \gamma_{F,\text{loc}}^*)\) fulfil the following inequality:
$\gamma_{F,loc}^* < \gamma_{F,EN}^*$. Design criteria based on the partial coefficients $\left(\gamma_{M,loc}^*; \gamma_{F,EN}^*\right)$ may therefore be less economical than in case of using $\left(\gamma_{M,loc}^*; \gamma_{F,loc}^*\right)$, the values being optimally calibrated on the basis of complete reliability simulations.

3. CALIBRATION OF PARTIAL RESISTANCE FACTORS FOR EUROCODES IMPLEMENTATION AT NATIONAL LEVEL

The partial resistance factor $\gamma_M$ in the Eurocode’s approach to design is differentiated according to the type of resistance check. In the Polish National Annex of PN-EN 1993-1-1 [4], the values recommended in EN 1993-1-1 are adopted, namely $\gamma_{M0} = 1.00$ for the section resistance check, $\gamma_{M1} = 1.00$ for the member buckling resistance check and $\gamma_{M2} = 1.25$ for the ultimate strength resistance check (generally for connections). Authors of this paper carried out a comprehensive study in [5÷7] that led to the evaluation of partial factors in a way of ensuring the same safety level in reliability predictions according to Eurocode 3 implemented in Poland as PN-EN 1993-1-1 [4] and the replaced Polish National Code PN-90/B-03200:1990 [8], treated integrally with other related codes in both packages. The values of partial factors yielding from this study are summarized in Table 1 [9].

It is noticeable that both the partial section resistance factor and member buckling resistance factor are not of a constant value equal to unity. The partial section resistance factor $\gamma_{M0}$ changes value from 1.00 to 1.20 while the partial member buckling resistance $\gamma_{M1}$ from 1.15 to 1.25. One may notice the following sequence: the lowest value of $\gamma_{M0}$ is equal to that recommended in the Polish National Annex. For the member buckling resistance the highest value of 1.25 is equal to that recommended for the ultimate strength resistance in [4]. Since generally $\gamma_{M0} \neq \gamma_{M1}$, there is a need to take a closer look at the evaluation of the design value of the member buckling resistance for the estimation of which a variable value of the partial member buckling resistance factor is to be suggested. This is presented in the following.

The partial factor $\gamma_{M1}$ for the evaluation of member buckling resistance $S_{b,Rd}$ (where for compression $S_{b,Rd} = N_{b,Rd}$ and for bending $S_{b,Rd} = M_{b,Rd}$) is allocated to the characteristic buckling resistance $S_{b,Rk}$ at present formulation of EN 1993-1-1 [4]:

$$S_{b,Rd} = \frac{S_{b,Rk}}{\gamma_{M1}}. \quad (3.1)$$

which in turn is related to the characteristic section resistance $S_{c,Rk}$ (understood hereafter as a nominal value) being reduced by a buckling reduction factor $\chi_{bk}$.
\( S_{b,Rk} = \chi_{bk} S_{c,Rk} \).  

(3.2)

Table 1. Partial factors according to authors' proposal.

<table>
<thead>
<tr>
<th>Live to dead load ratio ( \psi )</th>
<th>Partial resistance factor for stocky elements ( \gamma_{M0} ) for stocky elements ( \bar{\lambda}_k \leq 0.2 ) (partial cross section resistance factor)</th>
<th>Partial resistance factor for slender elements (partial member buckling resistance factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sections of class 1 and 2</td>
<td>Sections with slender walls being supported and nonslender supporting walls</td>
<td>Sections with slender walls being supported and slender supporting walls</td>
</tr>
<tr>
<td>( \psi \leq 0.6 )</td>
<td>1,10</td>
<td>1,15</td>
</tr>
<tr>
<td>( 0.6 &lt; \psi \leq 1.5 )</td>
<td>1,05</td>
<td>1,10</td>
</tr>
<tr>
<td>( \psi &gt; 1.5 )</td>
<td>1,00</td>
<td>1,05</td>
</tr>
</tbody>
</table>

*) For sections of class 3, resistance partial factors \( \gamma_{M0} \) are interpolated between those for class 1 and 2, and for class 4.

The buckling reduction factor \( \chi_{bk} \) is a function of the nominal slenderness ratio \( \bar{\lambda}_k \) calculation of which involves the nominal values of the section resistance and the nominal value of elastic buckling resistance of perfect element \( S_{cr} \):

\[
\bar{\lambda}_k = \frac{S_{c,Rk}}{S_{cr}}.
\]

(3.3)

The buckling reduction factor is obtained as a solution of the so-called Ayrton-Perry equation for the nominal buckling resistance [10]:

\[
\left(1 - \chi_{bk}^2 \right) \left(1 - \chi_{bk}\right) - \eta \chi_{bk} = 0
\]

(3.4)

in which the so-called Maquoi-Rondal geometric imperfection coordinate taking the following form:

\[
\eta = \alpha_b (\bar{\lambda}_{k} - \bar{\lambda}_{k0})
\]

(3.5)
is adopted, while $\alpha_b$ is the imperfection factor of the Eurocode’s multiple buckling curve formulation, and the slenderness at the strain hardening region $\bar{\lambda}_{k0} = 0.2$.

Since the partial factor $\gamma_{M1}$ is not generally of a constant value and dependent upon the nominal slenderness ratio, the design member buckling resistance may be written down in a following symbolic way:

$$S_{b,Rd} = \frac{\lambda_{k0}(\bar{\lambda}_k) S_{c,Rk}}{\gamma_M(\bar{\lambda}_k)}$$

where $F(X)$ is referred to the function $F$ dependent upon the argument $X$.

The slenderness dependent partial member buckling resistance factor has to fulfill the following boundary conditions:

$\gamma_M(\bar{\lambda}_k) \rightarrow \gamma_{M0}$ for $\bar{\lambda}_k \rightarrow 0.2$

$\gamma_M(\bar{\lambda}_k) \rightarrow \gamma_{M1}$ for $\bar{\lambda}_k \rightarrow \infty$,

that are equivalent to the following relationships for the resistance:

$$S_{b,Rd} \rightarrow \frac{S_{c,Rk}}{\gamma_{M0}}$$

for $\bar{\lambda}_k \rightarrow 0.2$

$$S_{b,Rd} \rightarrow \frac{S_{cr}}{\gamma_{M1}}$$

for $\bar{\lambda}_k \rightarrow \infty$,

Instead of finding an interpolation function that fulfils the above stated boundary conditions, let us go back to the original Ayrton-Perry equation (3.4). This equation was developed for the Euler case of simply supported imperfect compression member having a sinusoidal initial bow and by assuming that the second-order force state in the most stressed member section would reach the limit state described by a linear interaction curve, regardless of the section class. Keeping up the Eurocode’s format, see equation (3.2), also for checking the design member buckling resistance, gives the following notation:

$$S_{b,Rd} = \chi_{bd} S_{c,Rk}$$

Introducing the partial factors in the original Ayrton-Perry equation (3.4), yields the following modified equation:

$$\left(1 - \gamma_{M1}\chi_{bd}\bar{\lambda}_k^2\right)\left(1 - \gamma_{M0}\chi_{bd}\right) - \gamma_{M0}\chi_{bd} = 0$$

Solution of the above equation is written down as given below:
\[ \chi_{bd} = \frac{1}{\phi_{\text{mod}}^2 + \left( \phi_{\text{mod}} \gamma_{M1} \gamma_{M0} \right)^2} \leq \frac{1}{\gamma_{M0}} \]  

(3.11)

where:

\[ \phi_{\text{mod}} = 0.5 \left[ 1 + \alpha_0 (\overline{\lambda}_k - 0.2) + \left( \overline{\lambda}_k \gamma_{M1} \gamma_{M0} \right)^2 \right] \]  

(3.12)

Let us modify the obtained solution (3.11) in order for the design member buckling resistance (3.6) to be expressed as:

- for members sensitive to overall instability ($\overline{\lambda}_k > 0.2$):

\[ S_{b, Rd} = \frac{\chi_{bk} (\overline{\lambda}_k) S_{c, Rk}}{\gamma_M (\overline{\lambda}_k)} = \frac{\chi_{bd, \text{mod}} (\overline{\lambda}_k) S_{c, Rk}}{\gamma_{M1}} \]  

(3.13a)

- for members insensitive to overall instability ($\overline{\lambda}_k \leq 0.2$):

\[ S_{b, Rd} = \frac{\chi_{bk} (\overline{\lambda}_k) S_{c, Rk}}{\gamma_M (\overline{\lambda}_k)} = \frac{S_{c, Rk}}{\gamma_{M0}} = S_{c, Rd} \]  

(3.13b)

where in the latter case, the design member buckling resistance coincides with the design section resistance $S_{c, Rd}$.

The modified buckling reduction factor takes the following form:

\[ \chi_{bd, \text{mod}} = \frac{1}{\phi_{\text{mod}}^2 + \left( \phi_{\text{mod}} \gamma_{M1} \gamma_{M0} \right)^2} \frac{\gamma_{M1}}{\gamma_{M0}} \]  

(3.14)

The slenderness dependent partial member buckling resistance factor $\gamma_M (\overline{\lambda}_k)$ may now be evaluated as follows:

\[ \gamma_M (\overline{\lambda}_k) = \frac{S_{b, Rk}}{S_{b, Rd}} = \frac{\chi_{bk} (\overline{\lambda}_k)}{\chi_{bd, \text{mod}} (\overline{\lambda}_k)} \]  

(3.15)

where $\chi_{bk}$ is the buckling reduction factor $\chi_{bd, \text{mod}}$ according to (3.14) evaluated for $\gamma_{M0} = \gamma_{M1} = 1.00$. 

\[ \chi_{bd} = \frac{1}{\phi_{\text{mod}}^2 + \left( \phi_{\text{mod}} \gamma_{M1} \gamma_{M0} \right)^2} \leq \frac{1}{\gamma_{M0}} \]  

(3.11)
Figure 2 illustrates the modification of the original Eurocode’s formulation proposed in this paper and represents values of dimensionless member resistances corresponding to multiple buckling curve formulation, for partial factors chosen values from Table 1. The values of $\gamma_{M0} = 1.10$ and $\gamma_{M1} = 1.25$ are selected. The characteristic dimensionless resistances $S_{b,Rk}/S_{c,Rk}$ are represented for all the buckling curves by dashed lines as functions of $S_{b,Rk}/S_{cr}$ while the design dimensionless resistances $S_{b,Rd}/S_{c,Rk}$ (according to authors’ proposal) by dotted lines as functions of $S_{b,Rd}/S_{cr}$. It can be noted that all the characteristic dimensionless resistances are placed within a square bounded by the horizontal $1,0$ and vertical $1,0$ while the design dimensionless resistances are within a rectangle bounded by the horizontal $1/\gamma_{M0} = 0.91$ and the vertical $1/\gamma_{M1} = 0.80$.

In Figure 3, the values of slenderness dependent partial resistance factor $\gamma_M$ are shown for the same set of partial factors used to represent dimensionless resistances in Fig. 2. Besides the slenderness $\bar{\lambda}_k$ changing from zero to infinity, the slenderness index $\beta$ changing from zero to unity is used that is in the following relation to the slenderness:

$$\beta = \frac{1}{1 + (\bar{\lambda}_k)^{\gamma}}$$  \hspace{1cm} (3.16)
From the results shown in Fig. 3 one can conclude that although there are differences in the value of partial factor $\gamma_M(\bar{\lambda}_k)$ in the intermediate values of the member slenderness $\bar{\lambda}_k$ for different buckling curves, they vanish for the low values of slenderness and large values of slenderness. In the former case, the partial resistance factor approaches $\gamma_{M0} = 1.10$ while in the latter case the same factor approaches $\gamma_{M1} = 1.25$.

**Fig. 3. Slenderness dependent partial resistance factor $\gamma_M$**

4. **BASIS FOR THE CALIBRATION OF RESISTANCE PARTIAL FACTOR WITH USE OF EXPERIMENTAL STATISTICS**

In order to evaluate resistance partial factors with use of resistance model based on experimental investigations of the subsystem prototype load-displacement characteristics, it is assumed that the load applied in tests is a deterministic quantity while parameters of the resistance model are random variables. One can adopt the calibration procedure of randomness of resistance model that is summarized in Appendix D of code [1] and referred to the calculation of characteristic and design resistances. In procedures recommended by the code, the following considerations for resistance model are adopted:

- Ultimate limit state function is the function of $n$ basic random variables $X_k$.
- Experimental investigations of random resistance of tested subsystem prototypes are of a sufficient number and they are representative for the
range of resistance model parameters met in practice (e.g. range of deterministically understood slendernesses of subsystem elements).

- Availability of data concerned with random character of basic model variables (i.e. geometric properties of structural steel products and material properties of structural steel) that may have an important impact on the random resistance of tested prototypes.
- Random variables accounted for the ultimate limit state function are statistically independent (no correlation of these variables).
- All the random variables concerned are of normal or log-normal probability distribution.

On the basis of above stated code procedure, the method of its practical application was implemented in [11] for the evaluation of partial factor for steel I-beams lateral-torsionl buckling and in [12] for the evaluation of partial factor for steel plate girders with an account for resistance statistics from test in technical scale. It encompasses the following steps:
1. Setting the computational model of theoretical resistance \( r_t \).
2. Conducting laboratory tests for the evaluation of experimental resistance \( r_e \).
3. Comparison of obtained experimental resistances \( r_{ei} \) with their theoretical counterparts \( r_{ti} \) in their representation on plane \( (r_{ti}, r_{ei}) \) where \( i = 1, \ldots, m \), as indicated in Fig. 4.

![Fig. 4. Comparison values \( r_e - r_t \)](image)

The systematic deviations from the line \( r_t = r_e \) constitute an average error of test procedure or adopted resistance function.
4. Evaluation of correction coefficient \( b \) (best fit coefficient) with use of least square method as well as presentation of probabilistic resistance model \( r \) as the ultimate limit state function dependent upon deterministic parameters \( b, r_{ti} \) and random measure of errors \( \delta_i \).
5. Evaluation of error measure coefficient of variation \( \nu_\delta \).
6. Estimation of conformability of populations of experimental results and theoretical results of computational resistance model (Kolmogorov-Smirnov test of the normality of error distribution).
7. Assignation of the coefficient of variation values of Basic variables \( \nu_{Xi} \) where \( i = 1, \ldots, n \) (e.g. using available statistics from investigations of quality tests of steel strength and structural steel product dimensions).
8. Evaluation of characteristic resistance \( r_k \) and corresponding design resistance \( r_d \) including the effect of numbers, if appropriate.
9. Estimation of resistance partial factor \( \gamma_M \) as the quotient of \( r_k/r_d \), corrected by a coefficient being a quotient \( r_n/r_k \) where \( r_n \) is the nominal resistance evaluated on the basis of nominal values of geometric and material properties of basic variables.

5. CONCLUDING REMARKS

In calibration of partial factors for National Annexes to Eurocodes one has to take into consideration not only the random parameters influencing both action and resistance of computational model but also the local tradition in design procedures. Important difference in Eurocode’s approach and the approach used in replaced national codes in Poland is concerned with calculation of class dependent cross section resistances [5].

In addition, clear explanation is needed for differentiation of resistance partial factors in case of application of different resistance models. In traditional approach, the ultimate limit state is based on the criterion of weakest chain link and related to the load effects evaluated from relevant type of analysis. In such a case, calibration of resistance partial factors is carried out on the basis of reliability analysis of single structural element (member or joint). Steel Eurocode [4] introduced design rules based on more complex resistance models than postulated in [8], since the limit state is referred to divergence stability conditions of subsystem or in general, as indicated by relationship (2.1) - is referred to the limit point on the equilibrium path of entire imperfect model of the system. Question appears whether in design based on more complex models of structural mechanics one may use the same resistance partial factors as in case of traditional design where they are specified for single structural elements, or not, and whether such an approach is safe, or not. Latter aspects are planned to be the subject of investigations conducted within the doctoral thesis of the first author.

REFERENCES


**WSPÓŁCZYNNIKI CZĘŚCIOWE W EUROKODOWYM MODELU NIEZAWODNOŚCI KONSTRUKCJI STAŁOWYCH**

**Streszczenie**

Stan graniczny nośności konstrukcji, projektowanej w tradycyjnym podejściu na podstawie efektów oddziaływania jest oceniany z niezawodnościowego modelu szeregowego. Kalibrację współczynników częściowych do nośności przeprowadza się wówczas na podstawie analizy niezawodności pojedynczego elementu konstrukcji (pręta...

Słowa kluczowe: kalibracja współczynników częściowych, model nośności, współczynnik częściowy do nośności, nośność obliczeniowa, współczynnik wyboczenia

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