INVESTIGATION OF BALL SCREWS FOR FEED DRIVE

The paper presents a method which enables the ball screw choice for machine tools feed drive with increasing cutting speed. It has been proven, that when the velocity deflection in the ball/balltrack area of the transitive phase of ball motion reaches a certain level, the repetitive shocks (impact forces) are generated. Digital simulation of these forces for variable length screws shaft and ball diameters in function of rotational speed has been performed. The research proves that impact forces increase not only as rotational velocity increases, but also with the increase of the ball diameters. The cases of the harmful accordance of the feed drive resonance vibrations with the frequency of the balls entering the load have been examined.

1. INTRODUCTION

Ball screws are widely used as elements of feed systems in NC machine tools (Fig. 1) due to their high rigidity and low sensitivity to externals perturbations. Definite operating condition for a design are [1]:
- the admissible axial pulling force $F_{\text{max}}$,
- unsupported length of ball screw shaft $l_s$,
- the selection of the ball screw shaft mounting condition,
- the speed of quick move of nut the $V_{\text{max}}$,
- the table mass $m$.

Fig. 1. Example of NC machine tool feed system: 1 – motor, 2 – ball screw assembly, 3 – bearing, 4 - table: $\Delta l$ – elastic displacement of feed system, $F$ – load acting (cutting force), $l_s$ – unsupported length of ball screw shaft, $l_s2$ – nut position, $m$ – payload mass
In the Institute of Machine Design Fundamentals a computer program has been designed facilitating selection of ball screws for machine tools feed system [1]. The program enables a selection and calculation of the new type of circulation of balls screw, so-called end cap (Fig. 2). These ball screws are applied most often in a NC feed system where the high feed speeds can be obtained.

![Fig. 2. Ball screw end-cap type ball circulation: 1 – the return element (end cap type), 2 – ball nut, 3 – balls return hole](image)

The first step in designing is to define a rotational speed of the screws \( n \) and basic structural parameters of thread dimensions such as: screw diameter \( d \), lead \( P \), ball diameter \( d_k \), contact angle \( \alpha \), number \( I_z \) of loaded turns. The upper limit of rotational speed is obtained using the equation:

\[
n_{\text{lim}} = \frac{K}{d}
\]  

where: \( K \) – value of rotational speed parameter.

The \( dn \) value express the velocity of the balls moving in circulation circuit. Current standard of \( K \) value are defined within the range 70 000-80 000, but for innovative ball screws could reach to value of 150 000. As the driven shaft in the mechanism operates at high rotating speed, in transient phase appear the impact forces which are considerably larger the than forces \( N \) (Fig. 3) which come into being during quasi static entry in load. In case when the frequency of appearing of the impact forces (definite with angle \( \theta \)) agrees with the frequency of the vibration resonance of the drive, the growth of vibration of machine tool feed system will happen. Therefore it necessities the examining of what turns can set this phenomenon.

2. ANALYSIS OF FORCES AND BALL MOTION

The full cycle of the circulation of each ball consists of the following three phases: the working phase (the loaded balls roll between the coils of the screw and the nut), the transient phase (the balls come in to the working circulation subjected to the growing load or they leave it and the load diminishes to zero), and the phase of the return movement - the non-loaded balls come back to the beginning of the working circulation (Fig. 3). Fig. 3a shows a normal section in the ball/balltrack area. The axial deflection in the ball/balltrack area is defined by the following formula:
\[ \Delta l_{bt} = \frac{w}{\cos \phi \sin \alpha} \]  

where:  \( w = \Delta l_{sb} + \Delta l_{nb} \) – deflection in the ball/balltrack area in normal direction  
(Fig. 3a), \( \phi \) – lead angle, \( \alpha \) – contact angle.

\[ w = \frac{kw^2}{3N^{2/3}} \]  

where \( k \) – coefficient depending on radii of ball, ball track area, nut track, as well as 
Young’s modulus of elasticity and Poisson’s ratio \( \nu \) described in [1],[11].

Fig. 3b shows the position of the ball (movement tangent to helix) during entry into the working circuit. To simplify discussion, it was assumed that total contact deflection \( w \) is realized by deflection of the ball. Angle \( \varepsilon \) determines the acting direction of the variable force \( N_\varepsilon \).

\[ N_\varepsilon = k\varepsilon^{3/2} \]  

where \( w_\varepsilon \) is function of angle \( \varepsilon \) determined in [1].

Fig. 3 and 4a (see section A-A in the Fig. 3a) show the kinematic and geometric relations inside the ball screw. The velocity of the ball center can be described as follows:
The frequency of the balls entering in loading is calculated from [2],[3]:

\[ f_{\theta} = \frac{6n}{\theta} \]  

where: \( n \) – rotating speed [rpm], \( \theta \) – screw rotation angle corresponding to a ball rolling a distance equal to its diameter (Fig. 4).

3. THE IMPACT FORCES SIMULATION

In the transit phase (Fig. 3b) the speed of the elastic ball deflection \( V_0 \) acc. To (7) is a vector perpendicular to the surface of the ball and at the same time perpendicular to the component of the vector of center ball speed \( V \) (moving tangent to helix).

\[ V_o = \frac{wV}{L} \]  

where: \( V_o \) – velocity of ball deflection , \( L = \rho \sin \varepsilon \) – for small values of the angle \( \varepsilon \) (Fig. 3b).

This analysis is based on the elastic impact of the spherical bodies (Hertz theory). The detailed description of the transient phase of the dynamic phenomena is given in [1],[4]. According to this theory the contact deflections can be described as:

\[ w = \left[ \frac{5V_0^2 m_m n_s m_n}{4k (n_s + m_n)} \right]^{2/5} \]
where: $V_o$ – velocity of ball deflection, $m_{s,n}$ – mass of screw and nut respectively.

In the central moment of the impact, which is at maximum approach (for $\varepsilon = 0$) you will find the maximum force of impact:

$$N_{u1} = \left( \frac{5k^{2/3}k_m}{4} \right) \left( \frac{1}{m} \right) \left( \frac{1}{n_w} \right)^{3/5}$$

where: $k_m$ – coefficient depending on mass of bodies (screw and nut) during impact,

$k_p$ – coefficient depending on ball screw size [1].

A simulation of impact forces in function of the rotational speed was made. The results of the simulation are shown in Fig. 5 for ball screw with shaft diameter $d=40$ mm, lead $P=10$ mm and variable ball diameters $d_k$. For calculation we took the axial force $F$ equals 0.45 $C_a$ (dynamic axial load ratings). For mass calculations we took a length of the ball screw shaft $l_s=1000$ mm and length of a nut for the 4 turns of the thread ($l_z=4$). The dashed line $n_{lim}$ shows the upper limit of rotational speed for $K=150000$. The points 1 and 2 show the critical rotational speed in which $N_{u1} > N$. The research proves that impact forces increase not only as rotational velocity increases, but also with the increase of the ball diameters. The results of simulation show, that the value of impact forces for higher rotational speeds are considerably higher than the normal forces $N$ [2],[3].

Fig. 6 shows the relationship between the impact forces and velocity of ball deflection for variable ball diameters $d_k$. The research proves that impact forces increase not only as rotational velocity increases, but also with the increase of the ball diameters.
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4. THE RESONANCE FREQUENCY CALCULATION

The investigation of the resonance frequency of the feed system is possible after calculation of the overall rigidity of ball screw and the feed system (Fig. 7). The static axial rigidity of the ball screws exerts a major influence on positioning accuracy of NC machine tools. The rigidity $R$ constitutes the resistance to deformation and denotes the force $\Delta F$ which is required to effect a component deflection $\Delta l$ (Fig. 1) in the axial direction on load application:

$$R = \frac{\Delta F}{\Delta l} \quad \text{(10)}$$

The rigidity of the system composed of a ball screw and the co-operating machine elements (e.g. machine tool) is defined in general by the axial deflection $\Delta l$ resulting from screw bearing system deflection $\Delta l_b$, nut mounting deflection $\Delta l_m$, deflections of screw shaft $\Delta l_s$, nut body and screw shaft $\Delta l_{n/s}$, and deflection of the ball/balltrack area $\Delta l_{b/t}$:

$$\Delta l = \Delta l_b + \Delta l_m + \Delta l_s + \Delta l_{n/s} + \Delta l_{b/t}$$

The total axial rigidity of the driving system is obtained using the equation [1], [8]:

$$\frac{1}{R} = \frac{1}{R_b} + \frac{1}{R_m} + \frac{1}{R_{bs}} \quad \text{(11)}$$

where: $R_b$ – rigidity of the support bearing, $R_m$ – rigidity of the nut bracket (table montage), $R_{bs}$ – overall rigidity of the ball screw.

The axial static overall rigidity of the ball screw is arrived at by adding the pertinent rigidity values of the components [6]:

Fig. 6. The relationship between the impact forces $N_{ul}$: a) for velocity of ball deflection for variable ball diameters $d_k$, b) for variable number of loaded turns $I_z$, $l_s$ – unsupported length of the ball screw shaft.
\[
\frac{1}{R_{bs}} = \frac{1}{R_s} + \frac{1}{R_{nusf}}
\]

where: \(R_s\) – rigidity of the screw shaft, \(R_{nusf}\) – rigidity of the ball nut unit takes into account the machining inaccuracies.

The axial rigidity of the screw shaft varies depending on the method for mounting the shaft. The rigidity of the ball nut unit is obtained from the following equation \([1],[5]\):

\[
\frac{1}{R_{nusf}} = \frac{1}{R_{n/s}} + \frac{1}{R_{b/sf}}
\]

where: \(R_{n/s}\) – axial rigidity of nut body and screw shaft, \(R_{b/sf}\) – axial rigidity of the ball/ball track area calculated with the corresponding geometry correction factor.

The axial rigidity of the screw shaft varies depending on the method for mounting the shaft (Fig. 7).

Fig. 7. Axial stiffness of preloaded ball screw for fixed-free and fixed-fixed bearing systems [6]

One of rigidity components is the balls/balltrack area rigidity \(R_{btr}\), dependent on Hertz’s deflections. The rigidity in the ball/ball track area is not linear (is correct for one load application only). However, due to machining inaccuracies, it’s are considerably lower than the theoretical ones and depends not only the tolerances but also on geometrical parameters on the thread \([1],[5]\). The calculation of balls/balltrack area rigidity is the main problem because in a result of machining inaccuracies this rigidity is considerably smaller from theoretical. In work \([5]\) the geometry correction factor was introduced in order to estimate the influence of machining inaccuracies and dimensions of ball screw on lowering the level of rigidity:

\[
R_{btr,sf} = R_{btr,sf}
\]

where: \(s_f\) - geometry correction factor:
\[ s_f = \frac{\Delta l_{b/s}}{\Delta l_{b/f,sf}} \]  

where: \( \Delta l_{b/s} \) – the axial deflection due to Hertz stress, 
\( \Delta l_{b/f,sf} \) – deflection due to machining inaccuracies.

The computer simulations, described in [1],[2],[5], made for 1st standard tolerance grade (according to ISO [11]) show that geometry correction factor depend not only on accuracy grade but also on geometrical parameters of the ball screw thread.

After calculation of total rigidity \( R \) it becomes possible to calculate the resonance frequency of team the table of machine tool - the ball screw [7],[10]:

\[ f_r = \frac{1}{2\pi} \left( \frac{R \cdot 10^6}{m} \right)^{1/2} \]  

where: \( f_r \) – resonance of the table-ball screw ensemble [Hz], \( R \) - total rigidity [N/\( \mu \)m], 
\( m \) – payload mass (of the work and table) [kg].

The maximum angular speed of the screw should be considerably smaller from critical speed, near which transverse resonance vibration can happen. Moreover one should check if there is no phenomenon of agreement of the resonance vibrations with the frequency of the balls entering the load, that is:

\[ f_r = f_\theta \]

where: \( f_\theta \) - frequency of the balls entering in loading acc. to (6).

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**Fig. 8.** The relationship between the resonance frequency of the table-ball screw ensemble and the balls entering the load frequency \( f_\theta \) for variable balls diameters \( d_k \) and variable masses \( m \): \( l_s \) – unsupported length of ball screw shaft, \( n_{lim} \) – upper limit of rotational speed
Fig. 9. The relationship between the resonance frequency of the table-ball screw ensemble and the balls entering the load frequency $f_\theta$ for balls diameter $d_k=6.360$ mm and variable unsupported length $l_s$ of the ball screw shaft: $n_{lim}$ – upper limit of rotational speed unsupported length [2]

Fig. 8 shows the results of frequency calculation for divers masses and ball diameters. The ball screw shaft is mounting at both ends, and for the minimum of rigidity $l_{s2}=0.5l_s$. For investigation made in paper [2] the value of drive system resonance $f_r\approx 70$ Hz for a mass $=1000$ kg and $f_r\approx 218$Hz for a mass $=100$ kg. The points P1 and P2 (for $d_k=10$mm) show the agreement of the resonance vibrations with the frequency $f_\theta$ of the balls entering the load. The points marked as n1 and n2 represent a rotational speed threat for a work stability of a drive system. For calculation it was admitted the minimum of $R_s$ rigidity (the nut is in the central position). The lowering of the rigidity due to machining inaccuracies was not considered ($s_f=1$).

Fig. 9 shows the results of frequency calculation for a ball diameter $d_k=6.350$, mass $m=100$ kg and divers length of the screw [2]. The points P1 and P2 show the agreement of the resonance vibrations with the balls frequency $f_\theta$ entering the load. The points marked as n1 and n2 represent the rotational speed threat for a work stability of a drive system. The calculations were made for geometry correction factor $s_f=0.75$.

5. CONCLUSIONS

For certain levels of high rotational speed the repetitive shocks caused by the balls entering into load (impact forces) make growth of vibration and noise [1],[7],[9]. The frequency of impact forces can not be equal to the resonance frequency of driving system because it could cause a incorrect work of the system and damage to the ball screw mechanism. The investigations show, that for definite diameter of the screw and the definite turns the frequency of impact forces can change by selection of diameter of balls. The rigidity of system depends on machining errors and on the position of the nut. The simulation make it possible to predict the rigidity of the ball screws in the early stages of design.
REFERENCES