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**ELECTROMOTIVE FORCE INDUCED IN A LOOP CONDUCTOR UNDER A POWER LINE CONDUCTOR**

The study presents methods of the calculation of electromotive force (emf) induced in a mitigation loop located under an overhead power line conductor. The loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal to the earth surface.

Exact and simplified methods of the determination of the emf are presented. The exact method bases on the earth return circuit theory, whereas the simplified method allows one to calculate the induced emf under the assumption neglecting earth currents.

KEYWORDS: electromotive force, loop conductor, overhead power line, earth return

1. **INTRODUCTION**

The topic of mitigating the magnetic fields produced by overhead power lines is gaining more significance in recent years. In this context, efforts are continuously being done in order to maximize the utilization of the available line corridors without exceeding the tolerable limits of the lines’ magnetic fields.

The use of conductive shields (passive non-compensated or series capacitor compensated loops) to mitigate extremely low frequency magnetic fields generated from power lines has been proposed [1 - 8]. Their behavior principle is based on the electromagnetic induction law: time varying, primary magnetic fields, generated by AC sources, induce electromotive forces driving loops currents – additional field sources, which modify and reduce the primary magnetic field.

In the study exact and simplified methods of the determination of the electromotive force (emf) induces in a loop conductor hanging under a straight overhead current carrying conductor of infinite length are presented. The loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal to the earth surface.

The exact method bases on the earth return circuit theory taking into account the earth current induced in the earth, whereas the simplified method allows one to calculate the induced emf under the assumption neglecting earth currents.

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2. GENERAL EQUATIONS

2.1. Vector potential of an infinitely long current carrying conductor

An infinitely long conductor is placed at height \( h_k \) above the earth surface, Fig. 1, and carries the current, which flows in direction of the \( x \)-axis. The current varies with the time as \( \exp(j\omega t) \) where \( \omega \) is the radian frequency. The \( x, y \) plane is considered to be the earth surface. It is assumed that the earth is an isotropic, homogeneous medium of finite conductivity \( \gamma \). The magnetic permeability of the soil and of the air is \( \mu_0 \). The displacement currents in both regions: the air and the earth are neglected.

![Fig. 1. An infinitely long current carrying conductor above the earth surface](image)

The vector potential in the system has only one component \( A_x(y, z) \). The \( x \)-component of the vector potential satisfies the Poisson equation in the air.

The vector potential in the air takes the form [9]:

\[
A_x(y, z) = \frac{I}{2\pi} \int_0^\infty \left[ \frac{e^{-|z-z_k|}}{u} + \frac{(u - \alpha)e^{-(z-z_h)u}}{u(u + \alpha)} \right] \cos[(y - y_k)u] \, du \quad z \geq 0 \tag{1}
\]

where:

\[
\alpha = \sqrt{u^2 + k^2} \tag{2}
\]

and

\[
k^2 = j\omega\mu_0\gamma \tag{3}
\]

2.2. Mutual impedance in a system of parallel conductors with earth return

Consider the system shown in Fig. 2, consisting of two overhead conductors. In the system of rectangular co-ordinates the \( x, y \) plane is the surface of the earth. The infinitely long overhead conductors are located parallel along the \( 0x \) axis at heights \( h_k \) and \( h_m \) respectively. The current in the conductor \( k \) varies with time as \( \exp(j\omega t) \). The horizontal distance between conductors is \( a_{km} \). The earth conductivity is \( \gamma \).
The induced electric field intensity in the air is:

$$\overline{E_i} = -j \omega \overline{A}$$  \hspace{1cm} (4)

The mutual impedance of two conductors is the negative ratio of the electromotive force induced in the second conductor to the current in the first conductor \([9]\):

$$Z_{km} = -\frac{1}{I_k} \oint_C \overline{E_i} \cdot d\overline{l}$$  \hspace{1cm} (5)

whereas the integration path \(C\) lies on the axis of the second conductor.

It follows from the eqns (5), (4) and (1) that the unit-length mutual impedance in the system shown in Fig. 2 is given by:

$$Z'_{km} = \frac{j \omega \mu_0}{2\pi} \int_0^\infty \left[ \frac{e^{-u(h_k - h_m)}}{u} + \frac{(u - \alpha)e^{-u(h_k + h_m)}}{u(u + \alpha)} \right] \cos(a_{km}u) du$$  \hspace{1cm} (6)

The improper integral appearing in (6) has no a closed-form solution.

If the distance between conductors \(s_{km} < 0.3\Delta\), where:

$$s_{km} = \sqrt{(h_k - h_m)^2 + a_{km}^2}$$  \hspace{1cm} (7)

and

$$\Delta = \frac{1.85}{\sqrt{\omega \mu_0 \gamma}}$$  \hspace{1cm} (8)

the unit-length mutual impedance can be approximately calculated from the relationship \([10]\):

$$Z'_{km} = \frac{\omega \mu_0}{8} + j \frac{\omega \mu_0}{2\pi} \ln\frac{\Delta}{s_{km}}$$  \hspace{1cm} (9)

The application of the simplified formula (9) when \(s_{km} > 0.3\Delta\) can cause considerable errors in the mutual impedance calculations. In this case, the mutual
impedance has to be calculated from the eqn. (6) and the integral can be solved numerically or it can be expanded into infinite series.

3. ELECTROMOTIVE FORCE INDUCED IN A LOOP

3.1. Exact solution

Consider the magnetic flux passing through a surface of the horizontal, rectangular loop located underneath an infinitely long overhead conductor with a current, as in Fig. 3.

The vector potential in the air given by eqn.(1) can be now written in the form:

$$A_y(y, z) = \frac{\mu_0 I_y}{\pi} \left\{ \frac{1}{4} \ln \frac{(z + h_y)^2 + (y - y_k)^2}{(z - h_y)^2 + (y - y_k)^2} + \int_0^\infty (n - a - jb)e^{-(z+h_y)k|p|} \cos[(y-y_k)k|p|] \, dp \right\}$$

where:

$$u = \|p\|$$

$$\sqrt{n^2 + j} = a + jb$$

$$a = \sqrt{\frac{n^3 + 1 + n^2}{2}}, \quad b = \sqrt{\frac{n^3 + 1 - n^2}{2}}$$

Magnetic flux penetrating through the loop (horizontal location of the loop, Fig. 3) is obtained by applying the equation:

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{l} = I A_y(y = y_f', z = h_f) - I A_y(y = y_f'', z = h_f)$$

Fig. 3. Current carrying conductor and a loop
where \( l \) denotes the length of the longitudinal loop-conductor.

Finally, the electromotive force (emf) induced in the loop takes the form:

\[
emf = -\frac{j\omega \mu_0 I l}{\pi} \left[ \ln \left( \frac{(h_k - h_i)^2 + (y_j - y_k)^2}{(h_k + h_i)^2 + (y_j - y_k)^2} \right) \right] + 
\int_{0}^{\infty} be^{-(h_i + h_k) + h} \left[ \cos(y_j - y_k) \cos h - \cos(y_j - y_k) \cos h' \right] dn + 
\int_{0}^{\infty} (n-a) e^{-(h_i + h_k) + h} \left[ \cos(y_j - y_k) \cos h' - \cos(y_j - y_k) \cos h' \right] dn
\]

\[15\]

3.2. Approximate solution

If the effects of the currents induced in the earth on the total magnetic field produced under the overhead conductor are negligible as compared with the effects due to the current flowing in the overhead conductor only, the magnetic flux passing the loop can be determined from the relation (16):

\[
\Phi_{\text{approx}} = \int_{y_i}^{y_i'} B_z \text{approx} (y, z = h_i) dy
\]  

(16)

In the relation (16) \( B_z \text{approx} (y, z) \) denotes the \( z \) component of the magnetic flux density produced by current carrying infinitely long conductor in homogeneous medium (air) [11]:

\[
B_z \text{approx} (y, z) = \frac{\mu_0 I}{2\pi} \frac{y - y_k}{(z - h_k)^2 + (y - y_k)^2}
\]

(17)

Thus

\[
\Phi_{\text{approx}} = \frac{\mu_0 I}{4\pi} \ln \frac{(h_i - h_k)^2 + (y_j - y_k)^2}{(h_i - h_k)^2 + (y_j - y_k)^2}
\]

(18)

The electromotive force can be next obtained from eqn. (19) in the simplified form:

\[
emf_{\text{approx}} = -\frac{j\omega \mu_0 I l}{4\pi} \ln \frac{(h_i - h_k)^2 + (y_j - y_k)^2}{(h_i - h_k)^2 + (y_j - y_k)^2}
\]

(19)

The other possibility to derive the approximate formula for the induced emf consists in the calculation of the emf with the application of mutual impedances defined in the earth return circuit theory [9]. Consider the equivalent circuit of a loop as shown in Fig. 4.
The active elements of the circuit represent voltage sources with the source voltages:

\[ E'_l = -I_k Z'_l l, \quad E''_l = -I_k Z''_l l \]  \hspace{1cm} (20)

where \( E'_l \) and \( E''_l \) denote the induced electric field intensities along the longitudinal conductors of the loop, \( I_k \) is the current in the overhead conductor, \( l \) is the length of the longitudinal loop conductor, and \( Z'_l \) and \( Z''_l \) are the unit-length mutual impedances between the overhead conductor and the longitudinal loop-conductors given by the relation (9).

The passive element \( Z_s \) of the circuit shown in Fig. 4 is the self impedance of the longitudinal loop-conductor. The equation for the unit-length self impedance of the conductor \( k \) with the radius \( r_k \) is [12]:

\[ Z'_s = j \omega \frac{\mu_0}{2\pi} \ln \left( \frac{2(h_k + p)}{r_k} \right) \]  \hspace{1cm} (21)

where:

\[ p = \frac{1}{\sqrt{j \omega \mu_0}} = \frac{\delta_k}{2} (1 - j) \]  \hspace{1cm} (22)

and

\[ \delta_k = \frac{1}{\sqrt{\pi \mu_0 \gamma}} \]  \hspace{1cm} (23)

is the penetration depth.

The resultant driving \( emf \) in the system shown in Fig. 4 takes the form:

\[ emf_{approx} = I_k l (Z'_l - Z''_l) = -I_k \frac{j \omega \mu_0 l}{2\pi} \ln \frac{s_{kl}}{s_{kl'}} \]  \hspace{1cm} (24)
It should be noted that the approximate formula (24) obtained for the resultant electromotive force in the loop is identical with the relation (19).

4. EXEMPLARY CALCULATIONS

Exemplary calculations of the emf induced in the rectangular loop hanging under a power line conductor as shown in Fig. 3 have been carried out assuming that the conductor with the unit current $I_k = 1$ A is located horizontal to the earth at height $h_k = 10$ m; $y_k = 0$ m and $\gamma = 10^{-2}$ S/m. The results of calculations with the use of the exact method and the approximate method are shown in the Table 1. The results show an excellent agreement.

Table 1. Induced electromotive force

<table>
<thead>
<tr>
<th>Loop parameters</th>
<th>Induced emf [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exact method</td>
</tr>
<tr>
<td>$L = 300$ m; $h_l = 8$ m, $y'_l = 2$ m; $y''_l = 10$ m</td>
<td>$\mathbf{-j0,024}$</td>
</tr>
<tr>
<td>$L = 300$ m; $h_l = 8$ m, $y'_l = 2$ m; $y''_l = 30$ m</td>
<td>$\mathbf{-j0,0445}$</td>
</tr>
<tr>
<td>$L = 300$ m; $h_l = 8$ m, $y'_l = 2$ m; $y''_l = 50$ m</td>
<td>$\mathbf{-j0,0541}$</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The design of installation generating low-frequency magnetic and electric fields requires access to effective analytical and computational tools. The paper presents procedures of determining the electromotive force produced in a current loop under power line conductor.

Exact and approximate methods are developed for analyzing electromotive force in a loop located in the vicinity of an overhead conductor with the current having prescribed values. The loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located horizontal to the earth surface.

The exact method is based on the theory of earth return circuits and takes into account the earth currents. The approximate methods assume that effects of earth currents on the emf are negligible.

It is shown that the use of the approximate method of the calculation of the induced emf can be successfully used in the practice.
The results derived can be used as the foundation for almost every study on the power line fields for conditions that are almost always satisfied for power engineering applications.

REFERENCES


