This paper presents an evaluation of the Hypoplastic Clay constitutive model for finite element analysis of deep excavations and displacements induced by excavations in the influence zone. A detailed description and formulation of the Hypoplastic Clay soil model is included. A parametric case study of a deep excavation executed in Pliocene clays is presented. FE analysis was performed using several soil models (Mohr-Coulomb, Modified Mohr-Coulomb, Drucker-Prager, Modified Cam-Clay, Hypoplastic Clay) and the results were compared to in-situ displacements measurements taken during construction. Final conclusions concerning the suitability of the Hypoplastic Clay model for deep excavation modelling in terms of accurate determination of horizontal displacements of the excavation wall, the uplift of the bottom of excavation, and, most importantly, vertical displacements of the terrain in the vicinity of the excavation are presented.

Keywords: deep excavation, influence zone, FE modelling, soil models, hypoplastic model

1. INTRODUCTION

In Poland, in recent years, a big development in numerical modelling of geotechnical structures using Finite Element Method (FEM) has been observed. In the case of the design of retaining structures, the determination of internal forces and horizontal displacements of the excavation wall, the more time-effective dependent pressures method is usually applied [7]. When detailed analysis is necessary, e.g. evaluation of the influence of excavation on surrounding structures in order to assure its safety, the Finite Element Method is used [1], [6]. The choice of the most appropriate,
constitutive soil model, together with correct definition of its parameters, is then the greatest challenge [4].

Over the years, Mohr-Coulomb is the most popular and widely used elastic-perfectly plastic constitutive model, both for the design and scientific purposes, due to the simplicity of its parameters. However, it should be noted that it has several limitations, and when applied to deep excavation analysis tends to result in excessive deformation of the wall and excessive settlements of the surrounding terrain, which is not observed in practice. Common observations for over-consolidated soils prove that terrain around the excavation rises during excavation (unloading) [1], while in numerical analysis settlements are obtained. Concepts of applying the Hardening Soil model in such cases are noted [13]. The author decided to evaluate the suitability of the hypoplastic model – Hypoplastic Clay, implemented in GEO5 FEM software [14], for modelling soil response during excavation basing on the experience described in [8] of the effective use of a hypoplastic model for tunnel excavation.

2. DESCRIPTION OF THE HYPOPLASTIC CLAY MODEL

The hypoplastic clay model is applicable for modelling of fine grain soft soils [14]. It belongs to the family of standard phenomenological models. As for description of soil response, it falls into the group of critical state models (Cam-Clay, Generalized Cam-Clay), and allows for reflecting a different stiffness in loading and unloading, and softening or hardening, in dependence with the soil compaction and the change of volume in shearing (dilation, compression). The current stiffness depends not only on the load direction, but also on the current state of soil given by its porosity. Unlike the Cam-Clay model, it strictly excludes tensile stresses in soil, see Fig. 1 (left).

![Fig. 1. State boundary of hypoplastic model: left - in the meridional plane, right - in the deviatoric plane](image-url)
In the case of the Hypoplastic Clay model, the standard yield surface is replaced by a so-called boundary state surface. Its projection into the deviatoric plane is shown at Fig. 1 (right). The non-associated flow rule results into a non-symmetric stiffness matrix.

Mathematical formulation of the Hypoplastic Clay constitutive model [5].

Hypoplastic models are generally described by a single nonlinear tensorial equation yielding the stress rate $\mathbf{T}$ as a function of stretching rate $\mathbf{D}$ [8]. The general stress-strain relation is as follows:

\begin{equation}
\mathbf{T} = f_x L : \mathbf{D} + f_y f_0 \mathbf{N} \|\mathbf{D}\| 
\end{equation}

\begin{equation}
\mathbf{N} = L : \left( -\mathbf{Y} \frac{\mathbf{m}}{\mathbf{m}} \right) 
\end{equation}

The hypoelastic tensor $\mathbf{L}$ is

\begin{equation}
\mathbf{L} = 3(c I + c_2 a^2 \mathbf{\hat{T}} \otimes \mathbf{\hat{T}}) 
\end{equation}

where $\mathbf{1}$ is the second order unity tensor, $\mathbf{I} = \frac{1}{2} (\mathbf{1}_{ij} \mathbf{1}_{jk} + \mathbf{1}_{ij} \mathbf{1}_{kj})$ is a fourth-order unity tensor and

\begin{equation}
\mathbf{1} = \text{tr} \mathbf{T} : \mathbf{1}, \quad \mathbf{\hat{T}} = \mathbf{T} / \text{tr} \mathbf{T}, \quad \mathbf{\hat{1}} = \mathbf{\hat{T}} - 1/3 
\end{equation}

\begin{equation}
a = \frac{\sqrt{3} (3 - \sin \varphi_c)}{2 \sqrt{2} \sin \varphi_c} 
\end{equation}

The degree of nonlinearity $\mathbf{Y}$, with the limit value $\mathbf{Y}=1$ at Matsuoka-Nakai failure surface is calculated by

\begin{equation}
\mathbf{Y} = \left( \frac{\sqrt{3} a}{3 + a^2} - 1 \right) \frac{(I_1 I_2 + 9I_3) (1 - \sin^2 \varphi_c)}{8 I_3 \sin^2 \varphi_c} + \frac{\sqrt{3} a}{3 + a^2} 
\end{equation}

With stress invariants $I_1, I_2, I_3$,

\begin{equation}
I_1 = \text{tr} \mathbf{T}, \quad I_2 = \frac{1}{2} \left( \mathbf{T} : \mathbf{T} - (I_1)^2 \right), \quad I_3 = \text{det} \mathbf{T} 
\end{equation}

The tensorial quantity $\mathbf{m}$ defining the hypoplastic flow rule has the following formula:
Barotropy and pyknocropy factors $f_s$ and $f_d$ are

$$f_s = \frac{\text{tr}T}{\lambda} \left( 3 + a^2 - 2a^2 a^{3/2} \right), \quad f_d = \left[ -\frac{2\text{tr}T}{3p_r} \exp\left( \frac{\ln(1+e) - N}{\lambda} \right) \right]^{\alpha}$$

where $p_r$ is the reference stress 1 kPa and the scalar quantity $\alpha$ is calculated by

$$\alpha = \frac{1}{\ln 2} \ln \left[ \frac{\lambda + \kappa^*}{\lambda' + \kappa} \left( \frac{3 + a^2}{a^{3/2}} \right) \right]$$

Finally, factors $c_1$ and $c_2$ are calculated as follows:

$$c_1 = \frac{2(3 + a^2 - 2a^2 a^{3/2})}{9r}, \quad c_2 = 1 + (1 - c_1) \frac{3}{a^2}$$

Details regarding the development of the model and its formulation can be found in [5].

The Hypoplastic Clay model requires defining five constitutive parameters: angle of internal friction for constant volume $\phi_{cv}$, slope of swelling line $\kappa^*$, slope of the normal consolidation line $\lambda^*$, origin of the normal consolidation line $N$, and ratio of unit and shear modulus $r$.

Basic parameters $\kappa^*$, $\lambda^*$ and $N$ may be determined from a bilinear diagram of isotropic consolidation in a log-log scale, Fig. 2 (left). In the GEO5 FEM software, however, they may be also calculated.
from parameters of the bilinear Cam-Clay model: slope of swelling line $\kappa$, slope of normal consolidation line $\lambda$ - in semi-logarithmic scale, Fig. 2 (right), void ratio $e_{\text{max}}$ for normal isotropic consolidation by pressure of 1 kPa. Detailed information on the interpretation of other model parameters may be found in [5] and [14].

![Bilinear diagram of isotropic consolidation](image)

**Fig. 2.** Bilinear diagram of isotropic consolidation: left - Hypoplastic Clay, right – Cam-Clay

### 3. Hypoplastic Clay Model Evaluation - Case Study

#### 3.1. General Information

This case study concerns the analysis of a 14.6 m deep excavation executed in Pliocene clays [9], Fig. 3. The cut and cover method was used for the construction of the metro station; the stability of diaphragm walls during excavation was ensured by two levels of grouted anchors and one level of steel struts. The excavation was 20 m wide and approx. 150 m long. The following construction stages were distinguished and modelled in numerical FE analysis, Fig. 4:

- **Stage 1** – Greenfield
- **Stage 2** – Construction of the peripheral diaphragm walls
- **Stage 3** – Excavation - 4.55 m below ground surface (bgs)
- **Stage 4** – Installation and prestressing of the first row of anchors at -3.73 m bgs, spacing – 2.4 m
- **Stage 5** – Excavation – 8.85 m bgs, stressing of anchors ($F = 80\% \text{ of } F_D$, $F = 400$ kN)
- **Stage 6** – Installation and prestressing of the second row of anchors at -7.85 m bgs, spacing – 1.3 m
- **Stage 7** – Excavation – 11.85 m bgs, stressing of anchors ($F = 80\% \text{ of } F_D$, $F = 480$ kN)
- **Stage 8** – Installation of steel struts at -10.85 m bgs ($\phi 508/14.2\text{mm}$, spacing 2.0 m)
- **Stage 9** – Final excavation – 14.6 m bgs.
Further construction stages - including construction of foundation plate and underground slabs - were not modelled. The author decided to omit it as the results obtained in these stages are usually realistic; the loading is effectively modelled by geotechnical FE software.

Fig. 3. Construction site

Fig. 4. Typical cross section including construction stages and geotechnical conditions
3.2. GEOTECHNICAL CONDITIONS

In Warsaw, the subsoil is composed of Tertiary clayey deposits covered with a complex of Quaternary formations from the Pleistocene and Holocene periods. In the area of the analysed excavation, Tertiary, the Poznań formation clay layer is strongly elevated (due to glaciitectonic and erosion processes). Detailed information about the Tertiary clay layer in Warsaw may be found in [2], [3], [10], [11], [12].

Soil conditions at the construction site and in the vicinity are as follows, Fig. 4:
- directly under the surface, 2.5 m thick anthropogenic soils (so called artificial fills, fills) occur,
- then, up to great depths, Poznań formation clay occurs. In the model, this layer is divided into two layers taking into account the change in its parameters with depth.

There is no general groundwater table in the analysed area. The groundwater table is discontinuous and carries low quantities of water. Water is located mostly in sand lenses and pockets within the clay body. As a result, water was not taken into consideration in the numerical model. Basic geotechnical soil parameters of all soil layers specified above (considered in the numerical analysis) are compiled in Tab. 1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$\phi'$ (°)</th>
<th>$c'$ (kPa)</th>
<th>$E$ (MPa)</th>
<th>$K_0$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill</td>
<td>18</td>
<td>25</td>
<td>0</td>
<td>25</td>
<td>0.577</td>
<td>0.30</td>
</tr>
<tr>
<td>Clay 1</td>
<td>20.7</td>
<td>18</td>
<td>10</td>
<td>80</td>
<td>0.917</td>
<td>0.35</td>
</tr>
<tr>
<td>Clay 2</td>
<td>20.7</td>
<td>18</td>
<td>15</td>
<td>100</td>
<td>0.783</td>
<td>0.35</td>
</tr>
</tbody>
</table>

3.3. NUMERICAL ANALYSIS

Finite element plain strain analysis was carried out using GEO5 FEM software [14]. One typical geotechnical and structure geometry cross-section was modelled. The following basic assumptions were adopted: final excavation depth – 14.6 m, diaphragm wall height – 20.7 m, diaphragm wall thickness – 0.8 m, two rows of anchors and one row of steel struts supporting the diaphragm wall. Construction stages and geometry details as specified in chapter 3.1, Fig. 4, geotechnical conditions and soil parameters as specified in chapter 3.2, Fig. 4, Tab. 1.
Model dimensions are 40 x 100 m. Finite element mesh and model are shown at Fig. 5. FE model mesh, generated automatically, was built of 7048 nodes, 4189 elements (2641 15-nodes triangle surface elements, 387 beam elements, and 1161 contact elements). Anchors and struts are elements added in construction stages, after mesh generation.

For the basic model, an elastic perfectly plastic constitutive material model Mohr-Coulomb was chosen (calculation Series 1). Then the model was modified several times to: Modified Mohr-Coulomb (Series 2), Drucker-Prager (Series 3), Modified Cam-Clay (Series 4), and, finally, Hypoplastic Clay (Series 5). The parameters of Mohr-Coulomb, Modified Mohr-Coulomb, and Drucker-Prager models are given in Tab. 1. Parameters for clay layers defining the Modified Cam-Clay and Hypoplastic Clay models are given in Tab. 2 and Tab. 3, respectively. All parameters except $e_{\text{max}}$, $\varphi_{c}$, and $r$ for the HC model where established based on laboratory and field tests carried out at the Department of Geotechnics and Underground Structures of the Warsaw University of Technology [3], [10], [11], [12]. Parameters $e_{\text{max}}$, $\varphi_{c}$, and $r$ were taken as average for similar soil types [13]. Precise specification of all parameters of the Hypoplastic Clay model for Poznań formation clays will be a part of future study.

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$\nu$</th>
<th>$K_0$</th>
<th>OCR</th>
<th>$M_{\text{Is}}$</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$e_{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kN/m$^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay 1</td>
<td>20.7</td>
<td>0.35</td>
<td>0.917</td>
<td>2.5</td>
<td>0.710</td>
<td>0.019</td>
<td>0.071</td>
<td>0.57</td>
</tr>
<tr>
<td>Clay 2</td>
<td>20.7</td>
<td>0.35</td>
<td>0.783</td>
<td>1.5</td>
<td>0.710</td>
<td>0.019</td>
<td>0.071</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Table 3. Parameters of Hypoplastic Clay model

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$K_0$</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$e_0$</th>
<th>$e_{\text{max}}$</th>
<th>$\phi_{\text{cv}}$</th>
<th>$r$</th>
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<td>0.019</td>
<td>0.071</td>
<td>0.57</td>
<td>2.5</td>
<td>27</td>
<td>0.3</td>
</tr>
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<td>Clay 2</td>
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<td>0.019</td>
<td>0.071</td>
<td>0.57</td>
<td>2.5</td>
<td>27</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The results of all calculation series are further presented in form of graphs compared to geodesic in-situ measurements in terms of: horizontal displacements of the top of the excavation wall in construction stages – Fig. 6, the uplift of the bottom of the excavation – Fig. 7, vertical displacements of terrain behind the excavation in four construction stages (representing successive excavation): stage 3 - Fig. 8, stage 5 - Fig. 9, stage 7 – Fig. 10, stage 9 – Fig. 11.

![Fig. 6. Horizontal displacements of the excavation wall](image)

![Fig. 7. The uplift of the bottom of the excavation](image)
Fig. 8. Vertical displacements of terrain – construction stage 3

Fig. 9. Vertical displacements of terrain – construction stage 5

Fig. 10. Vertical displacements of terrain – construction stage 7
4. SUMMARY AND CONCLUSIONS

Based on the results of the analysis of the parametric case study it may be concluded that simple elastic perfectly plastic constitutive material models (Mohr-Coulomb, Modified Mohr-Coulomb, Drucker-Prager), with accurate calibration of their parameters may be used for the design of retaining walls embedded in Poznań formation clays. These models provide good estimates of both horizontal displacements of excavation walls (Fig. 6) and the uplift of the bottom of the excavation (Fig. 7).

It should be highlighted that appropriate, realistic results may only be obtained providing precise definition of soil parameters, especially the modulus of elasticity, resulting from special, dedicated soil testing campaigns for small strain ranges [2], [3], [11], [13]. On the contrary, these models do not perform very well in terms of the analysis of the influence of the excavation on surrounding terrain and structures. As mentioned before, in-situ displacement measurements usually show a raising of terrain behind the excavation walls, whereas in the same area the analysis (Series 1-3) performed using these models tends to result in settlements (Fig. 8-11).

Further observation based on the case study is that the critical state Modified Cam-Clay model (Series 4) is not suitable for modelling of pre-consolidated Poznań formation clays. This model significantly overestimates all analysed parameters (vertical and horizontal displacements of the structure and the soil body).
Finally, as the main point of interest, it may be stated that the Hypoplastic Clay model (Series 5) proved to be fully suitable for modelling excavations made in Poznań formation clays specific for the area of Warsaw, both in terms of the design of excavation walls (Fig. 6) and the prediction of vertical displacements of terrain (Fig. 8-11). It should be pointed out that the use of this model resulted in a very good mapping of the slight raise of terrain observed in reality in all construction stages.

Though it may be concluded that the Hypoplastic Clay constitutive model is suitable for modelling deep excavations executed in Poznań formation Pliocene clays in Warsaw.

REFERENCES


Received 18.09.2016
Revised 15.11.2016
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Tab. 3. Parametry modelu Hypoplastic Clay
OCENA MODELU HYPOPLASTIC CLAY DO MODELOWANIA GŁĘBOKICH WYKOPÓW

Słowa kluczowe: głębokie wykopy, strefa wpływu, MES, modele materiałowe gruntów, model hypoplastyczny

STRESZCZENIE

W artykule przedstawiono ocenę modelu konstytutywnego stanu krytycznego Hypoplastic Clay (II hypoplastyczny) do analizy metodą elementów skończonych głębokich wykopów i oceny przemieszczeń w strefie ich wpływu. W pracy zawarto również szczegółowy opis i sformułowanie modelu Hypoplastic Clay. Zaprezentowano parametryczne studium przypadku dotyczące głębokiego wykopu wykonanego w Warszawie, w ilach plioceńskich formacji poznańskiej. Analizowany obiekt to wykop stacji metra o wymiarach 20x150m i głębokości 14.6m wykonywany metodą odkrywkową z zastosowaniem dwóch rzędów kotew iniekcyjnych i jednego poziomu rozpar stalowych jako rozparcia. Analizę modelu przeprowadzono z zastosowaniem metody elementów skończonych, programem GEO5 MES, z uwzględnieniem kilku modeli materiałowych gruntu (Mohr-Coulomb, Modified Mohr-Coulomb, Drucker-Prager, Modified Cam-Clay, Hypoplastic Clay), a wyniki porównano z geodezyjnymi pomiarami przemieszczeń in-situ wykonanymi w trakcie budowy. Wnioski końcowe dotyczące przydatności zastosowanych modeli, a w szczególności modelu Hypoplastic Clay, do modelowania głębokich wykopów pod względem dokładnego określenia poziomych przemieszczeń ścian wykopu, odprężenia dna wykopu i co najważniejsze pionowych przemieszczeń terenu w sąsiedztwie wykopu są następujące:

- Proste modele sprężysto idealnie plastyczne (Mohr-Coulomb, Modified Mohr-Coulomb, Drucker-Prager) mogą mieć zastosowanie do projektowania obudów głębokich wykopów posadowionych w ilach plioceńskich formacji poznańskiej pod warunkiem prawidłowej kalibracji ich parametrów (zwłaszcza precyzyjnego określenia wartości modułu odkładzenia na podstawie właściwie dobranych badań in-situ, w zakresie małych odkształceń). Modele te właściwie odzwierciedlają wartości przemieszczeń poziomych ścian wykopu a także odprężenia dna wykopu. Nie powinny mieć jednak zastosowania do analizy wpływu wykopu na otoczenie ponieważ nie odzwierciedlają właściwie odprężenia związanego z odciążeniem wywołanym wykopem. W efekcie, zamiast obserwowanego zwykle unoszenia terenu sąsiadującego z wykopem w obliczeniach z zastosowaniem tych modeli uzyskuje się jego osiadanie.

- Model stanu krytycznego Modified Cam-Clay znacząco przeszczerza wartości wszystkich analizowanych parametrów (przemieszczeń poziomych i pionowych, ścian i terenu), nie powinien mieć więc zastosowania do analizy wykopów wykonywanych w prekonsolidowanych ilach plioceńskich rejonu Warszawy.

- Wyniki analizy wykonanej z zastosowaniem ocenień, szerzej omówionego, modelu Hypoplastic Clay wykazały bardzo dużą zgodność z wynikami pomiarów, zarówno w odniesieniu do wartości poziomych przemieszczeń ściany wykopu jak i w odniesieniu do pionowych przemieszczeń terenu. Należy podkreślić, że wyznaczone teoretyczne przemieszczenia pionowe terenu wokół wykopu bardzo precyzyjnie odzwierciedlają delikatne unoszenie terenu zaobserwowane w trakcie budowy we wszystkich fazach głębiania wykopu. Stwierdza się więc, że model ten może mieć zastosowanie do modelowania głębokich wykopów posadowionych w ilach plioceńskich.