Mathematical model for digital simulation of steam turbine set dynamics and on-line turbine load distribution

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Abstract

The general structure and purposes of the distributed control systems (DCS) are reminded in connection with the capability to use them for on-line optimization of the load distribution in large, multiunit power stations. The basic assumptions, equations and structure of a model of backpressure extraction turbine are presented. The tasks, realized using this model in collaboration with the DCS’s measuring and archive systems, are defined. The principles of using and correcting the input data for the considered tasks are described. Model calculation results and recorded measuring data are compared. The input data quality and influence of the DCS record mode on the abilities to use the model appropriately are discussed.

Keywords: Mathematical modeling; Steam turbine; Online optimization

Nomenclature

\begin{align*}
E &= \text{pressure coefficient} \\
G &= \text{mass flow rate, kg/s} \\
N &= \text{generated capacity, kW} \\
\dot{Q} &= \text{heat flux, kW} \\
T &= \text{temperature, } ^\circ\text{C} \\
X &= \text{pressure factor} \\
a &= \text{coefficient} \\
f &= \text{non-linear function} \\
h &= \text{specific enthalpy, kJ/kg} \\
j &= \text{normal enthalpy, MJ/kg}
\end{align*}

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1 Introduction

The development of modern methods of collecting control and measuring data brought new capabilities of computer applying in power plants. At the present time, all the power units under construction are being equipped with digital distributed control systems (DCS). In the older power stations, the traditional, analog control systems and automatics have been recently reconstructed to digital ones.

Apart from the advantages resulting from transmitting information, digital systems enable implementing various application software [1, 8], e.g., designed for monitoring operation or supporting operators’ tasks. In Poland, the majority of large power systems were equipped with such systems several years ago [9]. Within a typical configuration of such systems, presented on Fig. 1, signals from multiple distributed measuring systems (connected with various facilities, being parts of the controlled installation) are transmitted to a common, main plant network path (system highway). Those signals become then available for the process automatic, operation, engineer, archive and other computer stands (stations), realizing special tasks and collaborating with the main path. Mentioned signals become available for the following elements, cooperating with this highway:

- controllers (automatic processing stations), acquiring measuring data via input/output modules and through digital control of individual elements
of the system, according to chosen algorithms. They are simply specialized computers equipped with interfaces which ensure cooperation with the highway;

- operator’s stations (operator’s terminals), i.e., computers, usually of industrial type, operating in multiuser operating system. They are designed to visualize operating conditions of the object being controlled, by the operator’s entering the results of measurements of operating parameters of the controlled system, transmitted from the system highway, in selected system diagrams. Using a terminal, the operator can control system parameters, is informed about breakdown conditions, can follow not only current values of selected parameters, but also – on the basis of the records in the memory – their changes. Therefore, operator’s stations act as integrated consoles, including indicators and recorders of changeable configuration and alarm systems of a traditional control room;

- engineering stations (process supervising stations), i.e., computers designed to diagnose the state of DCS, as well as reconfigure, modify and tune control systems and construct systems diagrams;

- archiving stations which are computers equipped with disc memories of large capacity. They enable recording information concerning operating parameters of the supervised and controlled system;

- object part of the system (measuring apparatus and automatics systems), i.e., digitally controlled automatics systems, sensors and measuring slotted lines. Basic feature of the object part is a possibility of controlling executive elements, by getting information from a common highway. Therefore, a control signal can be generated in different sections of DCS.
Together with providing convenient mode of information transfer and storage, DCS enable implementation of various application software products [1]. For instance, the availability of the system memory resources enables online complex analysis of the main power facilities (boilers, turbine generators).

The basic assumptions and dependences of a mathematical model of back-pressure extraction steam turbine, used to determine the machine performance, are presented in the paper. In online collaboration with a DCS, the calculation results were used for optimization of the load distribution in a large combined heat and power plant with characteristic, complex technological output structure.

2 The idea of digital simulation

The important feature of DCS’s is their ‘open’ architecture which allows adding extra devices, playing quite new functions, or similar to the old ones. Using suitable tools, it is possible to transmit information outside the system. It is also possible to equip the operator’s station with a computer which will supply signals, similar to those from a selected device or subassembly of the system controlled by DCS. These signals may be generated by an appropriate computer program.

Simulator is a device being a model of the selected real object, recreating its features (usually only chosen ones), with its aid, one may conduct experiments, not using the real object. This possibility is important especially if the cost of operating the simulator is substantially lower than that associated with using the real object. Mathematical model usually gives such possibilities. Development of digital computers enabled simulation technique to spread out as a tool used to examine the properties of newly designed industrial equipment and systems, including power systems.

The following simulators can be mentioned [1]:

- testing simulators designed to examine the properties of new systems, machines and equipment;
- training simulators used to train people operating real objects.

Originally, training simulators appeared where an error of the operator can result in a dangerous situation, involving life hazard or serious material losses, e.g., when training pilots or maintenance personnel in nuclear power plants. In order for a training simulator to fulfill its purpose, it ought to provide good cooperation with an operator, which should be as similar to a real object as possible. Information should be transmitted in the same way as in the case of a real object.

Spreading of digital control systems in the power industry provided possibilities of building training simulators. ‘Classic’ control room is replaced by an operator’s terminal, i.e., a computer communicating with the operator with the
aid of monitor and a keyboard. Such cooperation with the system, currently applied in a typical real object, is also a natural method of exchanging information between computers and their users. Therefore, a simulator can be a computer program, supplying the operator’s station with information of the same structure as a simulated device, or, more precisely, a set of measuring sensors watching its operation. Calculations made by a simulator should be performed within time period shorter than real time. Some time reserve is necessary to communicate with the system (receiving information about parameters being input values and transmitting the results of the calculations, converted into digital signals). Fulfilling this condition is an essential constraint \[1, 8, 10\]. It defines a class of models that can be applied in the program.

3 Model assumptions and structure

Simulator, being a computer program, can be made of independent elements (modules), interconnected by an input and output system. A natural structure, corresponding to the structure of a power unit, is applying separate modules, simulating operation of a boiler, together with auxiliary devices, and a turbine set. When discussing the idea of the industrial simulator of a steam turbine set presented below has been confined to the description of processes with medium and high load (we have omitted the analysis of start-up and outage, in particular from the point of view of thermal and resistance effects), associated directly with the realisation of a basic technological process in:

- turbine and (within limited range) an electric generator,
- heat exchangers cooperating with the turbine,
- feed water path,
- external steam intake (collectors, bleed steam), cooperating with control systems.

The considered turbine has two uncontrolled and one controlled extraction. The mass flow rates of the inlet and outlet steam are measured at the points, marked on the flow diagram, presented in Fig. 2. The measurements are performed separately in the two parallel pipelines. At the uncontrolled extractions, the pressure and mass flow rate are measured. As for the considered purposes, this measuring system is not uniform: it is insufficient in relation to pressure and temperature data (e.g., no information about the steam parameters between the turbine stage groups is provided); and on the other hand, it provides full information about all the inlet and outlet steam flows, except the gland leaks. The main nominal parameters of one of the modeled turbine generators are set in Tab. 1.
Figure 2. Thermal diagram of a turbine set being modelled, including basic parameters of the working medium under calculated operating conditions (WH-1, WH-2 – feed water heaters).

Figure 3. Scheme of the turbine mathematical model, corresponding to the flow diagram on Fig. 2.

The model was used for realization of two part and consequent tasks:

- simulation: calculation of the turbine performance, including the generated electric capacity, $N_{el\,calc}$, and stage group internal efficiencies, basing on the measuring data;
- data reconciliation: correction of the measured steam mass flows, converging
the calculated capacity, $N_{el \, calc}$, to the measured generated electric capacity, $N_{el \, meas}$, and closing the mass balance.

The model and corrected data were next used for optimization of the load distribution, the results from which are beyond this paper as a task.

Module structure of the turbine mathematical model was introduced. The following separately modeled elements (modules) of the flow path were distinguished:

- three turbine stage groups in the high-pressure (HP) part,
- one stage group in the medium-pressure (MP) part,
- governing stages of the HP and MP parts,
- control-valve systems of the HP and MP parts.

### Table 1. Exemplary main nominal parameters of modelled turbine generator.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal rotational speed</td>
<td>$n$</td>
<td>rpm</td>
<td>3000</td>
</tr>
<tr>
<td>Nominal generated capacity</td>
<td>$N_{el}$</td>
<td>kW</td>
<td>55000</td>
</tr>
<tr>
<td>Live steam pressure</td>
<td>$p_0$</td>
<td>MPa</td>
<td>13</td>
</tr>
<tr>
<td>Live steam temperature</td>
<td>$T_0$</td>
<td>°C</td>
<td>535</td>
</tr>
<tr>
<td>Live steam mass flow rate</td>
<td>$G_0$</td>
<td>kg/s</td>
<td>100</td>
</tr>
<tr>
<td>Extraction 1 pressure</td>
<td>$p_{E , 1}$</td>
<td>MPa</td>
<td>4.0</td>
</tr>
<tr>
<td>Extraction 2 pressure</td>
<td>$p_{E , 2}$</td>
<td>MPa</td>
<td>2.95</td>
</tr>
<tr>
<td>Controlled extr. pressure</td>
<td>$p_E$</td>
<td>MPa</td>
<td>1.75</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>$p_{out}$</td>
<td>MPa</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The model diagram with determined connections between these elements and balance nodes, at which stream splits take place, is presented on Fig. 3.

The turbine mathematical model was based on the mass and energy balance equations, with the mass and energy accumulation neglected in all the modules (‘static’ model):

\[
\sum_{i=1}^{n} G_i = 0 ,
\]

\[
\sum_{i=1}^{n} G_i h_i + Q - N = 0 ,
\]

and characteristics of the above mentioned elements.

For instance, the model of turbine stage group (in the considered case, impulse type) is described by [1, 2]:
mass flow rate equation (output quantity: pressure), based on the similarity principles and determined [3] for constant in time, rated rotational speed:

$$\bar{G} = \frac{G}{G_0} = \frac{p_\alpha}{p_{\alpha 0}} \sqrt{\frac{j_{\alpha 0}}{\alpha E}} ,$$  

where

$$j = \frac{k}{k - 1} p \nu ,$$  

$$E = \sqrt{1 - \left(\frac{\pi - \pi_{cr}}{1 - \pi_{cr}}\right)^2} , \text{ for } \pi > \pi_{cr}$$  

or

$$E = 1 , \text{ for } \pi \leq \pi_{cr}$$  

and

$$E_0 = \sqrt{1 - \left(\frac{\pi_{0} - \pi_{cr 0}}{1 - \pi_{cr 0}}\right)^2} , \text{ for } \pi_{0} > \pi_{cr 0}$$  

or

$$E_0 = 1 , \text{ for } \pi_{0} \leq \pi_{cr 0} ,$$  

$$\pi = \frac{p_\omega}{p_o} ;$$

• internal efficiency equation, based on approximation, dependent on the operation conditions and design solution [4–6]:

$$\bar{\eta} = \frac{\eta}{\eta_0} = f(a, X) ,$$

where

$$a = f(X, z),$$

$$X = \frac{\frac{1}{\pi} - 1}{\frac{1}{\pi_0} - 1} .$$

In general two basic approaches to determining turbine stage group characteristics under off-design operating conditions by calculations can be distinguished: the approach based on expanded mathematical models of the group, requiring detailed data about the group, and the approach based on approximation models, using simpler dependencies of integral type [5]. The second of these approaches, in which a general lack of proper data and information occurs, is considered in this paper. Eq. (8) is used for internal efficiency calculation. Procedures shown in [4, 5] for determination of coefficient a in Eq. (9) are used for all stage groups of the turbine.

The next step of analysis included:
transformation of the working medium in the analysed devices, to determine
the values of its parameters (for successive instants) at characteristic points
of the system,

• simulation of the variability of other characteristic quantities being mea-
ured, resulting from the course of basic technological process, e.g., power
at the generator terminals, rotor speed, thermal elongation of the turbine
rotor with respect to the frame, degree of opening of control valves.

Diagram of modelled extracting back pressure turbine set of the rated power
of 55 MW is presented in Fig. 2. Simple regeneration system, lack of steam con-
denser and linkage between the turbine and technological supply points resulted
in limiting the number of system elements being modelled and, consequently, the
number of equations and time used to calculate the model. Symbol used in Fig. 2
are: $t_{FW}$ – feed water temperature, $G$ – mass flow.

Factors determining the course of events covered by simulator within the tur-
bine set are thermal and flow processes, associated with the flow of the working
medium within the turbine and regeneration system. As for feed water heaters,
the following effects are also of importance: heat exchange between the working
medium and metal walls and change of state (condensation of steam). Change
of thermal fields, deformations and stress do not determine directly the course of
technological process, however they are constraints that, if exceeded, may lead to
defects. In the turbine set model the following volume modules have been distin-
guished [1]: regions between groups of stages, extraction steam chambers, main
steam lines, turbine exhaust, water and steam regions within feed water heaters,
and water regions in the feed water path.

Mathematical models of these regions, taking into account accumulation ca-
pacity of mass and energy, have been prepared using state coordinates which in
a typical case are pressure and temperature. In case of steam regions of heat ex-
changers, state parameters are (saturation) pressure and condensate level. When
constructing the models, equations of mass and energy balance have been used.
In a general case they have the following form:

$$\frac{dm}{d\tau} = G_\alpha - G_\omega ,$$  \hspace{1cm} (11)

$$\frac{dH}{d\tau} = \sum G_\alpha h_\alpha - \sum G_\omega H_\omega - \nu \frac{dp}{d\tau} + \dot{Q} ,$$  \hspace{1cm} (12)

where: $G$ – mass flow; $H, h$ – enthalpy and specific enthalpy, respectively; $p$ –
pressure; $Q$ – heat flux through the balance shield, $\tau$ – time; indices $\alpha$ and $\omega$ refer
to inflowing and outflowing working medium flow, respectively.
4 Application and results from measurements and calculations (steam turbine)

The presented steam turbine mathematical model was applied for online simulation of the turbine performance under the conditions of unit connection with a boiler and header connection from the technological output side [1, 7]. The steam mass flow rates, pressures and temperatures, and generated capacity (see Fig. 2) were used as input quantities. Their full list is as follows: \( G_{0L} + G_{0R} \), \( G_{E1} \), \( G_{E2} \), \( G_{EL} \), \( G_{ER} \), \( G_{outL} \), \( G_{outR} \), \( T_0 \), \( p_0 \), \( p_{out} \), \( N_{el} \) with \( T_0 \), \( p_0 \) and \( p_{out} \) averaged from the corresponding values, measured at the left and right steam pipelines. The output quantities were: pressures between the particular stage groups, extraction and outlet steam temperatures, and generated capacity \( N_{el,calc} \).

In the industrial measuring systems of power installations, considerable errors in online measurements may occur. The main reasons are the lack of current equilibrium condition of the installation, caused by operation parameter changes, and incorrectness of the measuring paths. Unlike under laboratory conditions, the moment of realization, and often the measurement conditions, cannot be selected (e.g., it may be difficult to find straight pipeline section long enough for accurate measurement). Most easily considerable inconsistency is observed in measurements of mass flow rates. Before the realization of the presented task, analysis of consistency of registered measurements of inlet, extraction and outlet steam mass flow rates was made for the considered turbine generator. Taking the glands into account, it appeared [1] that the (variable in time) difference may reach even 24% of the live-steam mass flow rate.

Additional results were obtained by realizing the reconciliation task – Eq. (13). The steam mass flow rates \( G_{0corr} \), \( G_{E1corr} \), \( G_{E2corr} \), and \( G_{ote,corr} \) were determined by correcting the corresponding measured values counter-proportionally to their level of reliability, expressed using the relation \( SD_i/G_i \). The reconciliation was to provide mass and energy balance, and calculated generated capacity equal to the measured one, using the condition:

\[
\sum_{i=1}^{n} \left( \frac{G_i - G_{icorr}}{SD_i} \right)^2 \rightarrow \min ,
\]

where \( SD_i \) permissible values of the standard deviation of measured steam mass flow rates \( G_i \).

The considered model was used as a component of a machine-hall mathematical model. The latter is a part of calculation procedure, collaborating with an optimization program. The models of the rest of the steam turbines were created in analogous way.

Comparisons between measurement and calculation results are presented on
Figs. 4–7. These results were registered during special session, but during normal operation. In Fig. 4, the lower (thicker) line illustrates the changes in the generator load during measuring. These changes are expressed by relating the measured electric capacity, $N_{el \text{meas}}$, to the rated generator capacity, $N_{el0}$. The mode of collaboration with the measuring and archive systems conditioned the reading (and writing) time step to 3 min. The registration of the steam mass flow rates started with a 30-minute delay to the generated capacity and caused lack of data for the beginning period in Fig. 5 and 6.

Four circumstances caused the differences between the measurement and simulation results, appearing in the considered entertainment: firstly, the model is inaccurate; secondly, the measurement (signal transmission) paths happened to fail; thirdly, the measurements themselves have certain range of accuracy, which causes lack of balance even in cases of relatively long steady operation; and finally, such steady conditions are hardly obtainable during normal operation, and all the data were processed online.

As it is visible in Fig. 4, in the case of the generated capacity the divergence did not exceed 10% (the exception in one of the points is incidental). A correction of the live-steam mass flow rate of similar range appeared to be sufficient (see Fig. 5 and 6a). The differences were significantly larger in the cases of the extraction- and outlet-steam mass flow rates (Figs. 6b–6c).

The very large correction in the case of outlet steam mass flow rate needs additional explanation. In order to avoid imbalance, this measuring data were rejected as input data for the simulation task, and the outlet steam flow rate
Figure 5. Changes of inlet, controlled extraction and outlet steam mass flow rate – comparison between measurement results and corrected data, used in the model. \( G_{00} \) – rated live steam flow rate.

resulted directly from the other flow data. For the reconciliation purposes, the outlet steam mass flow rate was assumed to be least reliable, as the operating conditions of the corresponding measuring devices were far from the rated or required.

The time, needed for realization of the calculation tasks within the subsequent time steps, depended mainly on the quality of the measurement reading. In the cases of especially bad reading data, the calculation results could not be obtained fast enough to start the next scheduled reading. Then the old calculation results were left in the system and the program proceeded to calculations for the next time step. In the cases of failed reading of the generated capacity, the data correction was impossible. The above-mentioned cases are marked and commented in Figs. 4 and 5. The cases of lack of correction are marked also in Fig. 6. Simulation results illustrating relative divergence between measured and calculated generated capacity (electrical output) of the turbine set are shown in Fig. 7.

5 Example of simulation results (turbine set)

As was mentioned the results have been obtained from a simulator operating in a PC environment, without communication with DCS, i.e., above all with control systems of individual devices. Such an approach made it possible to trace characteristic behaviour of the mathematical model under the influence of simulated external changes in the form of disturbance of the values of selected parameters,
Figure 6. Relative divergence between the measured and corrected mass flow rates of the: live steam (a), steam from the controlled extraction (b), outlet steam (marked points (c) – no correction results).
but without affecting the behaviour of the object, caused by control processes.

When applying the above procedure, it was not possible to represent all external influences in case of a real object. For instance, full simulation of steam supply points influencing the turbine set was not possible. To do this, the computer program should be supplied with the models of such supply points for all pressure levels and the models of other devices (turbine sets, reduction stations etc.), cooperating with these supply points. Simulation program was only supplemented with the model of bleeding collector, simulating constant in time and independent of the conditions steam input \( G_{\text{col}} = \text{const} \), responding to condition changes in the extraction chambers, controlled by suitable changes of pressure and temperature of the medium.

The chosen example of simulated process refers to the change of turbine load. It was effected by changing the position of control valves of the high-pressure section (HP). Within the model, this position is represented by \( Z_1 \), i.e., degree of opening of the equivalent HP valve. Simulation was made for constant in time position \( Z_2 \), of control valves of the intermediate-pressure (IP) section. \( Z_1 \) values changed from 1000 to 1100 s, starting from the moment of activating the simulation, with the speed of 0.5%/s, starting from the initial value of \( Z_2 = 80\% \), corresponding to earlier, steady working conditions of the turbine. The effect of this change on the power developed by the turbine set and mass flow of live steam, taken from the controlled bleeder and of exhaust steam is presented in Fig. 8. Other parameters describing behaviour of the turbine and regenerative exchangers are presented in Figs. 9–11. Diagram of the turbine set (Fig. 2) shows points corresponding to simulated parameters of the medium, exhibited by the characteristics.
Water level in regenerative exchangers and in feed water container (Fig. 9), as distinct from all other presented values, does not stabilise during calculations. This is the result of pressure drop in the turbine bleeders, which leads to similar changes in the feed water heaters WH-1 and WH-2, but with constant pressure in the deaerator. Lower pressure at the steam side of the exchangers results in the drop of the condensation point, and with unchanged flow of the feed water, its temperature decreases after the exchangers, in accordance with the curves shown in Fig. 11. Water level in regenerative exchangers and in feed water container (Fig. 9), as distinct from all other presented values, does not stabilise during calculations. This is the result of pressure drop in the turbine bleeders, which leads to similar changes in the feed water heaters WH-1 and WH-2, but with constant pressure in the deaerator. Lower pressure at the steam side of the exchangers results in the drop of the condensation point, and with unchanged flow of the feed water, its temperature decreases after the exchangers, in accordance with the curves shown in Fig. 11.

Mass flow of steam taken from the controlled bleeder, initially reduced due to a disturbance resulting in pressure drop, returned to its original value. It is the result of the above mentioned constant power input from the 1.8 MPa collector. In case of the second bleeder (Fig. 10), there is an instantaneous (and in case of the first bleeder, full) stoppage of steam uptake, till the end of the simulated process. The reason is pressure drop in the bleeder, below its value in the collectors receiving steam. Pressure values have been inputs in the model.
Figure 9. Changes of the condensate level in the regenerative exchangers and in feed water container, due to limited power of the turbine set, as shown in Fig. 8.

Figure 10. Changes of mass flow of steam within the turbine set, due to limited developed power, as shown in Fig. 8.

This example illustrates the possibility of real time simulating thermal and flow processes within basic systems of the steam turbine set. It has been adapted to calculating capabilities of modern personal computers. Description of the mathematical model of the turbine set, used to build a turbine set simulator, is presented in [1]. The idea of a steam power unit simulator for a large industrial thermal electric power station is put forward in [9]. Selected results concerning power unit simulator, including boiler and turbine set modules, are given in [8] and [10].
6 Summary

A comparison of calculation results from a steam turbine mathematical model for load optimization in large industrial combined heat and power station, used for simulation with initial data, continuously and directly collected from the digital distributed control system (DCS), is presented. Problems, occurring with such using of measuring data, resulting from such implementation, are pointed.

The basic problem with any model, in which current measuring data are being used as input data, is to interpret that data appropriately, i.e., to eliminate the improper values, to balance the mass and energy flows, etc. The presentation of the principles of using and correcting the input quantities was made in [1], but is beyond this paper as a task. Nevertheless, the presented results illustrate very well the importance of the input data quality. The above problems do not disqualify the attempts to use measuring data for online simulation. The optimization program, realized with using the presented model, discovered considerable capabilities of overall plant performance improvement in the form of increasing the generated electric capacity with unchanged technological output, live-steam parameters and overall demand.

Another approach, alternative to the presented model formulation, could be based on characteristics, approximating the dependence between the internal capacity and mass flow rate for the particular, properly differentiated parts of the turbine. This could significantly simplify the mathematical description. The initial attempts to introduce such model formulation for the considered task gave promising results and further investigation in that direction is held.

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Received 15 October 2014

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