Vibrations of a Rotating System Containing a Textile Element

Abstract
The study examined the behaviour of a rotating system containing a textile element. A mathematical model is formulated and a numerical investigation is performed. It has been found that the layer of fibres limits the mass motion and stabilises the rotary motion, thereby preventing a decrease in the speed of the system.

Key words: nonlinear vibration, rotating body, fibre compression.

Introduction

The behaviour of fibre assemblies under compression has been studied in works [1 - 3]. In work [4] the author proposed a mathematical model of a layer of fibres submerged in a fluid. The properties of the layer were assumed to be determined with respect the axis of rotation and the resistance to fluid flow that is squeezed by the bending elasticity of fibres and the static eccentricity. In paper [5] it was shown that the textile layer significantly restricts the amplitude of vibration of a supported mass. The application of textile items in supports of rotating bodies was studied in paper [6]. The stability of rotation of a disc carrying a mass supported with a cubic spring and moving along its diameter was investigated in works [7, 8]. In this paper, the behaviour of a mass moving along the groove of a rotating rod containing a textile element is studied. The paper is a continuation of works [6, 7].

Equations of motion

The system, shown in Figure 1, consists a rotating bar with a moment of inertia of \( B \) driven by motor torque \( M \), a slider of mass \( m \) that can move along the bar in a groove and a textile resilient support. \( w \) denotes the coordinate of the slider along the bar and \( w_0 \) the static eccentricity.

The sum of projections of forces on the \( w \)-axis, that of the moments of forces with respect the axis of rotation and the equation of motor torque have form of the set of Equation 1.

\[
\begin{align*}
(B + m(v_0 + v)) &\frac{d^2\alpha}{dt^2} + \\
+ 2m &\frac{d\alpha}{dt}\left(w_0 + w\right) - M = 0, \\
m &\frac{d^2w}{dt^2} - m\left(\frac{d\alpha}{dt}\right)^2 \left(w_0 + w\right) + c \frac{dw}{dt} \frac{dw}{dt} + k \frac{dw}{dt} + k w = 0, \\
\frac{c}{M} &\left(\frac{dw}{dt}\right)^2 \frac{dw}{dt} \frac{dw}{dt} + k w = 0,
\end{align*}
\]

\[
\begin{align*}
\frac{dM}{dt} &= C\left(1 - \frac{\alpha}{\Omega}\right) - M.
\end{align*}
\]

Here, \( \alpha \) is the angle of rotation of the bar, \( (T,C,\Omega) \) the motor constants, and \( (k, L, c, H) \) are the resilient element material constants, as defined in work [5].

With the non-linear elastic force and negative centrifugal force (2) we can associate energy function (3).

\[
F_\alpha = \frac{\partial E}{\partial \alpha} = k w\left(1 - \frac{w}{L}\right)^2 - \frac{d\alpha}{dt}\left(w_0 + w\right)^2, \\
E = k w^2 - \frac{1}{2}m \left(\frac{d\alpha}{dt}\right)^2 \left(w_0 + w^2\right).
\]

Numerical results and discussion

The set of Equations 1 was numerically integrated using the Runge-Kutta method for mass \( m = 5 \) kg, inertia \( B = 0.001 \) kgm², eccentricity \( w_0 = 0.001 \) m, motor idle speed \( \Omega = 100 \) rad/s, motor...
time constant \( T = \frac{2\pi}{\Omega} \), material parameters \( H = 0.01 \text{ m}, L = 0.01 \text{ m}, \) and \( k = \frac{\Omega^2}{25} = 2000 \). The initial conditions were taken as follows: \( \alpha = 0, \frac{d\alpha}{dt} = \Omega, M = C(\Omega - \frac{d\alpha}{dt}) \) and various values of \( w \) and \( \frac{dw}{dt} \). In Figure 2 the radial velocity \( \frac{dw}{dt} \) of mass and energy \( E \) versus the mass displacement \( w \) are shown. Figures 2.a and 2.b show the results for a freely rotating system \( (C = 0) \) without damping \( (c = 0) \), and Figures 2.c and 2.d show the results for the motor characteristic slope \( C = 1 \text{ kgm}^2/\text{s} \) and damping coefficient of the material \( c = 0.05 \text{ kg/m} \). The energy plot shown in Figure 2.b is bounded by vertical asymptotes at \( w/L = \pm 1 \). The angular velocity \( \frac{d\alpha}{dt} \) of the rotating disc versus the angle of revolution \( \alpha \) is shown in Figure 3, (a) for a linear spring of stiffness \( k \) and (b) for a layer of fibres of stiffness \( \frac{k}{(1-w/L)^3} \). It can be seen that the angular velocity could be maintained for a nonlinear restoring force, while it was not the case for a linear spring.
The behaviour of the nonlinear system can be understood by looking at the energy curve in Figure 2.b. The energy $E$ has three extreme points that represent the state of equilibrium. There is one maximum and two minimums, one deeper than the other. The maximum represents an unstable state of equilibrium. The deeper minimum represents a stable equilibrium state. The other minimum is metastable, that is stable for sufficiently small disturbances. If, however, the disturbance goes over the maximum, then the point will move to the deeper minimum. The corresponding trajectories in the phase space are shown in Figure 2.a, where we see three types of closed curves representing periodic orbits, and two not closed homoclinic orbits. The behaviour of the system subjected to a damping force and driven by an electric motor is shown in Figures 2.a and 2.b. It can be seen that the system when starting from a slightly different initial condition moves toward one of two attracting points associated with energy minimums. The results shown in Figure 2 resemble the behaviour of the solutions of Duffing’s equation.

Conclusions

1. As opposed to a linearly elastic spring, the fibrous element limits the mass motion, which stabilizes the rotation of the system, not allowing for a decrease in the rotational speed of the driving motor.

2. The rotating system containing a textile element that can moves along its diameter exhibits three states of equilibrium: metastable, unstable and stable.

3. Restricting the motion on the rotating plane [6] to that along the line on the rotating body changed the unstable saddle to a meta-stable attractor.

4. The model of textiles under compression analysis can be applied for studying the behaviour of fabric handling by manipulators [9, 10].

References


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