Analytical description of the functioning of electromagnetic pulser of pairwise action

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Abstract. The article analyses the distribution of forces in statical equilibrium on the example of electromagnetic pulser of pairwise action. It views specifications of two stable states of equilibrium corresponding to the cycles of sucking and compression in the operation of the milking unit. The research suggests the way of determining durations of transition states of a pulser which directly impacts the quantities of technical characteristics of a milking unit and the nature of machine milking of cows.

Key words: electromagnetic pulser, vacuum-gauge pressure, valve, membrane, gauged hole, the constant of time.

INTRODUCTION

Automatization of milk production requires an employment of electromagnetic pulser in milking units for cow milking both in the stalls and in milking rooms. Electromagnetic pulser guarantee control over technological process of machine milking of cows.

Specific constructive and technological properties of structure and running of pulsers influence the parameters of milking units running on the whole. As capabilities of milking units (correlation of cycles, cycles durations, frequency of pulsings, velocity of cycles shifting) are specified by pulsers, the nature of transition processes in pulsers and their influence upon the parameters of milking require more detailed research.

RECENT RESEARCH AND PUBLICATIONS ANALYSIS

Constructions of pneumomembrane pulsers have been examined and researched many times [1-4], in particular pulsers of the pairwise action [2, 4]. In addition, perspective directions of pulsers construction development were outlined [2]. Regimes of running and the character of modifications of vacuum-gauge pressure in working chambers are described in the main considering pneumatic pulsers of the coincidental and pairwise actions [5, 6] as well as electromagnetic or electrically controlled pulsers [7, 8]. The mentioned above papers, however, have not managed to present a comprehensive definition of interdependences caused by the input of vacuum-gauge or atmospheric pressures in the working chamber and dynamics of elementary compounds of electromagnetic pulsers. The researches [8-10] described in detail the operation of electromagnetic actuating element under the impact of vacuum-gauge pressure on the examples of electromagnetic vacuum-regulator and pulser or some of their constituents [11]. In addition, they presented the milking unit by means of classical methods of the theory of the system of automated regulation [12]. This method, however, requires the preceding determination and proving of quantities of coefficients of magnification.

The presented specific moments of cows milking operations and the requirements to pulsers [13, 14] point to the vital directions of the devices investigation. The construction and running peculiarities of the suggested pneumomagnetic pulser of pairwise action [15] and the milking unit employing it [16] allow to carry out pneumodynamical analysis and to develop analytical dependences for determining parameters of the transition processes.

OBJECTIVES

The article is aimed at discovering dependences of the electromagnetic pulser functioning in statics and dynamics as a pneumo-mechanical electrically controlled device of converting steady vacuum-gauge pressure into variable which secures simulation of the constant of time for transition processes in electromagnetic pulser of a milking unit.

MAIN PRESENTATION

When discussing functioning of a pulser one should mention two steady position of equilibrium of movable element of a pulser which alternate with each other in time and, correspondingly, provide both cycles of a milking unit running (sucking and compression).
Let us consider distribution of forces influencing the moving elements of a pulser during the cycle of sucking (Fig. 1).

**Fig. 1.** Distribution of forces during the cycle of sucking:

- $V_I$ – volume of a chamber with constant vacuum-gauge pressure; $V_II$ – volume of the working chamber; $V_III$ – volume of the regulating chamber; $F_{III}$ – force formed by the difference of pressures in the chambers III and II influencing the valve; $F_{II}$ – force of elasticity of a membrane; $F_{I} \equiv F_{III}$ – force formed by the difference of pressures in the chambers IV and II of a pulser; $F_{II}$ – force formed by the difference of pressures in gauged hole and above the armature space of electromagnet; $G_e$ – weight of valves and a rod; $G_{arm}$ – weight of an armature valve.

According to the constructive parameters, forces interactions and functional peculiarities of a pulser, it may be divided into two parts (regulative and principal).

The regulative part contains an electromagnet, an armature valve and a gauged hole while the principal part possesses the system of valves, a membrane and chambers of constant and variable pressures.

As the regulative part does not have any serious effect on the pulser time characteristics, we shall further focus on the principal part functioning.

When considering the principal part of the pulser we shall describe the forces applied to the center of weight of the valve and rod group with a movable rod. The upper end of the movable rod is fixed to the movable membrane while the upper and lower valves are stiffly fixed to the rod.

Forces directed from the membrane across the rod's axis to the lower valves provide the lower valve's shutting up of the hole of atmospheric pressure input to the working chamber and the upper valve's opening of the hole of vacuum gauge pressure input.

The forces of the weight may be calculated by means of the following formula:

$$ G_v = m_v \cdot g, \quad (1) $$

where: $m_v$ – mass of valves and a rod, kg.

Force effecting the membrane and curving it in the direction of the impact of weight together with a rod and valves is formed by the difference of the pressure $P_I$ and $P_{IV}$ in the chambers I and IV of a pulser. It is determined by means of the formula [18]:

$$ F_{str} = \frac{\pi \cdot (D_0 + d_0 + d_1)^2}{12} (P_{III} - P_{IV}), \quad (2) $$

where: $D_0$ – diameter of the membrane, m; $P_{IV}$ – pressure in the regulative chamber of the pulser IV, kPa; $d_0$ – diameter of the washer which fixes the membrane to the rod, m.

The next two forces impact the rod trying to move it upwards. The membrane with an inelastic center is characterized by a slight movement. The force of membrane elasticity is determined by the expression [15]:

$$ F_{el} = A_o \cdot \frac{R^2 \cdot h}{E \cdot \delta^3}, \quad (3) $$

where: $h$ – motion of membrane, m; $A_o$ – coefficient of constructive specifications of the membrane; $R$ – radius of the membrane, m; $\delta$ – thickness of the membrane, m.

Coefficient considering constructive dimensions is determined by means of the formula [15]:

$$ A_o = 3 \cdot (1 - \mu^2) \cdot \left[ \frac{c^2 - 1}{4c^2 - 1} \cdot \ln \frac{c}{c^2 - 1} \right], \quad (4) $$

where: $\mu$ – coefficient of Poisson; $c$ – coefficient of constructive dimensions of the membrane.

$$ c = \frac{R}{r_0}, \quad (5) $$

where: $r_0$ – radius of the inelastic center, m.

Then, the force of elasticity is determined by means of the expression:

$$ F_{el} = \frac{h \cdot E \cdot \delta^3}{A_o R^2}. \quad (6) $$

Force formed by the difference of pressures in the chambers III and II influence the lower valve with a diameter $d_{II}$. As the chamber III has constant atmospheric pressure, the equation for determining the force will have the following form:

$$ F_{III} = \frac{\pi \cdot d_{II}^2}{4} (P_{III} - P_{IV}), \quad (7) $$

where: $d_{II}$ – diameter of the lower valve, m; $P_{II}$ – pressure in the working chamber II of the pulser, kPa.

Thus, to provide the cycle of sucking, the positions of the upper and lower valves should be accurately determined by the following inequality:

$$ G_e + F_{str} > F_{el} + F_{III}. \quad (8) $$

The pulser will be in this state till the electromagnet receives the pressure of power supply. In this case the armature-valve will move upward having opened the gauged hole with a diameter $d_1$. In some time due to the open channel connecting the regulative chamber with the chamber of constant vacuum-gauge pressure, the former will get equilibrium (or close to equilibrium) pressure. The principal part will get redistribution of forces (Fig. 2). Such position of valves correspond to the cycle of compression.
Forces distribution in the principal part and their character will be soon altered. It should be said, the force of membrane elasticity \( F_{m,n} \) in such case is opposite-directed and facilitates opening the upper valve. The force of weight stays constant as the amount and of constituents have not been changed. The force directed upward is formed by the difference of pressures in the chambers I and II equals to:

\[
F_{at} = \frac{\pi \cdot d^2}{4} \cdot (P_n - P_i) .
\] (9)

Thus, the condition which guarantees maintaining all parts of the pulser in the situation corresponding to the cycle of compression for the principal part of the pulser will be:

\[ F_{at} > G_i + F_{m,n} . \] (10)

Let us consider specific characteristics of transition process in the pulser and then determine time characteristics.

After the armature valve has moved down and the hole connecting the chamber IV with atmospheric open, this chamber starts to fill with air which alters its pressure and density.

We shall further consider adiabatic process of air movement. Then, we carry out the following ratio [19]:

\[ \frac{P}{P_n} = \left( \frac{\rho}{\rho_n} \right)^{\frac{1}{k-1}} \text{, or } \left( \frac{\rho}{\rho_n} \right) = \left( \frac{P}{P_n} \right)^{\frac{1}{k-1}} . \] (11)

According to the Bernoulli theorem:

\[
\frac{P}{P_n} = \frac{P_{G_{1}}} {P_{G_{2}}} \text{, or } \left( \frac{\rho}{\rho_n} \right) = \left( \frac{P}{P_n} \right)^{\frac{1}{k-1}} .
\]

Fig. 2. Distribution of forces during the cycle of air compression

\[
u = \frac{2k}{k-1} \frac{P_s}{\rho_n} \left( 1 - \left( \frac{P}{P_n} \right)^{\frac{1}{k-1}} \right) .
\] (12)

where: \( P_n \) – atmospheric pressure, kPa; \( \rho_n \) – air density under standard conditions, kg/m^3; \( P, \rho \) – variable quantities of pressure and density; \( k \) – index of polytrope; \( u \) – velocity of air, m/s.

Let us take the derivative to time quantities from the equation (12):

\[
\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{P_{G_{1}}}{\rho_n} \left( \frac{\rho}{\rho_n} \right)^{\frac{1}{k-1}} \frac{d\rho}{dt} , \text{ or:}
\]

\[
\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{P_{G_{1}}}{\rho_n} \left( \frac{\rho}{\rho_n} \right)^{\frac{1}{k-1}} \frac{d\rho}{dt} .
\] (13)

Knowing the quantity of air mass entering the chamber during a certain unit of time and considering the expression (12) for determining velocity quantity and \( \frac{d\rho}{dt} = \frac{1}{V_0} \frac{\mathrm{d}m}{\mathrm{d}t} \), we insert these data into the expression (13) and receive the differential equation:

\[
\frac{\mathrm{d}}{\mathrm{d}t} \left( \frac{P}{P_n} \right) = K_1 \left( \frac{P}{P_n} \right)^{\frac{1}{k-1}} \sqrt{1 - \left( \frac{P}{P_n} \right)^{\frac{k-1}{k}}} .
\] (14)

The differential equation (14) is valid till pressure in the chamber modifies from the initial quantity \( P_i \) (vacuum-gauge pressure) to a certain quantity of \( P_i \), is calculated by means of the static-equilibrium equation:

\[
(P_i - P_i)S_i + G_i = (P_i - P_i)S_i = 0 .
\]

In the equation (14) we separate the variable quantities and carry out an integration:

\[
P = P_i + \frac{(P_{i} - P_i)S_i - G_i}{S_i} .
\] (15)

\[
\tau = 1 \int \frac{dz}{k \left( z \right)^{\frac{k-1}{k}}} \cdot \sqrt{1 - \left( z \right)^{\frac{k-1}{k}}} .
\] (15)

where: \( z = \frac{P}{P_n} \).

Duration \( \tau_i \) of this stage is determined by means of the expression (15) with pointing out the upper limit:

\[
\frac{P}{P_n} = \frac{P_i}{P_n}, \text{ i.e.}
\]

\[
\tau_i = 1 \int \frac{dz}{k_1 \left( z \right)^{\frac{k-1}{k}}} \cdot \sqrt{1 - \left( z \right)^{\frac{k-1}{k}}} .
\] (16)

After the increase of pressure in chamber \(( P > P_i \)) the valve begins to move. Its motion may be described by means of the differential equation:

\[
\frac{d^2x}{dt^2} = \left( zP_n - P_i \right)S_i + G_i - \left( P_n - P_i \right)S_z .
\] (17)

Differential equation (14) will be soon altered because of the perspective of enlargement of the chamber IV volume \( V = V_0 + xS_i \). Then:

\[
\frac{dz}{dt} = \frac{kV_0}{\left( V_0 + xS_i \right)} \left( x \right)^{\frac{k-1}{k}} \sqrt{1 - \left( x \right)^{\frac{k-1}{k}}} .
\] (18)

The system of differential equations should be integrated by numerical methods till transitions \( \theta \leq x \leq h_1 \), under initial conditions: \( t=0, \)

\[ a \]
\[
x = 0, \quad V = \frac{dx}{dt} = 0, \quad z = \frac{P}{P_a},
\]
\[
t = \frac{V_0 + h_1 S_1}{k_1 V_0} \int_{z_t}^{z_k} \frac{dz}{k \sqrt{1 - z^k}}.
\]  
(19)

Duration of this stage \( \tau_1 \) is determined by means of the expression (19):
\[
\tau_1 = \frac{V_0 + h_1 S_1}{k_1 V_0} \int_{z_t}^{z_k} \frac{dz}{k \sqrt{1 - z^k}}.
\]  
(20)

Effected by electromagnetic force, the armature valve moves upward and opens the hole connecting the chamber IV where \( P = P_4 \), with the chamber where vacuum-gauge pressure is constantly maintained (\( P = P_1 \)). The air from the chamber IV begins to enter another chamber and we again witness here the adiabatic way of air movement. The correlation:
\[
\frac{P}{P_1} = \left( \frac{\rho}{\rho_1} \right)^k, \quad \left( \frac{P}{P_1} \right) = \left( \frac{\rho}{\rho_1} \right)^{\frac{1}{k}},
\]  
(21)

becomes valid.

And according to the Bernoulli theorem:
\[
u^2 = \frac{2k}{k - 1} \frac{P}{\rho_1} \left( 1 - \left( \frac{P}{P_1} \right)^{\frac{k-1}{k}} \right).
\]  
(22)

Let us take the derivative to time quantities from the equation (22):
\[
\frac{dP}{dt} = \frac{P}{\rho_1} \left[ \frac{P}{P_1} \right]^{\frac{k-1}{k}} \frac{d\rho}{dt}, \quad \text{or:}
\]
\[
\frac{dP}{dt} = \frac{P}{\rho_1} \left[ \frac{P}{P_1} \right]^{\frac{k-1}{k}} \frac{d\rho}{dt}.
\]  
(23)

Having made the transformations analogical to those in case with opening the hole we shall receive the differential equation:
\[
\frac{dp}{dt} = 1 \frac{dm}{V_1 \frac{dt}{dt}} = -2 \pi d a h \sqrt{\left( \frac{P}{P_1} \right)^{\frac{k-1}{k}} - 1}.
\]  
(24)

Having inserted (23) into (24) we shall receive the differential equation:
\[
\frac{dy}{dt} = -K_2 y \sqrt{\frac{k-1}{k}} - 1,
\]  
(25)

where:
\[
K_2 = \frac{k \cdot 2 \pi d a h \sqrt{2k P_1}}{k - 1 \rho_1}.
\]

The differential equation is valid till pressure in the chamber IV modifies from the initial quantity \( P_0 \) to a certain quantity of pressure \( P_4 \). The quantity of \( P_4 \) is calculated by means of static-equilibrium equation:
\[
(P_4 - P_4) = \frac{c}{S_1} - (P_4 - P_1) S_1 = 0,
\]

therefore:
\[
P_4 = P_1 + \frac{c}{S_1}.
\]  
(26)

where: \( c \) – membrane inelasticity.

In the equation (25) we separate variable quantities and integrate:
\[
t = \frac{1}{k_2} \int \frac{dz}{\sqrt{z^{\frac{k-1}{k}} - 1}}.
\]  
(27)

Duration of this stage is determined having put \( y = y_e \), i.e.:
\[
\tau_2 = \frac{1}{k_2} \int \frac{dz}{\sqrt{z^{\frac{k-1}{k}} - 1}}.
\]  
(28)

The defined integrals in formulas (27) and (28) are calculated by means of numerical methods.

After the decrease of pressure in chamber IV \( P < P_2 \) the valve begins to move upward. Its motion is described by the differential equation:
\[
m_1 \frac{d^2 x}{dt^2} = c(h_1 - x) - G_x + (P_2 - P_1) S_2 - (y P_1 - P_1) S_1.
\]  
(29)

The differential equation describing pressure modifications will get the form:
\[
\frac{dy}{dt} = \frac{k_2 V_1}{(V_1 - x S_1)} y \sqrt{\frac{k-1}{k}} - 1.
\]  
(30)

We integrate the system of differential equations till \( 0 \leq x \leq h_1 \) under initial conditions: \( t=0, \ x=0, \)
\[
V = \frac{dx}{dt} = 0, \quad y = y_e = \frac{P_1}{P_1}.
\]
\[
t = \frac{V_0}{k_2 V_1} \int \frac{dz}{\sqrt{z^{\frac{k-1}{k}} - 1}}.
\]  
(31)

Duration of this stage \( \tau_5 \) is determined from (31), when \( \frac{P}{P_1} = 1, \ i.e.:
\[
\tau_5 = \frac{V_0}{k_2 V_1} \int \frac{dz}{\sqrt{z^{\frac{k-1}{k}} - 1}}.
\]  
(32)

Interdependence of the described above quantities and characteristics of the milking unit is indicated in Fig. 3. which presents the way of determining durations of phases and cycles of pulsations according to the requirements of the standard ISO 5707.1996 “Installation of milking unit. Construction and operating capabilities”. In addition, it suggests the way of determining time intervals \( \tau_1 \) and \( \tau_2 \), which have a colossal impact upon the efficiency of machine milking of cows and directly depend on the quantities of \( \tau_1, \ \tau_3 \), and \( \tau_2, \ \tau_4 \) correspondingly.
CONCLUSIONS

We proved the dependences of constructive parameters of an electromagnetic pulser and the intake vacuum-gauge pressure (15, 19, 27, 31), which allow to determine modifications of pressure in the working chamber of a pulser.

We discovered analytical dependences for determining the constants of time of electromagnetic pulser of a pairwise action (16, 20, 28, 32). These dependences directly determine durations of the leading and trailing edges of pulser of vacuum-gauge pressure of indicatory diagram of the pulser functioning.

REFERENCES


