THE INFLUENCE OF PARAMETERS OF SPATIAL ORIENTATION OF A SOLAR POWER RECEIVER ON ENERGETIC GAIN

The paper characterizes selected mathematical models for calculating power density of solar radiation incident on the plane inclined with regard to the base and directed at proper azimuth angle with regard to southern direction. Analysis of the presented methods is carried out and for purposes of further consideration the Liu-Jordan model has been adopted. The paper presents the results of power density of solar radiation obtained from computer simulation, with reference to a day, month, and year, in graphical form. Power gain at different spatial settings of the receiver was determined.

1. INTRODUCTION

Total solar radiation is a hemispherical radiation, achieving the receiver surface of any spatial orientation, from the solid angle in the range \(2\pi\) [sr], supplemented by the component reflected from the ground and from the objects surrounding the receiver. In particular case of a receiver located in parallel to the ground the reflected component does not exist. Direction of incidence of the direct radiation at an inclined surface is evident – it is identical to the direction of the radiation beam. In case of the receiver that is inclined with respect to the ground proper definition of the diffused radiation is the most difficult. Three components should be specified in this radiation:

- isotropic radiation;
- circumsolar radiation;
- the radiation coming from the bright horizon.

The first of these components is distinguished by the fact that it comes uniformly from the whole hemisphere. On the other hand, the circumsolar component is a result of diffusion of the solar radiation. Nevertheless, it is specifically related to the direct component. The third component is focused near to the horizon and observable in case of clear sky.

Additionally, in case of the receiver inclined at a certain angle \(\beta \neq 0\) to the ground the radiation reflected from the ground and surrounding objects should be
considered too, apart from the direct and diffused components of radiation, all being the components of the so-called total radiation.

Based on the above consideration, in case of an inclined surface one can write down [5]:

\[ G_r = G_b + G_f + G_d \]  

where \( G_b \) – the direct radiation; \( G_f \) – the reflected radiation; \( G_d \) – the diffused radiation, being a sum of three components:

\[ G_d = \sum_{\alpha} G_{d\alpha} \]  

The Authors of early computational models focused chiefly at determining the direct component, devoting considerably less attention to the other ones. The diffused radiation was considered as an isotropic one. Such an approach is appropriate, but only in case of the surfaces located in parallel to the surface of Earth.

One of the first methods, that partially took into account the effect of receiver inclination is the Liu-Jordan method [6]. The authors introduce correction factors into the calculation. Nevertheless, the method does not fully reflect the complex character of the diffused component, as in the proposed model it is considered as an isotropic one.

Since the time of development of the Liu-Jordan method, i.e. about 50 years ago, many mathematical models have been developed, that allowed to determine the total density of the solar radiation incident on the arbitrarily oriented receiver surface. However, they are more complex from the mathematical point of view.

The HKDR model seems to be particularly appropriate. It is a result of the methods developed by several authors, i.e. the models of J.E. Hay, J.A. Davies, T.M. Kluchera, and D.T. Reindl [5].

In the first of them the authors assume isotropic distribution of the diffused radiation, considering the diffused, isotropic, and circumsolar components. The effect of anisotropy was here taken into account only as a function of atmospheric transparency for the direct radiation. D.T. Reindl supplemented the mathematical apparatus by the component resulting from the bright horizon. T.M Klucher inserted the effect of clouding factor to the correction factor of the bright horizon of the equation derived by Reindl. The relationship modified by him explains the considered components with sufficient accuracy, even in case of the receivers inclined at large angles with respect to the ground [5].

2. THE LIU-JORDAN METHOD

Authors of the present paper used the Liu-Jordan method. Its advantage lies in rather simple mathematical apparatus [6].

Moreover, comparison of the results obtained by other authors [2] for the calculation carried out both with the method considering anisotropy of the diffusion radiation and ignoring its effect, has shown that the results of the
radiation power density in yearly scale differ only slightly. In case of the Liu-Jordan method the results are a little lower as compared to the ones obtained with consideration of anisotropy.

Specification of the example results obtained with the isotropic and anisotropic methods is presented in Table 1 [2].

Table 1. Maximum power density in yearly scale falling at an inclined plane directed at appropriate azimuth angle

<table>
<thead>
<tr>
<th>The angles $\beta$ and $\gamma$</th>
<th>$\beta=20^\circ$</th>
<th>$\beta=30^\circ$</th>
<th>$\beta=40^\circ$</th>
<th>$\beta=45^\circ$</th>
<th>$\beta=60^\circ$</th>
<th>$\beta=80^\circ$</th>
<th>$\beta=90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic computation model</td>
<td>3870 MJ/m²</td>
<td>3900 MJ/m²</td>
<td>3870 MJ/m²</td>
<td>3850 MJ/m²</td>
<td>3600 MJ/m²</td>
<td>3100 MJ/m²</td>
<td>2740 MJ/m²</td>
</tr>
<tr>
<td>Anisotropic computational model</td>
<td>4000 MJ/m²</td>
<td>4100 MJ/m²</td>
<td>4140 MJ/m²</td>
<td>4120 MJ/m²</td>
<td>3900 MJ/m²</td>
<td>3450 MJ/m²</td>
<td>3100 MJ/m²</td>
</tr>
</tbody>
</table>

According to the example data of Table 1, maximum of the function of solar radiation power density in yearly scale incident at the plane inclined and directed at appropriate azimuth angle occurs for the angles $\beta=30^\circ$, $\gamma=15^\circ$ for the isotropic model, or for $\beta=40^\circ$ and $\gamma=15^\circ$ in case of consideration of anisotropy.

Differences in the calculation results obtained with both methods usually are of the order of several percent, reaching even 1 percent in case of small inclination of the receiver with respect to the ground.

Maximum difference occurs in case of vertical orientation, i.e. for photovoltaic applications existing in case of façade BIPV solutions.

Under our geographical and weather conditions the Liu-Jordan method is well justified. In summer months, when higher factor of anisotropy may be expected, the optimal inclination angle is not large, due to power gain, reaching the values of 25-35°. On the other hand, in winter conditions the optimal inclination angle amounts to 55-60°. Therefore, the isotropic Liu-Jordan model may be used with sufficient accuracy, particularly in case of lower power systems [3,6].

Figure 1 show the example of the systems in case of the Polish conditions.
Total density $G_\beta$ of solar radiation flow is a sum of components [3, 6]:

$$G_\beta = G_b \cdot R_b + G_d \cdot R_d + (G_b + G_d) \cdot \rho_o \cdot R_o$$  (3)

where: $G_b$, $G_d$ – direct and diffusive component of density of solar radiation, and $\rho_o$ – coefficient of the bed reflectivity assumed from 0,07 (for dry asphalt) to 0,095 (for fresh snow), $R_b$, $R_d$, $R_o$ – the correction efficiencies defined below, related to direct, diffusive, and reflected components, respectively. Therefore:

$$R_i = \frac{\cos \theta_i}{\cos \theta_f}$$  (4)

$$R_d = \frac{1 + \cos \beta}{2}$$  (5)

$$R_o = \frac{1 - \cos \beta}{2}$$  (6)

Where: $\theta_f$ - is an angle of incidence of the radiation on a horizontal surface (the zenith angle) [3, 6]:

$$\cos \theta_i = \cos \varphi \cos \delta \cos \omega + \sin \varphi \sin \delta$$  (7)

and $\theta_\beta$ - is an angle of incidence of the radiation on a plane inclined at the angle $\beta$ to the ground, being a function of many variables:

$$\cos \theta_f = \sin \varphi \sin \delta \cos \beta - \cos \varphi \sin \delta \sin \beta \sin \gamma \cos \delta \sin \varphi \cos \beta \cos \omega + \cos \delta \sin \varphi \sin \beta \cos \gamma \cos \omega + \cos \delta \sin \sin \beta \sin \omega \sin \gamma$$  (8)

$\varphi$ - the angle of latitude, $\delta$ - the declination angle, $\omega$ - the hour angle, $\beta$ - the angle of receiver inclination to the ground.

The $G_b$ and $G_d$ values are assessed on the grounds of many years data obtained from weather stations [1, 8].

3. RESULTS OF COMPUTER SIMULATION

The above considerations and the relationship (1) to (8) served as a basis for a program developed with a view to making the calculation and computer simulation. The hourly distribution of radiation power density for recommended months, days and day hours 4 - 20 in Warsaw was determined.

Figure 2 shows results of hourly distribution in all months of year for direct and diffused components of solar radiation in Warsaw.

Figures 3, 4 show example results for particular months, considerably differing with regard to possible solar power gain, provided the receiver is set-up horizontally.

The next Figures 5, 6, present the plots of solar radiation power density falling of the surface of the power receiver for its varying spatial orientation (the angles $\beta$ and $\gamma$) on August 8, and the Figures 7, 8 - on January 15.
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Fig. 2. Results of computer simulation of hourly distribution in all months of year for direct and diffused components of solar radiation in Warsaw.

Fig. 3. Hourly distribution of radiation power density falling on a horizontal plane, for $\gamma = 0^\circ$, on August 8.
Fig. 4. Hourly distribution of radiation power density falling on a horizontal plane, for $\gamma = 0^\circ$, on January 15

Fig. 5. Hourly distribution of radiation power density falling on a plane inclined at the angle $\beta = 30^\circ$, for $\gamma = 0^\circ$, on August 8

Fig. 6. Hourly distribution of radiation power density falling on a plane inclined at the angle $\beta = 30^\circ$, for $\gamma = 15^\circ$, on August 8
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Fig. 7. Hourly distribution of radiation power density falling on a plane inclined at the angle $\beta = 65^\circ$, for $\gamma = 0^\circ$, on January 15

Fig. 8. Hourly distribution of radiation power density falling on a plane inclined at the angle $\beta = 65^\circ$, for $\gamma = 15^\circ$, on January 15

The Figure 9 presents the power gain in the hours from 4 a.m. to 8 p.m. at different spatial settings (the angles $\beta$ and $\gamma$) of the receiver on August 8.

Fig. 9. Power gain at different spatial settings of the receiver on August 8
4. SUMMARY

The above consideration and computer simulations allow to state the following:
- The hourly distribution of radiation power density is significantly affected by the
deciliation and hour angles that is clearly visible by comparing Figs. 3 and 4.
- Figures 3 and 6 enable comparing values of power density of solar radiation
reaching the receiver arranged horizontally and at an angle optimal with regard
to energetic gain. Radiation power density possible to be gained for optimal
panel angle $\beta = 30^\circ$ is 1,13 (12 a.m.) times bigger than for its horizontal set-up.
- For January, power density possible to be gained for optimal angle $\beta = 65^\circ$ is
nearly 1,7 (12 a.m.) times bigger than for its horizontal set-up, Figs. 4 and 8.
- The effect of azimuth angle of the energetic gain is not so important like the
one of the receiver set-up angle, which is visible in Figs.5 and 6 and - 7 and 8.
Nevertheless is should not be neglected in the calculation.

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