Small-signal input characteristics of step-down and step-up converters in various conduction modes

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Abstract. Small-signal input characteristics of BUCK and BOOST DC-DC power converters in continuous conduction and discontinuous conduction mode have been presented. Special attention is paid to characteristics in discontinuous conduction mode. The input characteristics are derived from the general form of averaged models of converters. The frequency dependence of input admittance and other input characteristics has been observed in a relatively low-frequency range. The analytical formulas derived in the paper are illustrated by numerical calculations and verified by experiments with a laboratory model of BOOST converter. A satisfying level of conformity of calculations and measurements has been obtained.

Key words: pulse converters, BUCK, BOOST, small-signal models, input characteristics.

List of symbols:

- $C$ – capacitance,
- $D$ – diode,
- $d_A, D_A$ – duty ratio and its steady-state value,
- $f, f_S$ – frequency, switching frequency,
- $G$ – load conductance,
- $H_s$ – transmittance of control subcircuit,
- $i$ – with subscripts in capital letters: instantaneous values of currents,
- $I$ – with subscripts in capital letters: D.C. values of currents,
- $I$ – with subscripts in small letters: small signal representations of currents in s domain,
- $K$ – ideal controlled switch (transistor),
- $L$ – inductance,
- $M_{VT}, M_I$ – voltage and current static transmittance,
- $R$ – load resistance,
- $T_S$ – switching period,
- $v$ – with subscripts in capital letters: instantaneous values of voltages,
- $V$ – with subscripts in capital letters: D.C. values of voltages,
- $V$ – with subscripts in small letters: small signal representations of voltages in s domain,
- $Y$ – small-signal input admittance,
- $\Gamma$ – quantity in dependence of input current on control signal,
- $\omega$ – angular frequency,
- $\theta$ – small-signal representations of duty ratio in s domain.

1. Introduction

A typical switch-mode DC–DC power converter contains power stage and control subcircuit working together as a feedback dynamic system. The procedure of designing control subcircuit is based on the knowledge of power stage dynamics, usually described in the form of a set of small-signal transmittances. These transmittances are derived by linearization of large-signal averaged models of power stage [1–7]. The transmittances describing the influence of input voltage and control signal on the output voltage of converters are most frequently used, but in some applications the input small-signal characteristics are important [8, 9]. The examples are converters working in power-factor correctors (PFC) [10, 11] and in maximum power point tracing (MPPT) circuits used in photo-voltaic systems [12].

Small-signal averaged models of the power stage of converter have the form of equation systems in s-domain or equivalent circuits in which the small-signal representation of duty ratio $\theta$, currents and voltages averaged over single switching period are used as variables [1–7]. The dependence of input current $I_g$ of converter on input voltage $V_g$ and duty-ratio representation $\theta$ may be obtained in the form:

$$ I_g = Y \cdot V_g + \Gamma \cdot \theta. $$

$Y$ is small-signal input admittance and $\Gamma$ describes the dependence of input current on control quantity $\theta$. The control subcircuit having transmittance $H_s$ delivers the control signal $\theta$ (after comparing the current sample with the proper reference signal):

$$ \theta = H_s \cdot I_g. $$

From (1) and (2) one obtains:

$$ I_g = \frac{Y}{1 - \Gamma \cdot H_s} \cdot V_g. $$
The proper design of Hs transmittance of control subcircuit is possible if frequency dependencies of Y and Γ are known.

Small-signal input characteristics of the most popular DC-DC converters: step-down (BUCK) and step-up (BOOST) are considered in this paper. The analysis is performed separately for converters working in continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The schemes of circuits under consideration are shown in Figs. 1 and 2, where K and D denote active and passive switch, respectively.

Special attention is paid to the analysis of converters working in DCM because some of the input characteristics of converters in CCM were discussed in the previous paper [9].

Small letters with capital subscripts denote instantaneous values of currents and voltages, capital letters with capital subscripts – quiescent values (D.C. values), and capital letters with small subscripts correspond to small-signal representations of currents and voltages in s-domain.

The analysis of input characteristics in s-domain for BUCK and BOOST converters is presented in Sec. 2 and 3, respectively. The selected dependencies in time domain and numerical examples are given in Sec. 4 and some concluding remarks in Sec. 5.

2. Step-down (BUCK) converter

2.1. Continuous conduction mode (CCM). Small-signal form of averaged model of BUCK converter power stage in CCM [5, 6] is expressed by following equations:

\[ sL \cdot I_i = D_A \cdot V_g + V_o \cdot \theta - V_o, \]  

(4)

\[ I_i = (G + s \cdot C) \cdot V_o, \]  

(5)

\[ I_g = I_i \cdot D_A + I_L \cdot \theta. \]  

(6)

The dependence of small-signal input current on input voltage and duty ratio \( \theta \) is obtained from equations (4–6) in the form:

\[ I_g = \left( D_A^2 \cdot G \frac{sCR+1}{s^2LC+sLG+1} \right) \cdot V_g + I_O \left( \frac{sCR+1}{s^2LC+sLG+1} + 1 \right) \cdot \theta. \]  

(7)

Equation (7) corresponds to general dependence given by (1) with:

\[ Y(BUCK) = D_A^2 \cdot G \frac{sCR+1}{s^2LC+sLG+1}, \]  

(8)

\[ \Gamma(BUCK) = I_O \left( \frac{sCR+1}{s^2LC+sLG+1} + 1 \right). \]  

(9)

Low frequency values of \( Y(BUCK) \) and \( \Gamma(BUCK) \) (for \( s \rightarrow 0 \)) are:

\[ Y_0(BUCK) = D_A^2 \cdot G, \]  

(10)

\[ \Gamma_0(BUCK) = 2 \cdot I_O. \]  

(11)

\( D_A \) and \( I_O \) are D.C. values of duty ratio and load current.

2.2. Discontinuous conduction mode (DCM). The starting point for derivation of input characteristics of BUCK converter in DCM is small-signal model presented in Fig. 3a obtained from [6]. In this case, quantities \( Y(BUCK) \) and \( \Gamma(BUCK) \), according to definition in (1), are obtained for \( \theta = 0 \) or \( V_g = 0 \), respectively, which corresponds to Figs. 3b and 3c. For \( \theta = 0 \) (Fig. 3b), one obtains:

\[ I_g = G_g \cdot V_g + \alpha_{g3} \cdot V_o = G_A(V_g - V_o), \]  

(12)

\[ \alpha_{g2}V_g = (G_w + G + sC) \cdot V_o, \]  

(13)

where \( G_A \) and \( G_g \) are:

\[ G_A = D_A^2 \cdot \frac{T_s}{2L}, \]  

(14)

\[ G_g = \frac{1}{R_g} = G_A, \]  

(15)

After excluding quantity \( V_o \) (output voltage) from (12) and (13), one obtains the dependence of \( I_g \) on \( V_g \) (for \( \theta = 0 \)) and, as a result, the admittance \( Y_A(BUCK) \) for this case:
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The additional subscript “d” in \( Y_d \) (BUCK) refers to DCM. Quantities \( G_w \) and \( \alpha_{l2} \), according to [6] are:

\[
G_w = G_A \frac{V_o^2}{V_G^2}, \quad (17)
\]

\[
\alpha_{l2} = G_A \left( \frac{2V_G}{V_o} - 1 \right). \quad (18)
\]

After introducing (17) and (18) into (16), we obtain the following expression for \( Y_d \) (BUCK):

\[
Y_d(BUCK) = G_A \frac{G_w + G + sC - \alpha_{l2}}{G_w + G + sC}. \quad (16)
\]

\[
M_J = \frac{I_o}{I_G} = \frac{1}{2} \left( \sqrt{1 + \frac{4G}{G_A}} + 1 \right). \quad (19)
\]

The expressions for \( \omega_{ad} \) and \( \omega_{pd} \) - characteristic angular frequencies in (19) are:

\[
\omega_{ad} = 2\pi f_{ad} = \frac{1}{C} \left[ G_A (M_I - 1)^2 + G \right]. \quad (20)
\]

\[
\omega_{pd} = 2\pi f_{pd} = \frac{1}{C} \left[ G_A M_I^2 + G \right]. \quad (21)
\]

After excluding \( V_o \), it is obtained:

\[
I_g = \alpha_{g3} \frac{V_o}{G_A} \cdot \alpha_{g1} \theta, \quad (22)
\]

\[
V_o = \alpha_{g1} \cdot \theta \cdot \frac{1}{G_w + G + sC}. \quad (23)
\]

Fig. 3 corresponds to the condition \( V_g = 0 \), and in this case the following equations are obtained:

\[
\Gamma_d(BUCK) = \alpha_{g1} + \frac{\alpha_{g3} \cdot \alpha_{g1}}{G_w + G + sC}. \quad (24)
\]
\[ \alpha_{i1} = 2 \cdot V_G \cdot \frac{G_L}{D_A} \left( \frac{V_G}{V_o} - 1 \right), \]  
(27)  

\[ \alpha_{e1} = 2 \frac{G_A}{D_A} \cdot (V_G - V_o), \]  
(28)  

\[ \alpha_{e3} = -G_A. \]  
(29)

From (26–29), the expression for \( \Gamma_d (BUCK) \) quantity is obtained in the form:

\[ \Gamma_d (BUCK) = \Gamma_{d0} (BUCK) \frac{s/\omega_{z1} + 1}{s/\omega_{z1} + 1}, \]  
(30)

where:

\[ \Gamma_{d0} (BUCK) = 2 \frac{G_A}{D_A} \cdot V_G \cdot (1 - M_I^3) \cdot \frac{G_A \cdot M_I \cdot (M_I - 1) + G}{G_A \cdot M_I^2 + G} \]  
(31)

and:

\[ \omega_{z1} = 2 \pi f_{z1} = \frac{1}{C} \left[ G_A \cdot M_I (M_I - 1) + G \right]. \]  
(32)

The pole of \( \Gamma_d (BUCK) \) is the same as of \( Y_d (BUCK) \).

3. Step-up (BOOST) converter

3.1. Continuous Conduction Mode (CCM). Small-signal form of averaged model of BOOST converter power stage in CCM can be expressed by equations (33–35) [6]:

\[ sL \cdot I_g = V_g - V_o \cdot (1 - D_A) + V_o \cdot \theta, \]  
(33)

\[ I_d = (sC + G) \cdot V_o, \]  
(34)

\[ I_d = I_g \cdot (1 - D_A) - I_L \cdot \theta, \]  
(35)

where \( I_g \) is the input current and the inductor current as well, \( I_d \) is the current of a diode. After excluding \( I_d \) and the output voltage \( V_o \) one obtains:

\[ Y(BOOST) = \frac{G}{(1 - D_A)^2} \left( 1 + \frac{s \cdot CR}{(1 - D_A)^2} + \frac{s \cdot LG}{(1 - D_A)^2} + 1 \right), \]  
(36)

\[ \Gamma(BOOST) = \frac{2I_o}{(1 - D_A)^2 \left( \frac{LC}{(1 - D_A)^2} + \frac{LG}{(1 - D_A)^2} + 1 \right)}, \]  
(37)

For small frequencies (when \( s \rightarrow 0 \)) quantities \( Y (BOOST) \) and \( \Gamma (BOOST) \) are equal to:

\[ Y_0 (BOOST) = \frac{G}{(1 - D_A)^2}, \]  
(38)

\[ \Gamma_0 (BOOST) = \frac{2I_o}{(1 - D_A)^2}. \]  
(39)

3.2. Discontinuous Conduction Mode (DCM). The general small-signal scheme of a BOOST converter in DCM [6] shown in Fig. 4a can be used to derive dynamic input characteristic of the converter. Quantities \( Y_d (BOOST) \) and \( \Gamma_d (BOOST) \) for this case are obtained for \( \theta = 0 \) or \( V_g = 0 \), respectively, which corresponds to Figs. 4b and 4c.

For \( \theta = 0 \) (Fig. 4b), one obtains:

\[ I_g = G_{gT} \cdot V_g + \alpha_{gT3} \cdot V_o, \]  
(40)

\[ V_o = \alpha_{d2} \cdot V_g \frac{1}{G_d + G + sC}. \]  
(41)

After excluding output voltage \( V_o \), one obtains the dependence of \( I_g \) on \( V_g \) (for \( \theta = 0 \)) and, as a result, the admittance \( Y_d \) (BOOST) for this case:

\[ Y_d (BOOST) = \frac{G_{gT} \cdot (G_d + G + sC) + \alpha_{gT3} \cdot \alpha_{d2}}{G_d + G + sC}. \]  
(42)

 Quantities \( \alpha_{gT3}, \alpha_{d2}, G_{gT} \) and \( G_d \) are [6]:
After substituting expressions for coefficients α, Gd into (54), we get the following equation:

\[
\Gamma_d(\text{BOOST}) = \Gamma_{d0}(\text{BOOST}) \frac{1 + s/\alpha_1}{1 + s/\omega_p},
\]

(57)

where low-frequency value \(\Gamma_{d0}(\text{BOOST})\) in (57) is:

\[
\Gamma_{d0}(\text{BOOST}) = \frac{2G_A \cdot V_G \cdot (G \cdot (M_{VT} - 1) \cdot M_{VT} + G_A)}{D_A} \cdot \frac{G \cdot (M_{VT} - 1)^2 + G_A}{G \cdot (M_{VT} - 1)^2 + G_A}.
\]

(58)

Angular frequency \(\omega_p\) in (57) is equal to pole \(\omega_{pt}\) in (51). Zero in equation (57) corresponds to angular frequency:

\[
\omega_{zt} = \frac{G \cdot (M_{VT} - 1) \cdot M_{VT} + G_A}{C \cdot (M_{VT} - 1) \cdot M_{VT}}.
\]

(59)

4. Characteristics in time domain and numerical examples for DCM

Frequency dependencies of quantities \(Y_d\) and \(\Gamma_d\) for discontinuous conduction mode have general form:

\[
H(s) = H_o \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p}.
\]

(60)

It may be written as:

\[
H(s) = H_o \left( \frac{\omega_p}{\omega_z} + \frac{1 - \omega_p/\omega_z}{1 + s/\omega_p} \right).
\]

(61)

The equivalent description in time-domain is the time response \(h(t)\) to step pulse excitation:

\[
h(t) = H_o \left( \frac{\omega_p}{\omega_z} - \frac{\omega_p}{\omega_z} - 1 \right) \left( 1 - e^{-\omega_p t} \right).
\]

(62)
been measured and calculated according to formula (62). The comparison of the results of measurements and calculations is presented in Figs. 5 and 6.

According to formulas derived in the paper, input admittances and Γ parameters describing the dependence of input current on input voltage and control signal in DCM are frequency-dependent in a relatively narrow region of low frequencies. These frequency-dependencies are determined mainly by the value of converter output capacitance.

From the small-signal characteristics in the frequency domain, the time-domain responses of input current to step-pulse excitation of input voltage or duty ratio may be easily derived. The exemplary time-domain characteristics are presented in the paper. The results of calculations performed with the use of presented formulas are consistent with measurements of the laboratory model of BOOST converter.

REFERENCES