An approximate method for calculating total ship resistance on a given shipping route under statistical weather conditions and its application in the initial design of container ships

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Abstract

During ship design, its service speed is one of the crucial parameters that determine its future operational profitability. As sufficiently exact calculation methods applicable to preliminary design stage are lacking, the so-called contract speed, the speed a ship reaches in calm water, is usually specified during the draft stage. The service speed obtainable by a ship under real weather conditions (mainly wind and waves) is one of the most important parameters influencing a ship’s profitability on a given shipping route. This paper presents a parametric model of calculating total ship resistance on a given shipping route under actual weather conditions (wind, waves, sea current), that could be useful in the initial design of container ships.

Introduction

During ship design, one of the parameters most crucial to the operational profitability of a ship is its service speed under seasonal weather conditions. The service speed and total resistance of a ship can be determined during its operation, or calculated on the basis of complete design documentation or from the results of model basin tests. An algorithm for estimating total ship resistance (resistance on calm water, plus resistance from wind, waves, sea currents and rudder adjustments) was presented in Szelangiewicz and Żelazny (Szelangiewicz & Żelazny, 2006). This method cannot, however, be used in preliminary stages of ship design, when important decisions must now be made only on the basis of the ship’s hull geometry, which has been the only information a ship designer typically has at his disposal during the initial design stages. The details of hull geometry are insufficient to allow service speed under expected weather conditions to be estimated.

This article presents an approximate method of estimating a ship’s resistance on calm water, as well as the additional resistance from wind, waves, sea currents and rudder adjustments that would be useful to know in the preliminary design of a container ship (additional resistance from waves was presented in Żelazny (Żelazny, 2015).

The total resistance of the vessel on a given shipping route

While a transport vessel sails on a given shipping route (Figure 1) under real weather conditions, acted on by wind, waves and currents, then additional resistance as well as drift moments appear and, in order to keep the vessel on a constant course, these drift moments must be counterbalanced by a rudder plane and resulting carrying helm.

The total resistance of the vessel under real weather conditions on a given shipping route is given by the following expression:
The resistance of the vessel on the water as affected by sea current and drift angle

The ship’s still-water resistance is usually measured by model tests or is calculated for the ship in rectilinear motion. In real weather conditions, the ship sails at a certain drift angle due to the oblique action of wind and waves and/or possible surface sea currents. Hence, the still-water response for the ship at a steady speed, is composed of the following elements:

\[ R_x = \frac{1}{2} \rho_w S V^2 R_x \cos \beta \]
\[ R_y = \frac{1}{2} \rho_w S V^2 R_y \sin \beta \]
\[ M_z = \frac{1}{2} \rho_w S L V^2 C_m \]

where:
- \( R_x, R_y, M_z \) – components of still-water resistance forces, moment of ship sailing at drift angle \( \beta \), and surface sea currents, respectively;
- \( \rho_w \) – water density;
- \( S \) – lateral projection of underwater ship hull surface onto ship’s plane of symmetry \( PS \);
- \( V_{RV} \) – relative ship speed;
- \( \beta_{RV} \) – relative drift angle;
- \( L \) – ship length;
- \( C_x, C_y, C_m \) – coefficients of resistance forces and moment.

In Eq. (3), the relative speed and relative drift angle are as follows:

\[ V_{RV} = \sqrt{V_{RVx}^2 + V_{RVy}^2} \]
\[ \beta_{RV} = \tan^{-1} \left( \frac{V_{RVy}}{V_{RVx}} \right) \]
\[ V_{RVx} = V_x - V_C \cos \beta_C \]
\[ V_{RVy} = V_y - V_C \sin \beta_C \]

where:
- \( V \) – absolute ship speed, (Figure 2);
- \( V_x = V \cos \beta, V_y = -V \sin \beta \) – absolute speed ship components;
- \( \beta \) – drift angle at absolute speed, (Figure 2);
- \( V_C \) – surface sea current velocity;
- \( \beta_C \) – sea current direction relative to ship;

\[ \beta_C = \gamma_C - \psi \]

\( \gamma_C \) – geographical direction of surface current, \( \gamma_C = 0^\circ \) northbound current, \( \gamma_C = 90^\circ \) eastbound current;

\( \psi \) – geographical course of ship \( \psi = 0^\circ \) northward course, \( \psi = 90^\circ \) eastward course), (Figure 2).

If the sea current velocity \( V_C = 0 \), then the absolute ship speed \( V \) and absolute drift angle \( \beta \) is valid for Eq. (3).

Wind influence on ship

The mean wind forces acting on a ship in motion can be calculated by using the following formulae:

\[ R_{xA} = -\frac{1}{2} \rho_A S x V_{RxA}^2 C_{Ar} (\beta_{RA}) \]
\[ R_{yA} = \frac{1}{2} \rho_A S y V_{RyA}^2 C_{Ar} (\beta_{RA}) \]
\[ M_{zA} = \frac{1}{2} \rho_A S L V_{RyA}^2 C_{Am} (\beta_{RA}) \]

where:
- \( \rho_A \) – air density;
- \( \beta_{RA} \) – reference angle.

Figure 1. Example shipping route and directions of wind, waves, current

Figure 2. Parts of total ship resistance \( R_x, R_y, M_z \) (water, wind, waves), course of ship, and direction of current, wind, waves

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Additional ship resistance due to passive rudder

When a ship sails in waves, especially when oblique wind and waves influence the ship’s motion, lateral forces and moments are generated which could cause a change of course if not corrected for by the rudder. In order to keep the ship’s course constant, the rudder blade must be inclined, and that produces the additional resistance $R_{sr}$.

In the literature on ship manoeuvring, several algorithms are given for calculating hydro-dynamical forces on a passive rudder, including those dealing with additional resistance (see Hirano, 1980; Inoue et al., 1981). According to (Inoue et al., 1981), the passive rudder forces can be calculated by using the formulae:

$$R_{sr} = |F_N \sin \delta_R|$$
$$R_{sr} = a_y F_N \cos \delta_R$$
$$M_{sr} = a_z F_N \cos \delta_R$$

where:

- $\delta_R$ – rudder angle (Figure 3 – rudder angle at boardside $\rightarrow \delta_R > 0$, the rudder angle at starboard $\rightarrow \delta_R < 0$);
- $a_y$ – coefficient of hull influence on the rudder force $R_{sr}$;
- $a_z$ – coefficient of hull influence on the rudder moment $M_{sr}$ on the rudder;

$$a_z = a_y \cdot x_R$$

$x_R$ – abscissa of rudder axis measured from the ship mass centre $G$ ($x_R < 0$);

$F_N$ – rudder normal force, (Figure 3);

$$F_N = \frac{1}{2} \rho_w \frac{6.13 \cdot \lambda}{\lambda + 2.25} A_R V_R^2 \sin \alpha_R$$

$\lambda$ – rudder aspect ratio;

$A_R$ – rudder surface area;

$V_R$ – water inflow velocity to rudder;

$\alpha_R$ – effective rudder angle of attack.

As a result of passive rudder inclination, a moment, $M_{sr}$, of force, $R_{sr}$, appears, and in order to maintain course, the rudder moment must have a value as that balances the resultant forcing moment due to the action of wind, water flow and waves:

$$M_T = -M_{sr}$$

where: $M_T$ is the total moment of the marine environment (wind, waves and current from the vessel with the angle of drift):

$$M_T = M_{sr} + M_{sw} + M_z$$
Hence, the value of the rudder angle, $\delta_R$, calculated by using Eq. (15) under the assumption that a constant course is maintained, depends on the wind and wave parameters and ship’s drift angle.

**Approximate method of ship resistance on calm water with sea current and drift angle**

After comparing a number of methods of estimating ship resistance for relative accuracy and simplicity of the calculations entailed, a multiple linear regression was chosen to estimate resistance. The data required by the regression function to estimate ship resistance in calm waters using the Holtrop–Mennen’s method (Holtrop & Mennen, 1982) was taken from 56 container ships – three datasets to test the model, and 53 to search for a model. The range of parameters (independent variables) examined for the targeted group of container ships is summarized in Table 1.

**Table 1. The range of examined parameters for container ships**

<table>
<thead>
<tr>
<th>$L$</th>
<th>$B$</th>
<th>$T$</th>
<th>$C_B$</th>
<th>$C_{BP}$</th>
<th>$CP$</th>
<th>$V$</th>
<th>$L/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td>[m/s]</td>
<td>[-]</td>
</tr>
<tr>
<td>max</td>
<td>380</td>
<td>56</td>
<td>15</td>
<td>0.77</td>
<td>0.85</td>
<td>0.78</td>
<td>214580</td>
</tr>
<tr>
<td>min</td>
<td>125</td>
<td>22</td>
<td>6</td>
<td>0.64</td>
<td>0.77</td>
<td>0.65</td>
<td>12420</td>
</tr>
</tbody>
</table>

A multiple linear regression takes the following general form:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \epsilon$$  \hspace{1cm} (20)

where:
- $\beta_i$ – model parameters;
- $\epsilon$ – random effects;
- $y$ – dependent variable (ship resistance here);
- $x_i$ – independent variables (in this case geometric parameters of the hull).

Models of approximate function examined here, have been drawn up on the basis of experience and intuition having analysed the influence of ship design parameters on its resistance. The approximating function (in the form below) has been chosen from the analysis carried out:

$$\frac{R_l}{V^2} = f(L_{WL}, B, T, C_B, V)$$  \hspace{1cm} (21)

where the dependent variable has the form of:

$$y = \frac{R_l}{V^2}$$  \hspace{1cm} (22)

Based on analyses of the influence of the individual parameters driving ship resistance in calm water, a detailed form of approximating function can be expressed as:

$$\frac{R_l}{V^2} = a_0 + a_1 L_{WL}^{1.5} + a_2 B^{0.5} + a_3 T^{2.5} +$$

$$+ a_4 \frac{\ln(C_B)}{C_B^2} + a_5 V^{0.5} \ln(V)$$  \hspace{1cm} (23)

where:
- $\nabla$ – ship displacement;
- $L_{WL}$ – ship length at waterline line.
Coefficient values of an approximating model for container ships are as follows:

\[
\begin{align*}
    a_0 &= -7.23061, \\
    a_1 &= -0.00071, \\
    a_2 &= 1.94147, \\
    a_3 &= -0.00765, \\
    a_4 &= 0.10817, \\
    a_5 &= 0.00587
\end{align*}
\]

\( (R^2 = 0.930; \text{ Std. error of estimate} = 1.64) \)

The coefficients \( C_a(\beta), C_r(\beta), C_m(\beta) \) for container ships can be calculated from the following formulae:

\[
\begin{align*}
    C_a(\beta) &= -0.0008 \beta^2 + 0.0661 \beta + 1 \\
    C_r(\beta) &= 0.000278 \beta^2 + 0.003208 \beta \\
    C_m(\beta) &= 0.002251 \beta
\end{align*}
\]  

(24)

The final form of the function approximating force and moment resistance on container ships with a specific angle of drift in calm water is as follows:

\[
\begin{align*}
    R_x &= (-7.23061 - 0.00071) \ell_{aw}^{1.5} + 1.94147 B^{0.5} + 0.00765 T^{2.5} + 1.10817 \ln(C_R) + 0.0587 W^{0.5} \ln(W) V^2 (-0.0008 \beta^2 + 0.0661 \beta + 1) \\
    R_y &= \rho_a L T^2 (1.36 \cdot 10^{-4} \beta^2 + 1.56 \cdot 10^{-3} \beta) \\
    M_z &= 1.10 \cdot 10^{-3} \rho_a T L^3 V^2 \beta
\end{align*}
\]  

(25)

Approximation of ship additional resistance from wind action

Forces \( R_{x,t} \) and \( R_{y,t} \), as well as \( M_{z,t} \), the moment of wind action on the parts of the ship above water, depend on the resistance of the ship’s shape above water (given by coefficients \( C_{x,t}, C_{y,t}, \) and \( C_{m,t} \)), as well as the surface area of the parts of the ship above water, \( S_x \) and \( S_y \). As a result, the coefficients \( C_{x,t}, C_{y,t}, C_{m,t}, S_x \) and \( S_y \) are approximated here for selected types of container ships (see Table 1).

The coefficients \( C_{x,t}, C_{y,t}, \) and \( C_{m,t} \) are either empirically determined by conducting model tests (Blendermann, 1990; 1991) of the above-water parts of the ship in an aerodynamic tunnel, or by calculations using approximation formulae (Isherwood, 1973; Vorajew & Guliev, 1988). These coefficients for a specific type of a ship, such as a container ship, depend on the size of the ship size to a limited degree. Coefficients \( C_{x,t}, C_{y,t}, \) and \( C_{m,t} \), measured in an aerodynamic tunnel for a container ship (Blendermann, 1990; 1991) have been approximated by a polynomial dependent only on relative wind direction, \( \beta_{RA} \), (see Figure 2). The following relationships were obtained:

\[
\begin{align*}
    C_{x,t}(\beta_{RA}) &= 0.47676 + 0.00991 \beta_{RA} + 2.5026 \cdot 10^{-4} \beta_{RA}^2 + 9.2900 \cdot 10^{-7} \beta_{RA}^3 \\
    R^2 &= 0.987 \\
    C_{y,t}(\beta_{RA}) &= 0.01991 \beta_{RA} - 1.1108 \cdot 10^{-4} \beta_{RA}^2 \\
    R^2 &= 0.981 \\
    C_{m,t}(\beta_{RA}) &= -0.0048 \beta_{RA} + 1.056 \cdot 10^{-4} \beta_{RA}^2 + 5.181 \cdot 10^{-7} \beta_{RA}^3 \\
    R^2 &= 0.980
\end{align*}
\]  

(26)

Having developed approximations for coefficients \( C_{x,t}, C_{y,t}, C_{m,t} \), as well as for surfaces \( S_x \), \( S_y \), \( S_z \), a set of equations for container ships take the form shown by set (27) below:

\[
\begin{align*}
    R_{x,t} &= -\frac{1}{2} \rho_a (467.7 + 0.0093 V) V_{RA}^2 (0.47676 + 0.00991 \beta_{RA} - 2.5026 \cdot 10^{-4} \beta_{RA}^2 + 9.2900 \cdot 10^{-7} \beta_{RA}^3) \\
    R_{y,t} &= \frac{1}{2} \rho_a (1848.1 + 0.0628 V) V_{RA}^2 (0.01991 \beta_{RA} - 1.1108 \cdot 10^{-4} \beta_{RA}^2) \\
    M_{z,t} &= \frac{1}{2} \rho_a (1848.1 + 0.0628 V) L V_{RA}^2 (-0.0048 \beta_{RA} + 1.056 \cdot 10^{-4} \beta_{RA}^2 - 5.181 \cdot 10^{-7} \beta_{RA}^3)
\end{align*}
\]  

(27)

Approximation of forces and moment on plane rudder

In order to work out a simple approximating formula, the following assumptions were made:

- From estimates of the drift angle \( \beta_{h} \) in the rudder area (Żelazny, 2005), and from the distribution of this angle performed during ship movements along a shipping route, it has been assumed that effective thrust angle (4) is given by:

\[
\alpha_R \equiv \delta_R
\]  

(28)

- Rudder area, \( A_R \), and aspect ratio, \( \lambda \), were made dependent on basic ship dimensions.

Apart ship geometric parameters and rudder angle in equation (18) it is also velocity \( V_R \). It has therefore been assumed that the approximating model for velocity \( V_R \) will take the form of:

\[
V_R = a + b V
\]  

(29)

Taking into account the approximation of rudder area, \( A_R \), and rudder elongation, \( \lambda \), final expressions for the forces and moment on rudder are given by the following expressions:
\[ R_{\text{SR}} = \left[ (0.0194LT + 2.1874) c (a + bV)^2 (\sin \delta_R)^2 \right] \]
\[ R_{\gamma R} = \frac{1}{2} (1.14 - 0.6 C_B) \left( (0.0194LT + 2.1874) \cdot c (a + bV)^2 \sin 2\delta_R \right) \]
\[ M_{\text{SR}} = \frac{1}{4} L (1.14 - 0.6 C_B) \left( (0.0194LT + 2.1874) \cdot c (a + bV)^2 \sin 2\delta_R \right) \]

where: \[ c = \left( \frac{1}{2} \frac{P_w}{\lambda} 6.13 \frac{\lambda}{\lambda + 2.25} \right). \]

Coefficient values \( a, b, \) and \( c \) for an adopted model for container ships are as follows:
\[
\begin{align*}
  a & = 5.333 \\
  b & = 0.329 \\
  \lambda & = 1.795
\end{align*}
\]

**Verification of approximation model of total resistance of container ships**

Factual verification of the model was carried out by comparing the results of the regression model with calculations made for illustrative ships whose basic parameters are given in Table 2, as well as by tank tests (for relative and absolute error). Results of the model’s verification in the form of relative error values (comparison of values obtained from the regression with the results of exact calculations for illustrative container ships) are shown graphically in Figures 4–7.

**Table 2. Basic parameters of illustrative ships used to verify the model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Container ships</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K1</td>
</tr>
<tr>
<td>Length of the vessel ( L ) [m]</td>
<td>140.14</td>
</tr>
<tr>
<td>Ship breadth ( B ) [m]</td>
<td>22.3</td>
</tr>
<tr>
<td>Draught ( T ) [m]</td>
<td>8.25</td>
</tr>
<tr>
<td>Block coefficient ( C_B ) [-]</td>
<td>0.641</td>
</tr>
<tr>
<td>Waterplane coefficient ( C_{WP} ) [-]</td>
<td>0.809</td>
</tr>
<tr>
<td>Displacement ( V ) [m³]</td>
<td>17290</td>
</tr>
<tr>
<td>Ship speed ( V ) [m/s]</td>
<td>8.44</td>
</tr>
</tbody>
</table>

**Figure 4. Resistance of container ship K2 on calm water within drift angle**

**Figure 5. Resistance (forces and moment) of container ship K1 on calm water with drift angle for different speeds \( V \)**

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Figure 6. Wind forces and moment for different wind speeds $V_A$ – container ship K2

Figure 7. Forces and moment on rudder calculated for different ship speeds $V$ – container ship K2
Conclusions

- The final form of the formulae approximating total ship resistance during a cruise on a specific sea route is quite complex. However, the resistance components $R_L$, $R_p$ and the torque $M_z$ depend on the basic geometric parameters of the ship available at the preliminary design stage - length $L$, width $B$, draft $T$, side height $H$, displacement $\nabla$, block coefficient $C_b$, and waterplane area coefficient $C_{WP}$.

- The number of basic geometric parameters used in the approximation formulae developed is significantly lower than that in other known approximate methods. Therefore, the approximations developed can be easily used at preliminary ship design stages.

- The tests show that the approximation model developed is accurate enough to be used in preliminary ship design. Also, a comparison of newly developed approximations of total resistance components with other approximations presented in the literature showed the former were more accurate.

- High accuracy of the final parametric model was achieved mainly by using a multiple criteria regression. Artificial neural networks were used only to approximate the additional resistance from the waves for which the highest accuracy was achieved.

- The individual components of the total resistance calculated with the approximation formulae may contain some errors, but ultimately the total resistance estimation error for model ships was no more than several per cent. The greatest relative errors were obtained when approximating the forces and the torque on the plane rudder; however, the impact of these components on total ship resistance is negligible.

References


