A METHOD OF COMPARATIVE EVALUATION OF CONTROL SYSTEMS BY THE SET OF PERFORMANCE MEASURES

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Abstract. It was proposed a method of comparative estimation of process control loops based on a complex of direct and indirect performance measurers taking into account stability margins. It was presented a comparison algorithm for complex performance parameter considering the influence of stability margins of a control loop on its properties. Features of the method of comparative estimation of process control systems performance have been considered as an example for the closed loop control feedback system with PI controller (4 variations) and the second order plus transport delay plant.

Keywords: feedback control systems, performance evaluation, stability, control

METODA PORÓWNAWEJ OCENY UKŁADÓW AUTOMATYCZNEGO STEROWANIA WYKORZYSTUJĄCA ZBIÓR WSKAŹNIKÓW JAKOŚCI


Słowa kluczowe: systemy sterowania ze sprzężeniem zwrotnym, ocena jakości, stabilność, sterowanie

Introduction

Characteristics of an automatic control system which has a specific purpose depend on many factors. Main of them are: the structure of the system (closed loop, with local feedbacks, cascade, combined, etc.), a mathematical model of the plant, control algorithm, calculation method and criteria of performance control, technological requirements for process control response (oscillating or aperiodic). As a result, control of the same object in general can be achieved by different structural schemes and algorithms with different adjustment parameters.

This raises the problem of comparative performance assessment of control loops in order to choose the optimal one for a given plant. Most often it is carried out by transient response analysis for setpoint and disturbance channels. If a priority is one of the performance measures, such as the maximum dynamic error, then there is no problem, but in practice many different systems have different performance indexes [2]. The differences may be minor or very significant. So it’s impossible to conclude confidently that one control loop is better than another, merely by comparing one or even several performance indexes. The task of comparative performance assessment becomes more difficult with increasing of the number of comparable indexes. Almost even with the same system structure and control algorithm using different methods of parametric synthesis it could be obtained a number of possible system parameters. They differ by controller parameters and performance measures. So the problem of choice appears. The problem of comparative assessment also occurs whenever it is necessary to make a comparison of control loops that have been synthesized by the same method, but with different control algorithms. However, it must be emphasized that the problem of comparative performance assessment in any case can be considered only for systems with the same purpose. Based on the fact that closed loop control system should compensate setpoint and disturbances changes, performance measures for both types of responses should be taken into account.

A number of control performance assessment techniques based on different performance measures were developed in recent decades [1; 2, 5, 6]. It is known that completely control loop performance can be characterized by a combination of direct and indirect performance indexes. Direct performance indexes can be represented as

\[ y_{max}, y_{off} \] - the maximum dynamic deviations of output variable; \( t_p, t_s \) - settling time (the time of entering to the area 5% or 2% deviation of the settled value of controlled variable).

This raises the problem of comparative performance assessment by the set of performance measures, which is that the smaller are their values, the better is the control system; however, in relation to \( M \) index this statement is true only to a certain extent, because this

1. Description of the method

It has been proposed a method for comparative evaluation of control systems by a set of performance indexes considering the stability margin [4]. The method is based on the following main assumptions:

1) an important common feature of direct and indirect performance indexes is that the smaller are their values, the better is the control system; however, in relation to \( M \) index this statement is true only to a certain extent, because this
parameter can only be applied when implementing the control system with underdamped responses. If it is necessary to implement a feedback control system with damped responses, then direct rates of the stability margins $A_w$ (gain margin) and $\phi_w$ (phase margin) should be used;

2) all the performance indexes are considered as equally important, e.g. they have the same "weight". So it is assumed that the loss in one or more performance indexes can be compensated by gain in the others. If some indexes of one type differ for different control systems (for example, by one order and more), it is possible (though not necessarily) to exclude this index from the comparative analysis and consider it in the final evaluation of the control system;

3) only the relative values of the performance indexes should be compared as they are very different by their nature (e.g., some of them are dimensionless, while others, such as settling time, are dimensional quantities) and by absolute values.

2. Algorithm of the method implementation

Taking into account these assumptions comparative performance assessment of control systems can be implemented by the following algorithm:

1) For comparable control systems relative values of the same type indexes (performance indexes are defined in dimensionless form) are calculated by dividing the absolute value of this index by its maximum value:

$$
\delta y_{gi} = \frac{y_{mi}}{(y_{mi})_{\text{max}}}, \quad \delta y_{fi} = \frac{y_{mf i}}{(y_{mf i})_{\text{max}}},
$$

$$
\delta t_{gi} = \frac{t_{gi}}{(t_{gi})_{\text{max}}}, \quad \delta t_{fi} = \frac{t_{fi}}{(t_{fi})_{\text{max}}},
$$

$$
\delta J_{gi} = \frac{J_{gi}}{(J_{gi})_{\text{max}}}, \quad \delta J_{fi} = \frac{J_{fi}}{(J_{fi})_{\text{max}}},
$$

$$
\delta M_{gi} = \frac{M_{gi}}{(M_{gi})_{\text{max}}},
$$

where $i = 1, 2, \ldots$ – serial numbers of compared systems. And all the indexes of the same type for compared systems are rescaled to the same scale. This, in fact, is a basic requirement for correct comparison.

2) Calculation of the sums of relative values of all the performance indexes for each of the compared control systems

$$
S_i = \delta y_{gi} + \delta y_{fi} + \delta t_{gi} + \delta t_{fi} + \delta J_{gi} + \delta J_{fi} + \delta M_{gi}.
$$

Decreasing of all the components in the expression (1) means improvement of the system and as a consequence the best control system by the set of performance indexes corresponds the minimum value of the sum $S_i$. Here should be taken into account not the absolute values of the sums $S_i$ but their relative values. However, it is more appropriate to rescale these values to a single range like it is done with the performance indexes. This means division the individual values by the maximum value among them. The result is an expression for optimality criterion of the control system by the set of performance indexes and stability margin which will be called a complex performance criterion

$$
J_{\text{com}} = S_i/(S_i)_{\text{max}} = \min
$$

It is also necessary to pay attention to the stability margin influence on the properties of the control system. If the stability margin is too small, it threatens the disability of the control systems by large disturbances or changes of plant parameters. On the other hand, excessive stability margin leads to increasing of dynamic variations and settling time in the control loop. So there is the problem of choosing between dynamic accuracy and stability of the system. In practice, it is usually preferred the stability that is a factor of security. The above conflict can be solved by finding the optimal balance between dynamic accuracy and stability margin of the control system. For this purpose, the method of comparative performance assessment (that is described above) by the $J_{\text{com}}$ criterion can be used in a somewhat simplified version.

The equation (1) can be written in a form that takes into account only the parameters of dynamic accuracy and stability of the system, i.e.

$$
S_i = \delta J_{gi} + \delta J_{fi} + \delta M_{gi}.
$$

Then the optimal ratio "dynamic accuracy / stability" will be reached in the control system for which

$$
J_{MB} = S_i/(S_i)_{\text{max}} = \min.
$$

Calculations show that control systems which are optimal by the $J_{\text{com}}$ criterion usually provide also the optimum or very close to it ratio of "dynamic accuracy / stability" which corresponds to the equation (4).

3. Example

Let’s consider the features of the proposed method of comparative performance evaluation of control systems by the following example. For example, a simple closed loop feedback control system with PI controller is discussed below. The controlled process is described by the second order plus time delay model $W(p)=K_p e^{-\tau}/(Ts+1)^2$. The four versions of the control system have been analyzed which were named respectively System 1, 2, 3 and 4. Parameters of these systems and their performance measures are given in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>System #</th>
<th>$K_p K_i$</th>
<th>$T_s/T$</th>
<th>Stability margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>System 1</td>
<td>1.16</td>
<td>40.3</td>
<td>5.631</td>
</tr>
<tr>
<td>System 2</td>
<td>1.345</td>
<td>44.8</td>
<td>3.256</td>
</tr>
<tr>
<td>System 3</td>
<td>1.16</td>
<td>36.4</td>
<td>5.11</td>
</tr>
<tr>
<td>System 4</td>
<td>1.478</td>
<td>58.8</td>
<td>4.394</td>
</tr>
</tbody>
</table>

The relative values of performance measures and their sums are shown in Table 3.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>System 1</th>
<th>System 2</th>
<th>System 3</th>
<th>System 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_{0}$</td>
<td>1.105</td>
<td>0.744</td>
<td>1.347</td>
<td>0.910</td>
</tr>
<tr>
<td>$\Delta y_{1}$</td>
<td>0.444</td>
<td>1.0</td>
<td>0.340</td>
<td>0.743</td>
</tr>
<tr>
<td>$\Delta y_{2}$</td>
<td>40.30</td>
<td>0.685</td>
<td>44.67</td>
<td>0.759</td>
</tr>
<tr>
<td>$\Delta y_{3}$</td>
<td>45.40</td>
<td>1.0</td>
<td>36.20</td>
<td>0.797</td>
</tr>
<tr>
<td>$\Delta y_{4}$</td>
<td>5.631</td>
<td>1.0</td>
<td>4.342</td>
<td>0.731</td>
</tr>
<tr>
<td>$\Delta y_{5}$</td>
<td>2.094</td>
<td>1.0</td>
<td>0.752</td>
<td>0.358</td>
</tr>
<tr>
<td>$\Delta y_{6}$</td>
<td>0.052</td>
<td>0.437</td>
<td>1.676</td>
<td>0.098</td>
</tr>
<tr>
<td>$\Delta y_{7}$</td>
<td>-5.870</td>
<td>-5.037</td>
<td>-5.486</td>
<td>-5.626</td>
</tr>
<tr>
<td>$\Delta y_{8}$</td>
<td>-1.0</td>
<td>-0.858</td>
<td>-9.258</td>
<td>-9.958</td>
</tr>
</tbody>
</table>

The numerical data presented in Table 3, for greater clarity, are shown in Fig. 1.

As we can see, there is a clearly defined minimum for the System 2 (Fig. 2) that indexes its optimality by the criterion $J_{\text{com}}$. So the problem has been solved uniquely.

The data from Table 4 is shown in graphical form on Fig. 2.
Table 4. The components of the complex performance criterion and the resulting assessment δS of the systems by the ratio "dynamic accuracy / stability"

<table>
<thead>
<tr>
<th>Indicator</th>
<th>System 1</th>
<th>System 2</th>
<th>System 3</th>
<th>System 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>J₁, δJ₁</td>
<td>abs 5.631 rel 1.0</td>
<td>abs 4.342 rel 0.771</td>
<td>abs 5.11 rel 0.907</td>
<td>abs 4.394 rel 0.780</td>
</tr>
<tr>
<td>J₂, δJ₂</td>
<td>abs 2.094 rel 1.0</td>
<td>abs 0.752 rel 0.558</td>
<td>abs 1.597 rel 0.763</td>
<td>abs 0.471 rel 0.225</td>
</tr>
<tr>
<td>M, δM</td>
<td>abs 1.052 rel 0.433</td>
<td>abs 1.676 rel 0.696</td>
<td>abs 1.152 rel 0.480</td>
<td>abs 2.402 rel 1.0</td>
</tr>
<tr>
<td>δS</td>
<td>abs 1.437 rel 1.0</td>
<td>abs 1.828 rel 7.50</td>
<td>abs 1.270 rel 0.882</td>
<td>abs 2.005 rel 0.825</td>
</tr>
</tbody>
</table>

From Table 4 and Fig. 2 it follows that the System 2 is also the best by the ratio "dynamic accuracy / stability".

Thus, as the results of the studies it was found that the worst by the $J_{\text{com}}$ criterion is the System 1, so it could not be considered. The properties of the other three control systems are illustrated in Fig. 3, where the solid lines correspond to System 2 which is optimal by $J_{\text{com}}$ criterion.

Fig. 3 clearly shows that improving the performance of setpoint response leads to the disturbance response deterioration. However, it could be seen that the system which is optimal by the $J_{\text{com}}$ criterion provides a certain compromise between the performance of setpoint response and disturbance response.

Conclusions

The described comparison algorithm can be easily programmed in different software packages for any number of comparable systems, but in practice it is sufficient to study three or four possible variants of control systems.

References


