THE THEORY OF MATRIX MAGNETOSENSITIVE SENSOR 
ON THE BASIS OF FERROPROBES

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Summary: The mathematical models of field, which allow to determine the interaction of ferroprobe cores in a matrix with the influence of a constant magnetic field of article are considered. The models allow to execute numerical calculation of an electromagnetic field in cores created by both the field of a defect, and the field excitation. The calculation allows to receive data for the rational arrangement of ferroprobe cores in the matrix, and also to determine the transformation function of the matrix sensor.

Key words: control of defect, field of defect, ferroprobe.

INTRODUCTION

The matrix disposition of ferroprobes for control of defect in ferromagnetic article has a number of advantages such as possibility of surface control without mechanical scanning, forming a three — dimensional information signal about defect with the help of computer, possibility of curvilinear surface control. The impulse schemes of excitation [Krotov L.N. 1985; Gaichenko V.Y. 1992] are applied simplification of the schemes of treatment of output signals of ferroprobes. The close disposition of elements in the matrix sensor (MS) influences the transformation function of every ferroprobe by the inductive connection of a coils of excitation. In this paper two problems are decided. One problem allows to determine the magnetization in the cores caused by the magnetic field of defect, the second one assesses the influence of MS elements on each other, caused by a current in windings of excitation.

OBJECTS AND PROBLEMS

For the construction of the mathematical model the following assumptions are made:

- the magnetization area, in which the defect is located, does not vary at a measuring of the MS field;
• the field strength of the source, which magnetized of an article is considered to be known.

The mathematical model of the field of defect represents an integral equation [Vinokurov V.E. 1991]

\[
\mathcal{H} = \frac{1}{4\pi} \int_{\Sigma} \left( \frac{M(R) \cdot n}{|R - \mathcal{R}|^3} dS + \frac{\text{div} M(R) (R - \mathcal{R})}{|R - \mathcal{R}|^3} dV' \right) + \mathcal{H}_0,
\]

where: \( M \) is the vector of a ferromagnetic material magnetization; \( R \) and \( \mathcal{R} \) are vectors of points of observation and sources; \( n \) is the normal to a ferromagnetic surface; \( \mathcal{H} \) is the magnetic field strength in a point of observation; \( \mathcal{H}_0 \) is the magnetized field strength.

By the approximation of area or a ferromagnetic material with elementary volumes (EV), which have the shape of rectangular prisms, inside which the vector of a magnetization is constant, the equation (1) will be transformed in a system of the algebraic equations

\[
\mathcal{H}_i = [A_{ij}] [M_j] (H_j) + \mathcal{H}_{0i},
\]

where: \( i, j \) are points of observation and source.

The elements of the matrix \( [A_{ij}] \) are determined by geometric parameters of the defect and the space adjoining to it and are calculated for each EV by the following formula:

\[
a_{ij} = \frac{1}{4\pi} \sum_{k=1}^{6} \frac{\pi(R_k - \mathcal{R}_j)}{|R_k - \mathcal{R}_j|^3} dS_k.
\]

The system of equations (2) is supplemented by the function of a nonlinear dependence of the module of the ferromagnetic material magnetization vector of an inspected article on the strength for the first and the second quadrant of the hysteresis loop

\[
M_j = f(H_j),
\]

which is approximated by the cubic splines. The system of equations (2) is solved by an iterative mode with the help of algorithm offered in [Shvedchicova I. 1996].

Electric circuit of the ferroprobe is shown on the fig. 1.

For excitation circuit it is possible to write down

\[
\frac{d}{dt} (\psi_{a1} + \psi_{b1}) + H_b \frac{L}{W} R = \epsilon(t),
\]

where: \( \psi_{a1}, \psi_{b1} \) — magnetic-flux linkage of excitement windings of ferroprobes semi-elements a and b; \( H_b \) — field intensity in the cores of semi-elements; \( \epsilon(t) \) — excitation voltage of ferroprobe.
Output voltage of ferroprobe is determined from the following expression

\[ u_2 = \frac{d}{dt}(\psi_{a2} - \psi_{b2}), \]  

(6)

where: \( \psi_{a2}, \psi_{b2} \) — magnetic-flux linkage of output winding of ferroprobe. The task of further theoretical construction is the determination of magnetic-flux linkages \( \psi_{a1}, \psi_{b1}, \psi_{a2}, \psi_{b2} \).

According to the theorem of reciprocity [Polivanov K.M. 1974] the magnetic-flux linkage in the winding of the semi-element is equal to

\[ \psi_u = \frac{1}{i_u} \left[ \sum_{u=1}^{U} \left[ \sum_{V} H_u \cdot M_u dV_u + \sum_{V} H_p \cdot M_p dV_p \right] \right] \]

(7)

where: \( M_u \) — magnetization in the cores of ferroprobes, caused by excitation voltage; \( M_p \) — magnetization of the defect area; \( H_u, H_p \) — field intensity, created by excitation current in the cores and in the area of defect location.

While dividing the ferroprobes cores into EV (each core is given as K EO located along the core length), ratio (7) is presented in the form of U-quantity of semi-elements in the group.

\[ \psi_u = \frac{1}{i_u} \left[ \sum_{u=1}^{U} \sum_{k=1}^{K} H_{uk} \cdot M_{uk} \Delta V_{uk} + \sum_{p=1}^{P} H_p \cdot M_p \Delta V_p \right] \]

(8)

where: \( \Delta V_{uk}, \Delta V_p \) — EV of corresponding areas; \( P \) — quantity of EV in the area of defects.
Functioning of MS takes place both under excitation of one ferroprobe, and under excitation of the group or all ferroprobes.

It is quite sufficient to consider the influence of a group of cores on each other and located near each other.

There are three variants of a group of cores location, influencing each other in the matrix MS (fig. 2) — groups 10, 6, 4 according to the number of cores of semi-elements in the group, surrounding one ferroprobe (the cores of the considered ferroprobe are lined, the direction of the excitation field are shown by crosses and dots).

As a mathematical model of the vector field of magnetization in the cores an integral equation (1) is used. It is represented by a system of algebraic equations while dividing the volume of ferroprobes cores into EV

\[
\bar{H}_i = \left[C_{ij}\right] \bar{M}_j + \left[D_{ij}\right] \bar{M}_p + \bar{H}_b,
\]

where: \(\left[C_{ij}\right]\) the matrix with dimension L×L , where \(L = K \cdot U\); \(\left[D_{ij}\right]\) — matrix for calculation of intensity in i-EV, created by j-m magnetized EV defect; \(\bar{H}_b\) — intensity vector of excitation field.

The system of algebraic equations (9) is added by the dependence of magnetization on the field intensity for ferromagnetic material of ferroprobes cores

\[
M_j = \varphi(H_j),
\]

The elements of the matrix \([C]\) and \([D]\) are calculated through the formula analogous to (3).

Transformation function of ferroprobe is determined by the following ratio:

\[
S = \frac{U_{2m}}{H_0},
\]

where: \(U_{2m}\) — amplitude value of outlet ferroprobe signal, \(H_0\) — intensity of the measured field.

While calculating the transformation function the system of equations (9) is simplified, as the vector \(\bar{H}_b\) is used instead of the vector \(\left[D_{ij}\right] \bar{M}_p\) in (9). Its direction coincides with longitudinal axis of ferroprobe.

The algorithm of the coefficient of ferroprobe transformation is the following.

The function of the excitement voltage of the ferroprobe is approximated by the function

\[
e(n) = e(n\Delta t) \cdot 1(t-n\Delta t),
\]

where: \(n = 1 \ldots N\); \(1(t-n\Delta t)\) — single function; \(\Delta t\) — the time of quantization; \(N\) — quantity of time intervals of quantization.

Differential non-linear equation (5) is solved through numerical method

\[
\psi_{a1}^{(n)} + \psi_{b1}^{(n)} = \psi_{a1}^{(n-1)} + \psi_{b1}^{(n-1)} + \left[e(n) - qH^n\right]M,
\]

where: \(q = \frac{l}{W} \cdot R\).
The intensity of magnetic field in the core of ferroprobe is calculated by the method of sequence approximations

\[ H^{(n)} = \frac{1}{q} e(n) - \frac{1}{q\Delta t} \left[ \psi_{a1}^{(n)} + \psi_{b1}^{(n)} \right] + \frac{1}{q\Delta t} \left[ \psi_{a1}^{(n-1)} + \psi_{b1}^{(n-1)} \right] \]  

(14)

the value of magnetic-flux linkage in the cores of ferroprobes is determined by formula (8) after the solution of algebraic equations (9). All the calculations being made with the account of the direction of current in the windings of semi-elements of ferroprobes.

In the usual of numerical calculations the values of magnetic-flux linkages of the excitement windings \( \psi_{a1}(n), \psi_{b1}(n) \) are obtained and it permits to determine the output voltage of ferroprobe from the ratio

\[ u_2(n) = \frac{1}{\Delta t} \left[ \psi_{a2}(n) - \psi_{b2}(n) \right] 1(t - n\Delta t) \]  

(15)
where: \( \psi_{a2}(n) = \frac{\omega_2}{\omega_1} \); \( \psi_{b2}(n) = \frac{\omega_2}{\omega_1} \); \( \omega_2 \) — number of turns of excitement windings and output winding.

### CONCLUSIONS

The output signal of the single ferroprobe which measured the vertical component of the scattering field of defect with a breadth \( 2b = 0.2 \) mm, depth \( t = 0.5 \) mm and length \( a = 10 \) mm on a flat surface of steel IIIX18 are calculated. The distance between ferroprobe and surface of steel \( h \) was \( 0.2 \) mm. The ferroprobe core has a geometric parameter \( 3 \times 0.5 \times 0.025 \) mm; a winding of excitation has \( \omega_e = 50 \) coils; an output winding — \( \omega_2 = 30 \) coils. The ferroprobe was excited by unipolar impulses with an amplitude \( 15V \) and duration \( 1 \) mks. In the fig. 3 the plot of an output signal single ferroprobe is shown at transition one above the defect (dashed line) and ferroprobe, which were in an environment 10, 6 and 4 ferroprobes in the matrix.

The distances between the cores of ferroprobe made up \( \alpha = \frac{d}{l} = 1 \). Relative change of the transformation coefficient of the ferroprobe in the groups 10, 6 and 4 form the value \( \alpha \) is shown on the fig. 4.

The charts show that the stronger change of the transformation coefficient is obtained at \( \alpha < 1 \) and depends on the quantity of cores which surround the ferroprobe. The obtained data correspond to experimental ones given in [Vinokurov V.E. 1991].

![Fig.3. The plot of output signal of ferroprobe for the three groups of cores 10, 6, 4 at \( \alpha = \frac{d}{l} = 1 \)](image-url)
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ТЕОРИЯ МАТРИЧНОГО МАГНИТОЧУВСТВИТЕЛЬНОГО ДАТЧИКА НА ОСНОВЕ ФЕРРОЗОНДОВ

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Аннотация: В статье рассматриваются математические модели поля, которые позволяют определять взаимодействие феррозондовых элементов в матрице с влиянием постоянного магнитного поля. Модели позволяют выполнять численное вычисление электромагнитного поля в элементах, созданных как областью дефекта, так и полем возбуждением. Вычисления позволяют получать данные для рационального использования феррозондовых элементов в матрице, а также для определения передаточной функции матричного чувствительного элемента.

Ключевые слова: контроль дефекта, область дефекта, феррозонд.