Estimation of the load carrying capacity of the timber beams strengthened with FRP strips

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Abstract: This article presents general rules of determining load carrying capacity of timber beams strengthened with FRP materials with accordance to the American Standard PCF-5100 with relevance to the European. In addition, the conclusions from the comparison of the theoretical load carrying capacity and deflections evaluated on basis of PCF-5100 and PCF-6046 with the results of the laboratory tests conducted on full-scale beams are presented.

Keywords: timber structures, timber beams, laboratory tests, FRP, strengthening

INTRODUCTION

Strengthening of timber structures with fibre reinforced polymers (FRP) strips is a solution which is based on analogous, commonly used strengthening of RC-structures. However, in case of strengthening of RC-structures, there are already many verified algorithms to determine the effectiveness of the strengthening. In case of timber structures these methods are not completed yet, because strengthening of timber is rarely used in Europe. Just two American standards PCF-5100 and PCF-6046 take into account strengthening of timber and glulam structures with FRP. These standards present the methods to determine the load carrying capacity and stiffness of beams reinforced at the stage of production and glulam elements strengthened in tension zone or at tensional and compressional fibres.

GENERAL RULES FOR CALCULATIONS

Standards PCF-5100 and PCF-6046 consider aramid, glass and carbon fibres. The total amount of reinforcement is limited by cross section from minimum 0.25 percent of the total cross-sectional area of the beam to maximum 2.0 percent (in case of single reinforcement) or max. 4.0 percent (in case of double reinforcement). FRP strips may be glued into the cross section of the beam (under the last layer) or may be glued to both: the bottom and the top surfaces of the reinforced glued laminated timber beam.

To determine the neutral axis of the cross-section under flexure [PCF-5100 and PCF-6046] the following assumptions apply:
- distinction in modulus of elasticity along the grains of timber in tension and compression,
- the equivalent ratio of the tension modulus of elasticity of timber to the modulus of elasticity of timber under axial compression:

\[ n' = \frac{E_{t,0,\text{mean}}}{E_{c,0,\text{mean}}} \]

Due to the lack of differentiation in Eurocode 5 between both modulus of elasticity of timber under tension and compression it is acceptable to assume \( n' = 1 \).
- the equivalent ratio of the tension modulus of elasticity of FRP to the modulus of elasticity of timber along the grains under axial compression:
\[ n = \frac{E_f}{E_{c,0,\text{mean}}} \]

where:

- \( E_{f,0,\text{mean}} \) - mean modulus of elasticity of timber under tension along the grains,
- \( E_{c,0,\text{mean}} \) - mean modulus of elasticity of timber under compression along the grains,
- \( E_f \) - modulus of elasticity of the FRP strip under tension,

For the equivalent modulus of elasticity, in case of FRP strip applied only in tensional zone, the depth of the neutral axis is equal to:

\[ a = a' + M_t - N_t \]

where:

\[ a' = h - \frac{\sqrt{n'}}{1 + \sqrt{n'}} \quad N_t = n' d_1 (n - 1)^{0.63} \frac{t_f}{a'} \quad M_t = n' (n - 1)^{0.63} t_f \]

- \( h \) - depth of beam,
- \( t_f \) - total thickness of FRP strip in tensional zone,
- \( d_1 \) - distance from the centerline of FRP strip to the edge of the beam.

After evaluation of neutral axis of the strengthened element, the equivalent moment of inertia \( I_z \) was calculated – parallel axis (Steiner’s) theorem,

\[ I_z = \frac{b}{3} a^3 + b n' h_1^3 + b n t_f \left[ \frac{t_f^2}{12} + \left( \frac{c - d_1}{2} \right)^2 \right] + b n' t_f \left[ \frac{t_f^2}{12} + \left( \frac{c - t_1}{2} \right)^2 \right] \]

where: \( h_1 = h - a - t_1 - t_f \), \( c = h - a \)

For such equivalent moment of inertia, the deflection is calculated from the well-known formulas of the theory of elasticity. It depends on the load scheme, with regards to the equivalent moment of inertia of the strengthened cross-section of the beam and mean value of the modulus of elasticity \( E_{0,\text{mean}} \).

The load carrying capacity of the strengthened element is calculated [PCF-5100 and PCF-6046] with assumption of yielding of the compression zone. The strength of the element is determined either by the depletion of capacity in the compression zone of timber or by the limit strength of FRP strip. It is assumed that the tensional stresses are transferred by the FRP strips, analogically like in RC-structures.
For abovementioned assumptions, the allowable bending moment for reinforced glulam cross-section is equal to:

\[ M_r = F_c \times z' \]

where: \( F_c = \sigma_c \times a \times b \), \( z' = \frac{a}{2} + g = \frac{a}{2} + (c - d_1) = h - \frac{a}{2} - d_1 \)

Axial tensile stress in FRP strip is equal to:

\[ \sigma_f = \frac{M_r \times (g + \frac{I_f}{2}) \times n}{I_z} \]

Allowable horizontal shear stress for reinforced glued-laminated beam is equal to:

\[ \tau_j = \frac{2}{3} \times f_v \times h \times b \times h \]

where: \( f_v' = f_v + 20 \ln(x) \)  \( f_v \) - shear strength of timber.

In many cases there is no need to strengthen the elements under flexure along the whole length. Due to this fact, in the Standard, the design of strengthening at mid-span of the beam – strengthening non-extended to the supports – is considered. For the economical design of the strengthening of the glulam beams, it is necessary to define the theoretical points between which the FRP reinforcement is required. For safety, an additional 500 mm of bonding length of FRP strip is required.

GENERAL DESCRIPTION OF THE TESTS AND THE TESTS’ RESULTS

In the laboratory of the Faculty of Civil Engineering of the Silesian University of Technology the tests were conducted on the influence of glass-aramid (GARP) strips as reinforcement on flexural capacity of glulam beams. Fifteen glulam beams with the cross section of 140 * 320 mm and the length of 6200 mm, made from spruce timber GL28h were tested.

The test beams were divided into three groups of 5 samples each and characterized as follows:
- first group: 5 elements without strengthening,
- second group: 5 elements strengthened with GARP strips with modulus of elasticity \( E = 50 \) GPa; GARP strips glued internally between the last and next to the last lamellas, as it is shown in Fig. 2.
- third group: 5 elements with GARP strips, in this case: glued to the bottom surface (Fig. 2).

![Figure 2. Cross-sections of the beams and the laboratory stand](image)

The beams were subjected to four-points flexural tests with the concentrated forces spaced at \( 6h = 960 \) mm at mid-span. The span of the beam was equal to \( 18h = 5760 \) mm. All
elements were tested under flexure with accordance to PN-EN 384. During the tests, the deflections for the specified load steps were measured and the ultimate force was registered. The five elements were subjected to flexural tests without any reinforcement in order to determine the strength and stiffness values to be compared to those of the reinforced beams.

In the case of the beams strengthened with GARP strips glued internally, inside the cross-section, the increase in the flexural capacity by 53.6% was observed in comparison to the unstrengthened beams. The increase in the load carrying capacity of the beams with GARP strips glued externally to the bottom surface of the beams was even higher – by 67.5% in comparison to the unstrengthened beams. The tests and its results were described in the article [Brol, 2009].

CONCLUSIONS OF COMPARISON BETWEEN THE THEORETICAL CALCULATIONS AND THE RESULTS OF LABORATORY TESTS

The following points summarize the results based on the theoretical calculations [PCF-5100 and PCF-6046] with the results of laboratory tests [Markowska, 2014].

1. Presented numerical calculations are correct in terms of the standardized (theoretical; based on the Standards) capacity of the element.
2. Due to the lack of differentiation in Eurocode 5 the modulus of elasticity of timber under tension and under compression it is acceptable to assume $n' = 1$.
3. The value of the ultimate force in the tests was much higher than it was determined by theoretical calculations. This shows significant reserves of load carrying capacity despite assuming full yielding in compression zone in numerical calculations.
4. Values of deflections obtained with accordance to PCF-5100 and PCF-6046 are slightly underestimated in comparison to the tests results. It is due to assumptions of PCF-5100 and PCF-6046 of fully-elastic model of material for deflection.
5. It would be appropriate to use rules of Eurocode 5 in estimating deflections, which take into account load – duration and moisture influences on the values of deflections.

REFERENCES:
1. BROŁ J. 2009: Strengthening of bent glued laminated (glulam) beams with GARP tapes at the stage of production. Wiadomosci Konserwatorskie - Journal of Heritage Conservation nr 26; 345-353
3. PN-EN 384:2011 Structural timber. Determination of characteristic values of mechanical properties and density
Streszczenie: Szacowanie nośności belek drewnianych wzmacnianych taśmami FRP. W artykule przedstawiono metodykę wyznaczania nośności belek drewnianych wzmacnianych FRP według amerykańskiej normy PCF-5100 z uwzględnieniem uwarunkowań Europejskich. Przedstawiono również wnioski z porównania wyznaczania nośności i wielkości ugięć wg norm PCF-5100 i PCF-6046 z badaniami laboratoryjnymi na belkach w skali technicznej.

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