Numerical and analytical analysis of the stress-strain relations of extended beech plywood

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Abstract: Numerical and analytical analysis of the stress-strain relations to extended beech plywood.

The study presents a theoretical and numerical analysis of problems connected with the determination of strength parameters in multilayer beech plywood with varied layer configurations. Analytical calculations were performed in accordance with the classical theory of thin plates lamination in view of the theory of elasticity taking into consideration the Kirchhoff-Love hypothesis. The analyses focused on models of composite layers in plane stress. Values of components of transformed stiffness matrices were estimated theoretically together with those of flexibility in the axial and non-axial configurations of individual laminate layers. A verification was made of analytical calculation results with computer simulation results made on numerical models using the finite element method (FEM). Results of investigations are presented in the form of tables, graphs and graphic visualization of FEM calculations.

Keywords: FEM calculations, lamina strength, wood composites.

INTRODUCTION

Wood in reality is a material with anisotropic properties and a heterogeneous structure, which in the analysis of strength parameters makes certain problems with their definite identification. Thus certain simplifications are adopted in calculation models. The assumption that wood in the macro scale is a material with isotropic properties is an oversimplification, while the assumption of the material being orthotropic is a sufficient approximation for the needs of analytical considerations.

Wood in the macroscopic scale when assumed to have properties of orthotropic material in its structure is similar to composite material (Bodig et al. 1982). A comparable structure is found e.g. in LVL glue-laminated wood, plywood and wood-based composites. Plywood may be produced from different wood species and thanks to an appropriate arrangement sequence of layers and their volumetric proportions modified in the production process it may have varied strength parameter values.

In the processes of strength calculations for composite materials the Kirchhoff-Love hypothesis is adopted for thin plates and small displacements. Non-dilatational strains in the plane perpendicular to the middle surface are neglected (Reddy 1997).

MATERIAL AND METHODS

Limit strength parameters of composite layers such as stress or strain may be determined experimentally in the principal directions of the material axis. Thus obtained numerical data with five or three values, when discarding differences between uniaxial tension and compression of the layer, the limit strength parameters. In practice most composites work in the complex state of stress. Such a situation is observed in the case when the direction of load does not coincide with any of the principal material directions of the layer figure 1. A presentation of deformation shape and stress distribution in the layer of there wood-like laminates with different layer orientation was the objective of the test. Computer methods with numerical models and Finite Element Method (FEM) were used in the analysis of the considered problems. Numerical calculations were performed employing a professional computer program developed by an American Company Algor Inc. This is computing system based on the finite element method FEM (Spyrakos et al. 1994). Numerical calculations by the finite element method were performed using the Thin Composite four-node isoparametric rectangular plate finite elements in the computer software by Algor. Each laminate layer
consisted of 400 finite elements with 451 nodes. The element has constant thickness \( t_k = 1 \text{mm} \), the other two dimensions are 4x5 (mm x mm) and each node has a total of six degrees-of-freedom. The element has twenty-four degree-of-freedom, that is six degree-of-freedom per node. Geometry and material constants in numerical simulations were assumed to be identical to those in theoretical calculations.

Figure 1 a) The structure and diagram of load in layer laminate with global and local systems of coordinates, b) Finite element mesh for model of laminate and applied loading

Theoretical and numerical calculations were performed for the wood-based laminate with material properties of European beech (\textit{Fagus sylvatica} \textit{L.}). Material constants were adopted after literature data (Keylwerth 1951, Neuhaus 1994), as mean values amounting to density of 690 kg/m\(^3\), 12\% moisture content, moduli of elasticity and Poisson’s coefficient \( E_1=E_{11}=14000 \text{MPa}, E_2=E_{22}=1160 \text{MPa}, G_{12}=G_{66}=1080 \text{MPa} \), (major) \( \nu_{12}=\nu_{12}=0.52 \), (minor) \( \nu_{12}=\nu_{21}=0.043 \). Geometrical dimensions of the layer were \( L \times b = 200.0 \text{mm} \times 40.0 \text{mm} \) and thickness \( t_k = 1.0 \text{mm} \). A general laminate has layers of different orientations \( \theta \) where \(-90^\circ \leq \theta \leq 90^\circ\). Three standard laminates were assumed for these analyses coded, the symmetric laminate \([0/45/-45/90/90/-45/45/0]\) is denoted as \([0/45/-45/90]\), the unsymmetrical is denoted as \([0/60/30/90/90/-30/-60/0]\) and cross-ply \([0/90/0/90/0/90/0]\) is denoted as \([0/90/0/90]\). The subscript “s” denotes a symmetric laminate. Laminates were loaded at tensile force \( F = 5.0 \text{kN} \).

**RESULTS**

Analytical and numerical calculations. The reduced stiffness matrix for a single laminate layer in the function of engineering constants according to equation is as follows (German 1996):

\[
\begin{bmatrix}
 m E_{11} & m v_{12} E_{22} & 0 \\
 m v_{12} E_{11} & m E_{22} & 0 \\
 0 & 0 & G_{12}
\end{bmatrix}
= \begin{bmatrix}
 Q_{11} & Q_{12} & 0 \\
 Q_{21} & Q_{22} & 0 \\
 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
 14.32 & 0.62 & 0 \\
 0.62 & 1.19 & 0 \\
 0 & 0 & 1.08
\end{bmatrix}
\text{[GPa]}
\] (1)

where: \( m=(1-\nu_{12} \nu_{21})^{-1} \)

These constants (\( E_{11}, E_{22}, \nu_{12}, G_{12} \)) are associated with the main orthotropic directions of a given layer. For the calculation of components of the transformed matrix of laminate stiffness \( Q^*_{ij} \), we need to perform calculations according the following form:

\[
Q^*_{11}=Q_{11} c^4 + 2(Q_{12} + 2Q_{66}) s^2 c^2 + Q_{22} s^4, \quad Q^*_{12}=Q_{12} c^4 + (Q_{11} + Q_{22} - 4Q_{66}) s^2 c^2 + Q_{22} s^4, \\
Q^*_{22}=Q_{22} c^4 + 2(Q_{12} + 2Q_{66}) s^2 c^2 + Q_{11} s^4, \quad Q^*_{66}=Q_{66} c^4 + (Q_{11} + Q_{22} - 2Q_{12} - Q_{66}) s^2 c^2 + Q_{66} s^4, \\
Q^*_{16}=(Q_{11} - Q_{12} - 2Q_{12} - 2Q_{66}) s^2 c + (Q_{12} + Q_{22} - 2Q_{66}) s^2 c, \quad Q^*_{26}=(Q_{12} - Q_{22} - 2Q_{12} + 2Q_{66}) s^2 c + (Q_{11} - Q_{12} - 2Q_{66}) s^3 c,
\]
where: \( s = \sin \theta, c = \cos \theta \)
Transformation dependence between components of a deformation of strain state for an orthotropic layer in any coordinate system determined by angle \( \theta \), assumes the form:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix} Q'_{11} & Q'_{12} & Q'_{16} \\
Q'_{12} & Q'_{22} & Q'_{26} \\
Q'_{16} & Q'_{26} & Q'_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix}
\]

(2)

Results of calculations are given in tables 1, 2 and 3:

**Table 1** Components of the transformed stiffness matrix of laminate \([0/45/-45/90]_k\).

<table>
<thead>
<tr>
<th>Number of layer and angle ( \theta )</th>
<th>( Q^*_{11} )</th>
<th>( Q^*_{22} )</th>
<th>( Q^*_{12} )</th>
<th>( Q^*_{66} )</th>
<th>( Q^*_{16} )</th>
<th>( Q^*_{26} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1; 8 ( 0^\circ )</td>
<td>14.32</td>
<td>1.19</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2; 7 ( 45^\circ )</td>
<td>5.27</td>
<td>5.27</td>
<td>3.11</td>
<td>3.57</td>
<td>3.28</td>
<td>3.28</td>
</tr>
<tr>
<td>3; 6 ( -45^\circ )</td>
<td>5.27</td>
<td>5.27</td>
<td>3.11</td>
<td>3.57</td>
<td>-3.28</td>
<td>-3.28</td>
</tr>
<tr>
<td>4; 5 ( 90^\circ )</td>
<td>1.19</td>
<td>14.32</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2** Components of transformed stiffness matrix of laminate \([0/60/30/90/-30/-60/0]_k\).

<table>
<thead>
<tr>
<th>Number of layer and angle ( \theta )</th>
<th>( Q^*_{11} )</th>
<th>( Q^*_{22} )</th>
<th>( Q^*_{12} )</th>
<th>( Q^*_{66} )</th>
<th>( Q^*_{16} )</th>
<th>( Q^*_{26} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1; 8 ( 0^\circ )</td>
<td>14.32</td>
<td>1.19</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 ( 60^\circ )</td>
<td>2.61</td>
<td>9.17</td>
<td>2.49</td>
<td>2.95</td>
<td>1.77</td>
<td>3.92</td>
</tr>
<tr>
<td>3 ( 30^\circ )</td>
<td>9.17</td>
<td>2.61</td>
<td>2.49</td>
<td>2.95</td>
<td>3.92</td>
<td>1.77</td>
</tr>
<tr>
<td>4; 5 ( 90^\circ )</td>
<td>1.19</td>
<td>14.32</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6 ( -30^\circ )</td>
<td>9.17</td>
<td>2.61</td>
<td>2.49</td>
<td>2.95</td>
<td>-3.92</td>
<td>-1.77</td>
</tr>
<tr>
<td>7 ( -60^\circ )</td>
<td>2.61</td>
<td>9.17</td>
<td>2.49</td>
<td>2.95</td>
<td>-1.77</td>
<td>-3.92</td>
</tr>
</tbody>
</table>

**Table 3** Components of transformed stiffness matrix of laminate \([0/90/0/90]_k\).

<table>
<thead>
<tr>
<th>Number of layer and angle ( \theta )</th>
<th>( Q^*_{11} )</th>
<th>( Q^*_{22} )</th>
<th>( Q^*_{12} )</th>
<th>( Q^*_{66} )</th>
<th>( Q^*_{16} )</th>
<th>( Q^*_{26} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1; 3; 6; 8 ( 0^\circ )</td>
<td>14.32</td>
<td>1.19</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2; 4; 5; 7 ( 90^\circ )</td>
<td>1.19</td>
<td>14.32</td>
<td>0.62</td>
<td>1.08</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on components of the transformed stiffness matrix we calculate the matrix of disc rigidity of laminate. The calculation procedure for this matrix is presented based on laminate coded \([0/60/30/90/90/-30/-60/0]_k\). The calculation consists in the summation of respective elements of the transformed matrix according to formula:

\[
A_j = \sum_{i=1}^{n} Q'_{ij} \cdot t_k
\]

in which the thickness of a single layer is \( t_k = 1\text{mm} \). Results of calculations are as follows:

\[
A_j = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\
A_{21} & A_{22} & A_{26} \\
A_{61} & A_{62} & A_{66}
\end{bmatrix} = \begin{bmatrix} 54.58 & 12.44 & 0 \\
12.44 & 54.58 & 0 \\
0 & 0 & 16.12
\end{bmatrix} \text{[GPa mm].}
\]

(3)

Further calculations consist in the determination of the disc flexibility matrix for the laminate, calculated from the rigidity matrix according to the scheme \( C_{ij} = [A_j]^{-1} \):

\[
C_{ij} = \begin{bmatrix} 20.45 & -4.66 & 0 \\
-4.66 & 20.45 & 0 \\
0 & 0 & 65.63
\end{bmatrix} \cdot 10^{-3} \text{[mm/kN]}
\]

(4)
At axial disc loading we assume a limited thickness of the layer in relation to the laminate. Assuming e.g. unidirectional tensile load in the layer plane their strains are identical to those of the whole composite. In individual laminate layers this causes plane stress. Strains are identical in each layer, thus stresses in the laminate may be defined as the averaged value of stresses at the cross-section. Disc load of a layer will be: \( N_x = F / b = 5 \text{kN}/40 \text{mm} = 0.125 \text{kN/mm} \). The procedure of calculations of deformation parameters, such as relative strains \( \varepsilon \) in the directions of principal axes of the laminate, percentage elongation \( \delta \), as well as transverse contraction \( \Delta b \), e.g. for composite \([0/60/30/90/-30/-60/0] \) is as follows: \( \{ \varepsilon \} = [C] \{ N \} \)

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
20.45 & -4.66 & 0 \\
-4.66 & 20.45 & 0 \\
0 & 0 & 65.63
\end{bmatrix} \times 10^{-3} [\text{mm}/\text{kN}] \begin{bmatrix}
N_x \\
0 \\
0
\end{bmatrix} \times [\text{kN}/\text{mm}], \quad (5)
\]

\( \varepsilon_{xx} = 20.45 \times 10^{-3} \text{ mm/kN} \cdot 0.125 \text{ kN/mm} = 2.556 \times 10^{-3} \); \( \varepsilon_{yy} = -0.582 \times 10^{-3} \); \( \gamma_{xy} = 0 \), \( \delta = L \cdot \varepsilon_{xx} = 200 \text{ mm} \cdot 2.556 \times 10^{-3} = 0.511 \text{ mm} \), \( \Delta b = b \cdot \varepsilon_{yy} = 40 \text{ mm} \cdot (-0.582 \times 10^{-3}) = -0.023 \text{mm} \).

It results from numerical analyses that the length and width of the laminate in the principal directions of orthotropy were changed. Strain in the form of elongation ranged from 0.405 to 0.523 mm, while transverse contraction ranged from -0.008 to -0.032 mm. Estimated deformation parameters of laminates are given in table 4.

**Table 4** Values of deformation parameters of laminates. FEM values were read from numerical models.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>Strain ( \times 10^{-3} )</th>
<th>Elongation [mm]</th>
<th>Contraction [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varepsilon_{xx} )</td>
<td>( \varepsilon_{yy} )</td>
<td>( \gamma_{xy} )</td>
</tr>
<tr>
<td>([0/45/-45/90]_s )</td>
<td>2.614</td>
<td>-0.748</td>
<td>0</td>
</tr>
<tr>
<td>([0/60/30/90/-30/-60/0] )</td>
<td>2.556</td>
<td>-0.582</td>
<td>0</td>
</tr>
<tr>
<td>([0/90/0/90]_s )</td>
<td>2.027</td>
<td>-0.162</td>
<td>0</td>
</tr>
</tbody>
</table>

Estimated analytical stresses according to dependence (2) at respective values from tables 1, 2 and 3, in individual layers of laminate are presented in fig. 2. Example results of analytical calculations for a layer with an orientation angle of 30° are as follows:

\[
\begin{align*}
\sigma_{xx} &= 9.17, 2.49, 3.92, 2.56, 22.03, \text{MPa} \\
\sigma_{yy} &= 2.49, 2.61, 1.77, -0.58, 4.86, \text{MPa} \\
\tau_{xy} &= 3.92, 1.77, 2.95, 0, 9.01, \text{MPa}
\end{align*}
\]

\( (6) \)

Stress values estimated in layers using the numerical method require transformations of components of the state of stress from the local to global systems (Makowski 2010). The algorithm of calculations consists in the reading of mean values \( \sigma_{11} \), \( \sigma_{22} \), \( \tau_{12} \) for example figure 3, in the medial zone from each layer in the stress map. The example presented below presents calculations of stresses in the layer with orientation angle \( \theta = 30^\circ \).

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = [T] \begin{bmatrix}
\sigma_{11_{xoy}} \\
\sigma_{22_{xoy}} \\
\tau_{12_{xoy}}
\end{bmatrix} = \begin{bmatrix}
0.75 & 0.25 & -0.87 \\
0.25 & 0.75 & 0.87 \\
0.43 & -0.43 & 0.5
\end{bmatrix} \begin{bmatrix}
23.40 \\
1.76 \\
-3.03
\end{bmatrix} = \begin{bmatrix}
20.63 \\
4.54 \\
7.79
\end{bmatrix} [\text{MPa}]
\]

\( (7) \)
Laminas are also varied and have values close theoretical values. In figure 2, estimated in layers of individual types of laminate are presented in figure 2.

\[
\begin{bmatrix}
  \cos^2 \theta & \sin \theta \cos \theta & 2 \sin \theta \cos \theta \\
  \sin \theta \cos \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\
  -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\]

- trasformation matrice

Results of analytical and numerical calculations of stresses in the plane stress, estimated in layers of individual types of laminate are presented in figure 2.

**Table 1**

<table>
<thead>
<tr>
<th>Laminate [0/45/-45/90]s</th>
<th>Layers and angle ( \theta )</th>
<th>( \sigma_{xx} ) [MPa]</th>
<th>( \sigma_{yy} ) [MPa]</th>
<th>( \tau_{xy} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>FEM</td>
<td>Theory</td>
<td>FEM</td>
</tr>
<tr>
<td>1.8</td>
<td>0°</td>
<td>41.51</td>
<td>36.97</td>
<td>0.96</td>
</tr>
<tr>
<td>2.7</td>
<td>45°</td>
<td>13.20</td>
<td>11.43</td>
<td>5.32</td>
</tr>
<tr>
<td>3.6</td>
<td>-45°</td>
<td>13.20</td>
<td>11.43</td>
<td>5.32</td>
</tr>
<tr>
<td>4.5</td>
<td>90°</td>
<td>3.04</td>
<td>2.64</td>
<td>-8.49</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Laminate [0/60/30/90/90/-30/-60/0]s</th>
<th>Layers and angle ( \theta )</th>
<th>( \sigma_{xx} ) [MPa]</th>
<th>( \sigma_{yy} ) [MPa]</th>
<th>( \tau_{xy} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>FEM</td>
<td>Theory</td>
<td>FEM</td>
</tr>
<tr>
<td>1</td>
<td>0°</td>
<td>36.30</td>
<td>36.77</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>60°</td>
<td>5.24</td>
<td>3.56</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>30°</td>
<td>22.03</td>
<td>20.63</td>
<td>4.86</td>
</tr>
<tr>
<td>4.5</td>
<td>90°</td>
<td>2.69</td>
<td>2.86</td>
<td>-6.72</td>
</tr>
<tr>
<td>6</td>
<td>-30°</td>
<td>23.03</td>
<td>18.36</td>
<td>4.86</td>
</tr>
<tr>
<td>7</td>
<td>-60°</td>
<td>5.24</td>
<td>4.53</td>
<td>1.06</td>
</tr>
<tr>
<td>8</td>
<td>0°</td>
<td>36.30</td>
<td>36.77</td>
<td>0.90</td>
</tr>
</tbody>
</table>

**Table 3**

<table>
<thead>
<tr>
<th>Laminate [0/90/0/90]s</th>
<th>Layers and angle ( \theta )</th>
<th>( \sigma_{xx} ) [MPa]</th>
<th>( \sigma_{yy} ) [MPa]</th>
<th>( \tau_{xy} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>FEM</td>
<td>Theory</td>
<td>FEM</td>
</tr>
<tr>
<td>1.8</td>
<td>0°</td>
<td>29.04</td>
<td>28.95</td>
<td>1.06</td>
</tr>
<tr>
<td>2.7</td>
<td>90°</td>
<td>2.33</td>
<td>2.31</td>
<td>-1.08</td>
</tr>
</tbody>
</table>

**Figure 2** Stress distribution along the laminate thickness. Dot. lines present FEM values.

Images of laminate deformation and stress distribution in selected layers are presented in figure 3. Values referring to stresses corresponding to stresses in the layers of individual laminas are also varied and have values close theoretical values.

**Table 4**

<table>
<thead>
<tr>
<th>Laminate [0/45/-45/90]s</th>
<th>Layer 1 (( \theta = 0° ))</th>
<th>( \sigma_{11} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Composite Stress</td>
<td>( \sigma_{11} = 36.97 )</td>
</tr>
<tr>
<td></td>
<td>[Composite Stress]</td>
<td>( \sigma_{11} = 36.97 )</td>
</tr>
</tbody>
</table>

Laminate displacement in the direction of the x axis [mm]
**CONCLUSIONS**

In the analysis of stress distribution in individual laminate layers in the control section, the results of calculations from both methods differed slightly, while the distribution was
similar. Minimal discrepancies in the calculation results may have been caused e.g. by the simplifying assumptions in the adopted calculation models. We may include e.g. neglecting the effect of the so-called Lechnicki (Lekhnitskii 1982) coefficients in non-axial configurations of layers. In the axial configuration of layers for example the laminate with the [0/90/0/90]s configuration does not have shear strains resulting from the occurrence of shear stresses. In the non-axial configuration of layers, apart from normal strains, the shear strains were found in laminates with the [0/45/-45/90]s, and [0/60/30/90/-30/-60/0] configurations in layers 2, 3, 6, 7. These layers under the influence of uniaxial tension are also subject to shear if an unidirectional external load is applied to the laminate. In layers 1, 4, 5 and 8, in the above mentioned laminates and in the laminate with the [0/90/0/90]s, layer configuration this effect is not found, since axes x y are also principal material axes. If an unidirectional external load is applied to the laminate with the [0/60/30/90/-30/-60/0] configuration, then it undergoes rotational deformation by a certain angle in relation to the original plane figure 3, while no such deformation is found for the other laminates.

High consistency of analytical calculation results with those of numerical calculations confirms accuracy of the adopted assumptions and model solutions.

Streszczenie: Numeryczna i analityczna analiza relacji naprężenia i odkształcenia w sklejce bukowej przy obciążeniu rozciągającym. W pracy przedstawiono analizę teoretyczną i numeryczną zagadnień związanych z określaniem parametrów wytrzymałościowych wielowarstwowego laminatu drewnopochodnego o różnej konfiguracji warstw. Obliczenia analityczne wykonano zgodnie z klasyczną teorią laminacji płyt cienkich w zakresie teorii sprężystości z uwzględnieniem hipotezy Kirchhoffa-Love’a. Przedmiotem analiz były modele warstw kompozytu znajdujące się w płaskim stanie naprężenia. Oszacowano teoretycznie wartości składowych transformowanych macierzy sztywności oraz podatności w konfiguracji osiowej i nieosiowej indywidualnych warstw laminatu. Wyniki badań przedstawiono w formie tabel, wykresów oraz graficznej wizualizacji obliczeń z MES. Analityczne obliczenia zweryfikowano numerycznymi modelami opartymi o metodę elementów skończonych. Wyniki badań analitycznych i numerycznych przedstawiono w postaci tabel i map graficznej wizualizacji wyników MES.

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