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# Modified model following control structure

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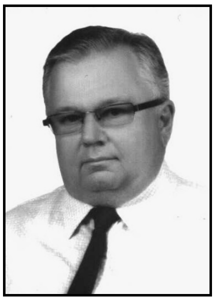

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**Abstract**

This paper presents a modified version of the control system with model following control (MFC). The auxiliary (corrective) controller has been designed for an input signal, which is the difference between the amplified state vectors of a model and process. The values of gain coefficients of the state variables of the model and process have been calculated according to the LQR *(Linear Quadratic Regulator*) procedure. It has been proved that a new controller improves the properties of the MFC system.

**Keywords**: model following control, uncertain system, fault tolerant control.

## Zmodyfikowana struktura ze śledzeniem modelu

**Streszczenie**

Artykuł dotyczy dwu pętlowego układu regulacji ze śledzeniem modelu typu MFC (*Model Following Control*). Układ taki posiada dwa regulatory: główny oraz korekcyjny. W klasycznym rozwiązaniu sygnałem wejściowym regulatora pomocniczego jest różnica sygnałów wyjściowych z modelu i procesu. Pozwala to na generowanie korekcyjnego sygnału błędu w przypadku zaistnienia różnic między procesem i modelem. Takie rozwiązanie nie pozwala jednak w jawny sposób kontrolować zmiennych stanu procesu. Dlatego też w artykule przedstawiono zmodyfikowaną wersję układu regulacji MFC, z regulatorem pomocniczym (korekcyjnym), którego sygnałem wejściowym jest różnica odpowiednio wzmocnionych współrzędnych stanu modelu i procesu. Analizowany układ opisano w przestrzeni zmiennych stanu, a regulator korekcyjny został zaprojektowany jako regulator typu LQR (*Linear Quadratic Regulator*). W dalszej kolejności przeproawdzono analizę porównawczą własności nowego i klasycznego układu MFC (wartości własne, stopień oscylacyjności, stabilności oraz funkcję wrażliwości). Przedstawiono, że układ MFC może być wykorzystywany również jako tolerujący uszkodzenia (a nie tylko w przypadkach zmian dynamiki procesu), co umieszcza go w klasie układów regulacji odpornych na uszkodzenia. Zostało to zilustrowane przeprowadzonymi badaniami symulacyjnymi.

**Słowa kluczowe**: regulacja ze śledzeniem modelu, systemy niepewne, regulacja tolerująca uszkodzenia.

## 1. Introduction

The faults (failures) of controlled systems occur very often during exploitation. These faults can occur in any component of the controlled system such as sensors, actuators. Plant, sensor or actuator faults may be total failures or partial faults. They could involve changes in input/output signals or in the parameters of the differential equations describing the system dynamics. However, control systems must work in such situations, despite the occurrence of different kind of failures or faults (abrupt, incipient, intermittent, actuator failures and sensor failures – inaccurate measurements, intermittent and major failures). Such systems are called fault tolerant control systems (FTCS). They offer the possibility of auto-adaptation of the damaged components and are capable of maintaining overall system stability and acceptable performance in case of such failures. Some degradation in control performance may be accepted, of course, and it is often called as "graceful performance degradation" [1].

This paper presents synthesis and analysis of the MFC/IMC (*Model Following Control/Internal Model Control*) structure, which has passive FTCS properties.

## 2. MFC/IMC description in state space

The MFC/IMC structure described by the transfer function is shown in Fig. 1 [2], where: $M(s)$ – model of nominal plant, $P(s)$ – plant, $R_m(s)$ – main controller, $R(s)$ – corrective (auxiliary) controller and $r(s)$ – set point value, $y(s) = y_p(s)$ – process controlled value, $y_m(s)$ – model output, $d(s)$ – disturbances.

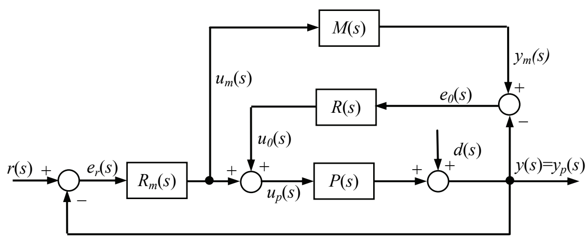


Fig. 1. Block diagram of classical MFC/IMC structure
Rys.1. Schemat blokowy klasycznego układu MFC/IMC

The main controller $R_m$ is often of the PI (or PID) type and auxiliary controller $R$ is proportional with the gain coefficient $k_p$. The goal is to find control input $u_p$ which can track the object state according to the reference model. Then the output signal $y_p$ will track $y_m$ automatically, despite faults in the system.

The state equations and output equations (1) of the whole system (Fig. 1) can be written as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}, \qquad (1)$$

where (for exemplary data):

$$A = \begin{bmatrix} A_p - B_p D_r C_p - B_p k_p C_p & B_p k_p C_m & B_p C_r \\ -B_m D_r C_p & A_m & B_m C_r \\ -B_r C_p & 0 & A_r \end{bmatrix};$$

$$B = \begin{bmatrix} B_p D_r \\ B_m D_r \\ B_r \end{bmatrix};$$

$$C = \begin{bmatrix} C_p & 0 & 0 \end{bmatrix};$$

$x(t) = \begin{bmatrix} x_p(t) & x_m(t) & x_r(t) \end{bmatrix}^T$; $u(t)=r(t)$ and state vectors $x_p \in R^n$, $x_m \in R^n$; input vectors $u_p \in R^p$ $u_m \in R^p$; output vectors $y_p \in R^q$ $y_m \in R^q$ and $A_p$, $B_p$, $C_p$, $A_m$, $B_m$, $C_m$, $A_r$, $B_r$, $C_r$, $D_r$ are constant matrices of appropriate dimensions of the plant, model and main controller. The pairs of matrices ($A_p$, $B_p$) and ($A_m$, $B_m$) are stabilizable and $A_m$ is stable matrix.

For the exemplary nominal data of process $P(s) = \dfrac{k_{ob}}{(T_p s + 1)^3}$ ($k_{ob}$= 1, $T_p$ = 3 sec) and its model $M(s) = \dfrac{k_m}{(T_m s + 1)^3}$ ($k_m$= 1, $T_m$ = 2.5 sec), main controller $G_r(s) = k_r\left(1 + \dfrac{1}{T_i s}\right)$ ($k_r$= 0.8, $T_i$ = 4 sec), and corrective controller $G_c(s) = k_p$ ($k_p$= 1.6), the considered system is controllable and observable. The parameters of controllers have been set according to the modified (for the needs of MFC) Ziegler-Nichols procedure, described in [3].

## 3. Modified MFC/IMC architecture and its properties

As it follows from Fig. 1, the auxiliary (corrective) controller R generates a correction signal uo based on the information contained in the error signal $e_o$, i.e. the difference between the output signals of: model $y_m$ and plant $y_p$. In this situation, the plant state variables are 'beyond control' and they do not follow the state variables of the model in a transparent manner. Tracking the state variables of the model, by the state variables of the process is carried out implicitly, because the state variables of both the model and process are included in the output signals $y_m$ and $y_p$, respectively. A character of the waveforms of state variables of the process is very important, for example in the control systems of drives. In these systems, is it necessary to provide the required waveforms of acceleration, velocity and position. Measurements of these physical quantities in those plants are possible, because these state variables are usually directly available. In the cases, when the state variables are unavailable, a possible approach to overcome this difficulty is to estimate the state $x_p$, of the process based solely on the measured output $y$, and use $\hat{x}_p$, where $\hat{x}_p$, denotes an estimate of the process state $x_p(t)$.

In order to track the state variables $x_m$ of the model directly by the process state variables $x_p$, a new control system has been proposed (Fig. 2). It is the MFC system with a proportional corrective controller, but its input signal is the difference of weighted state variables of the process and model (2)

$$u_o = K_m x_m - K_p x_p \,, \tag{2}$$

where $K_m$ and $K_p$ are the vectors of the gain coefficients of the model and process, respectively. The values of $K_m$ and $K_p$ have been designed according to the LQR procedure.

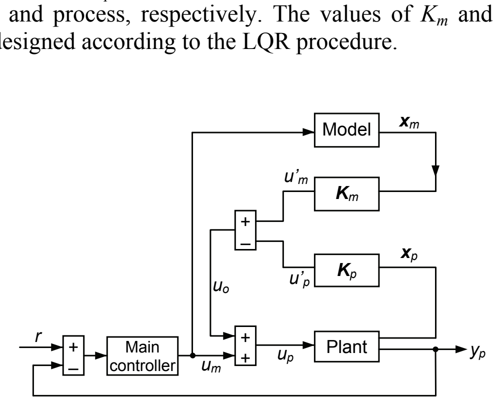


Fig. 2. Block diagram of MFC/IMC structure with state feedback
Rys. 2. Schemat blokowy układu MFC/IMC ze sprzężeniem od stanu

Now, the following matrices (3) describe the new system (Fig. 2) in the state space (for the exemplary data):

$$A_{new} = \begin{bmatrix} A_p - B_p D_r C_p - B_p K_p & B_p K_m & B_p C_r \\ -B_m D_r C_p & A_m & B_m C_r \\ -B_r C_p & 0 & A_r \end{bmatrix}$$
$$B_{new} = \begin{bmatrix} B_p D_r \\ B_m D_r \\ B_r \end{bmatrix} \quad C_{new} = \begin{bmatrix} C_p & 0 & 0 \end{bmatrix} \tag{3}$$

For exemplary 'nominal data' and $Q = 150 \cdot I_7$, $R = 0.01$ the values of gain coefficients are (4):

$$\begin{aligned} K_m &= [123.0044 \;\; 212.6487 \;\; 122.4105], \\ K_p &= [123.2038 \;\; 212.7949 \;\; 122.4375] \end{aligned} \tag{4}$$

The matrices ($A_{new}$, $B_{new}$) are controllable ($rank$=7) and ($Q$, $A_{new}$) are detectable ($rank$=7), matrices ($A_{new}$, $C_{new}$) are observable.

The eigenvalues $s_k$ for these data are following (5):

$$\begin{aligned} &-122.47; -0.87 + 0.50\,i; -0.87 - 0.50\,i; -0.66; \\ &-0.17 + 0.17\,i; -0.17 - 0.17\,i; -0.19 \end{aligned} \tag{5}$$

and the oscillation degree $\mu$ of the new system is $\mu_{new} = 1$, and its stability degree ($\eta = min|Resk|) - \eta_{new}$= 0.17 [4].

The proposed system is therefore asymptotically stable.

For an unmodified MFC system (Fig. 1) with the previous data, the pair of matrices ($A$, $B$) is controllable ($rank$ = 7) and ($C$, $A$) is observable ($rank$=7).

The eigenvalues for these data are following (6):

$$\begin{aligned} &-0.7145 + 0.1029i; -0.7145 - 0.1029i; \\ &-0.0663 + 0.2681i; -0.0663 - 0.2681i; \\ &-0.2052 + 0.3064i; -0.2052 - 0.3064i; -0.2280 \end{aligned} \tag{6}$$

and the oscillation degree of this system is $\mu$=4, and its stability degree – $\eta$ = 0.0663, therefore $\mu_{new}$ = 0.25$\mu$, $\eta_{new}$ = 2.56$\eta$, also the new system is less oscillatory (Fig. 3) and more stable (in classical system – the gain margin = 1.07 dB, the phase margin = 6.18°; in the modified control system – the gain margin = 16.6 dB, the phase margin= –180°). The settling time for the modified system is slightly bigger than 17 sec, and for the unmodified system, is greater than 45 sec (Fig. 3).

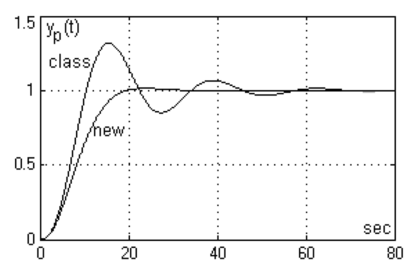


Fig. 3. Step responses in the proposed (IAE=8.88) and classical (IAE=11.04) system (both systems differed with corrective controllers only)
Rys. 3. Odpowiedzi skokowe w układzie proponowanym (IAE=8.88) i klasycznym (IAE=11.04); oba układy różnią się tylko regulatorem korekcyjnym

## 4. Parametric uncertainty

The parametric uncertainty occurs when the values of the model parameters are not precisely known or may vary (which may be a symptom of fault or failure).

Comparison of exemplary faults of the ordinary architecture of MFC (Fig. 1) and the new one with the proposed controller (Fig. 2) for different faults ( step changes in the set-point) is shown in Tab. 1 and in Figs. 4 and 5.

Tab. 1. Performance indices of sytems ($Q$ = 150, $R$ = 0.01)
Tab. 1. Wskaźniki jakości w układzie (dla $Q$ = 150, $R$ = 0.01)

| No | Exemplary faults | IAE | |
|---|---|---|---|
| | | MFC new | MFC class |
| 1. | x2+5sin(0.25) with saturation ±2 of control signal up | 18.48 | 75.09 |
| 2. | Delay (τ =2 sec) at the plant input | 11.81 | 13.13 |

The faults of the process or closed-loop control system can appear as changes of the current values (relative large) of the error in control loop [5]; this method has been used in this paper.

Both systems were compared on the basis of the performance index IAE (*Integral of Absolute Error*).

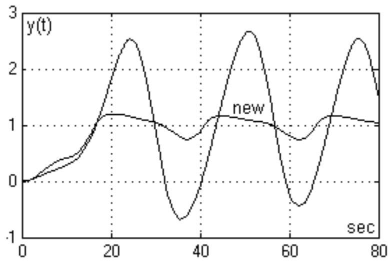


Fig. 4. Step response to the parametric uncertainty: Tab. 1, 1st point
Rys. 4. Odpowiedź skokowa na niepewność parametryczną – Tab. 1., punkt 1

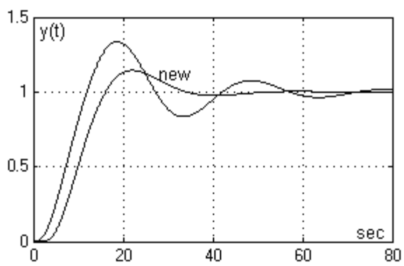


Fig. 5. Step response to the parametric uncertainty: Tab. 1, 2nd point
Rys. 5. Odpowiedź skokowa na niepewność parametryczną – Tab. 1., punkt 2

Magnitudes of the complementary sensitivity transfer functions $T(j\omega)$ for the classical and novel systems are presented in Fig. 6. As it is known, for good tracking performance, there is required that:

- $|T(j\omega)|$ is small for large $\omega$, so that the effect of the sensor noise is attenuated,
- $|T(j\omega)|$ is unity (0 dB) for small $\omega$, so that the (low-frequency) characteristics of the reference input are unaffected.

The new system satisfies these requirements better.

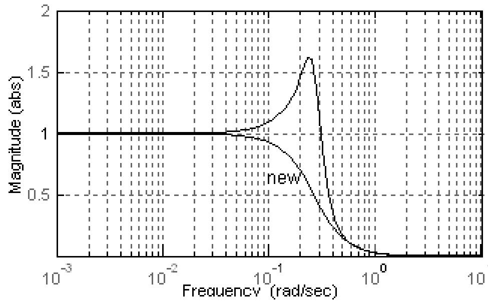


Fig. 6. $|T(j\omega)|$ for the classical and new system under nominal conditions
Rys. 6. $|T(j\omega)|$ dla układu klasycznego i nowego w warunkach nominalnych

## 5. Conclusions

This paper presents a new version of the model following a control system with the model of controlled process. Using the method of state variables, the proportional, differential state controller has been designed. The input signal of this controller is a difference between the relevant state variables of the model and process. The new control structure has been thoroughly analyzed from the point of view of fulfilling the conditions of stability and performance control. Then it was compared to the classical structure with the model following. Additionally, the performance of the new system has been analyzed in the presence of the parametric uncertainty. The effects and conclusions of this analysis suggest that the new system is better than the classical MFC.

The new system allows controlling the waveform parameters of the process state variables and it has better performance indices. Application LQR procedure to design of the correction controller enables using all advantages of the systems with these controllers. These advantages are: ease design of controllers, the guarantee of stability and quality performance control with bounded value of the control signal. Although the presented system is not adaptive, it works very well in the presence of the parametric uncertainty.

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