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STRESS SENSITIVITY COEFFICIENT OF A MATERIAL IN THE RANGE OF HIGH-CYCLE FATIGUE

Key words

Fatigue life, two-parametric characteristics, S355J0 steel.

Summary

In the calculation of fatigue life of structural components that have been subjected in service conditions to stochastic load, load spectra in the form of sinusoidal cycles of different asymmetry can be observed. Cycle asymmetry is distinguished by the stress ratio $R = S_{min} / S_{max.}$ In case of the load mentioned above, the R coefficient changes in broad boundaries from $-\infty$ to $+\infty$. In this work, the analysis of the material's stress sensitivity to cycle asymmetry of random load has been presented. In the references, analytical formulas for the fatigue strength range can be found. In this work, the formulas for High Cycle Fatigue (HCF) have been derived and the empirical formula for calculating stress sensitivity coefficient of a material of a random load, which facilitates fatigue calculations, has been given.

Nomenclature

A	–	elongation in %,
C	–	constant in formula describing the S-N curve for the fluctuating stress (R = 0),
C_0	–	constant in formula describing the S-N curve for the alternating stress (R = -1),

N	–	the number of cycles general designation fatigue life),
N_0	–	the number of cycles of fatigue life corresponding to fatigue limit
$R = S_{min}/ S_{max}$	–	the stress ratio,
R_e	–	material plasticity limit in MPa,
R_f	–	general fatigue limit designation in MPa,
R_m	–	tensile strength in MPa,
R_0	–	fatigue limit for fluctuating stress (R = 0) for the number of cycle N_0 in MPa,
R_0^N	–	fatigue limit for the sinusoidal fluctuating stress (R = 0) for the number of cycles N in MPa,
R_{-1}	–	fatigue limit for the alternating stress (R = -1) for the number of cycles N_0 in MPa,
R_{-1}^N	–	fatigue limit for the sinusoidal alternating stress (R = -1) for the number of cycles N in MPa,
S	–	general stress designation in the specimen in MPa,
$S_a = 0{,}5(S_{max} – S_{min})$	–	stress amplitude in the sinusoidal cycle in MPa,
$S_m = 0{,}5(S_{max} + S_{min})$	–	the mean stress in the sinusoidal cycle in MPa,
$S_{max} = S_m + S_a$	–	the maximum stress in the sinusoidal cycle in MPa,
$S_{min} = S_m - S_a$	–	the minimum stress in the sinusoidal cycle in MPa,
Z	–	contraction in %,
m	–	exponent in the formula describing the S-N curve for the fluctuating stress (R = 0),
m_0	–	exponent in the formula describing the S-N curve for the alternating stress (R = -1),
ψ	–	material stress sensitivity coefficient for $N = N_0$,
ψ_N	–	material stress sensitivity coefficient for $N \neq N_0$.

1. Formulation of the problem

The basis of the fatigue diagram determination in the system amplitude S_a – where the average value S_m of stress are the Wöhler curves, often designated in the references as S-N curves. They occur in various forms, e.g.: $S_a(N)$, $S_{max}(N)$, etc. Figure 1 presents the set of diagrams $S_a(N)$, drawn with assumption of stress ratio constant (R-const.). Privileged diagrams from the formula given are S-N curves for R = -1 (alternating stress where $S_m = 0$) and for R = 0 (fluctuating stress $S_{min} = 0$). For durability $N > N_0$ fatigue limits have been marked on the diagram with points respectively: 1-R_{-1}; 2-$R_{-0,5}$ 3-R_0 and 4-$R_{0,5}$. On that basis, the ultimate bearing capacity diagram 1-2-3-4 is presented in Fig. 2. Similarly, any constant durability line N = const. can be drawn. For N’ from diagrams depicted in Fig.1, one receives respectively points 1’,2’,3’,4’, moved to Fig. 2 give a line

1'-2'-3'-4'. The stress sensitivity coefficient of a material with reference to ultimate bearing capacity diagram is defined as follows:

$$\psi = tg\Theta \tag{1}$$

$$\psi = \frac{2R_{-1} - R_0}{R_0} \tag{2}$$

Data equal to coefficient value (ψ) for numerous sets of steel, aluminium alloys, cast steel and titanium alloys can be found in monographs [1] and [4] and in works of W. Schütz [2] and [3]. Analogically, for the range of limited durability, this coefficient is determined by the following formula:

$$\psi_N = \frac{2R_{-1}^N - R_0^N}{R_0^N} \tag{3}$$

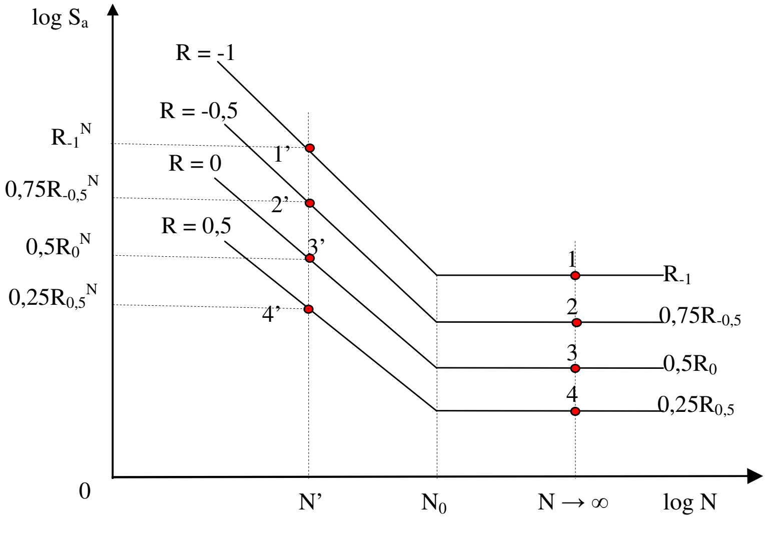

Fig. 1. Set of diagrams Sa drawn with assumption of the constant value of the stress ratio R = const

Fatigue diagrams for R = -1 and R = 0 are determined by formulas:

$$\left(R_{-1}^N\right)^{m_o} \cdot N = R_{-1}^{m_o} N_o = C_o \qquad \text{for R = -1} \tag{4}$$

$$\left(R_0^N\right)^m \cdot N = R_0^m N_o = C \qquad \text{for R = 0} \tag{5}$$

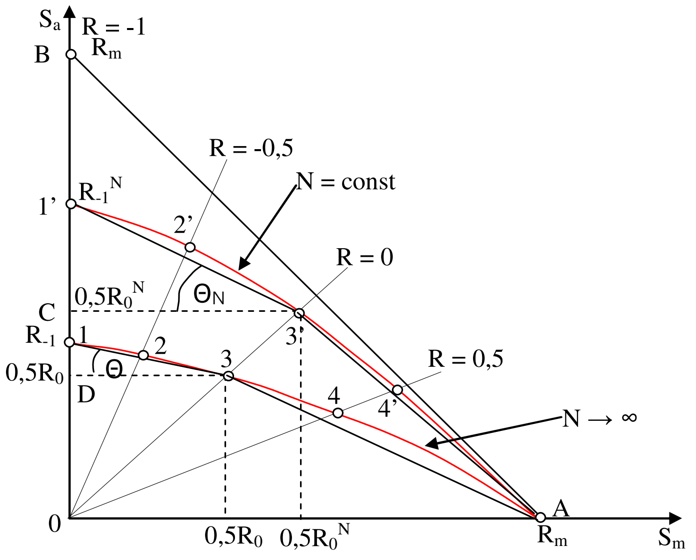

Fig. 2. Diagrammatic representation of two-parametric fatigue characteristics $N(S_a, S_m)$

After replacing formula (3) with formulas (4) and (5) and after transformations the following formula is obtained:

$$\psi_N = 2C_0^{\frac{1}{m_0}} C^{-\frac{1}{m}} N^{(\frac{1}{m}-\frac{1}{m_0})} - 1 \tag{6}$$

Calculated value:

$$\psi_N = tg\Theta_N \tag{7}$$

In order to calculate the stress sensitivity coefficient of a material according to Formula (6), the fatigue diagrams designated by Formulas (4) and (5) should be known.

Usually, as a part of the assessment of cyclic mechanical properties in the range of high-cycle fatigue, the diagram for r = -1 is determined, and in special cases (e.g. thin-walled metal plates subjected in service conditions to axial loads – floating tension), diagrams for R = 0 are determined.

The cases of simultaneous determination of the formula for R = -1 and R = 0 are rare, because fatigue research is time-consuming and expensive. An additional problem may pose the necessity of the determination of many diagrams on a large number of specimens due to considerable dispersion of fatigue research results, significantly influencing the accuracy of data C_o and C determination; therefore, the accuracy of the determination of stress sensitivity coefficient of a material is ψ_N .

2. Results of fatigue calculations

A means determining the stress sensitivity coefficient of a material is illustrated with fatigue research results of S355JO steel.

Static properties of this steel are the following: R_e = 500 MPa, R_m = 678 MPa, E = 208159 MPa, A_5 = 17,2% and Z = 60%.

The S-N curve for R = 1 with research results marked on it (Fig. 3) is determined by the following formula:

$$\left(R_{-1}^{N}\right)^{12,33} \cdot N = 1{,}156 \cdot 10^{36} \tag{8}$$

Whereas, the fatigue diagram for R = 0 with research results marked on it is shown in Fig.4 and determined by the following formula:

$$\left(R_{0}^{N}\right)^{15,92} \cdot N = 6{,}163 \cdot 10^{48} \tag{9}$$

As a result of the formulas above: m_0 = 12,33, m = 15,92, $C_0 = 1{,}156 \cdot 10^{36}$ and $C = 6{,}163 \cdot 10^{48}$.

For the data above, the values of stress sensitivity coefficient of S355JO steel for selected fatigue life value N, according to (6), have been calculated. The data has been correlated in Table 1.

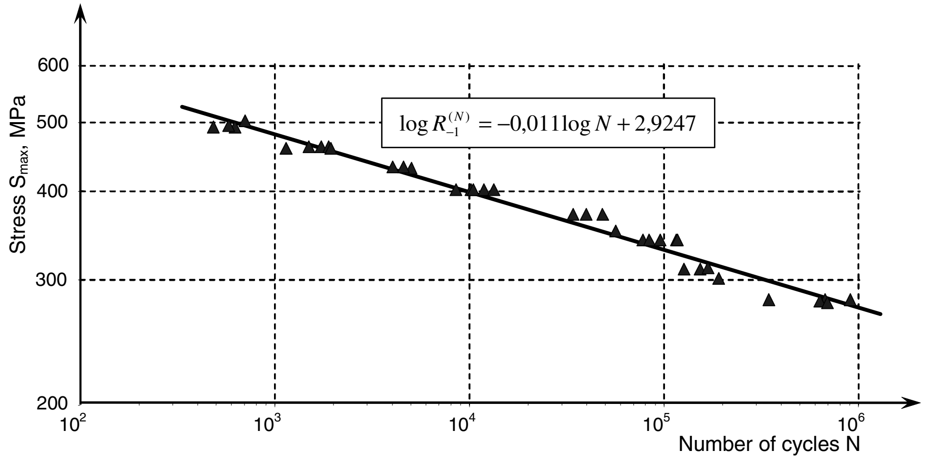

Fig. 3. The S-N curve for S355JO steel determined under alternating stress conditions (R = -1)

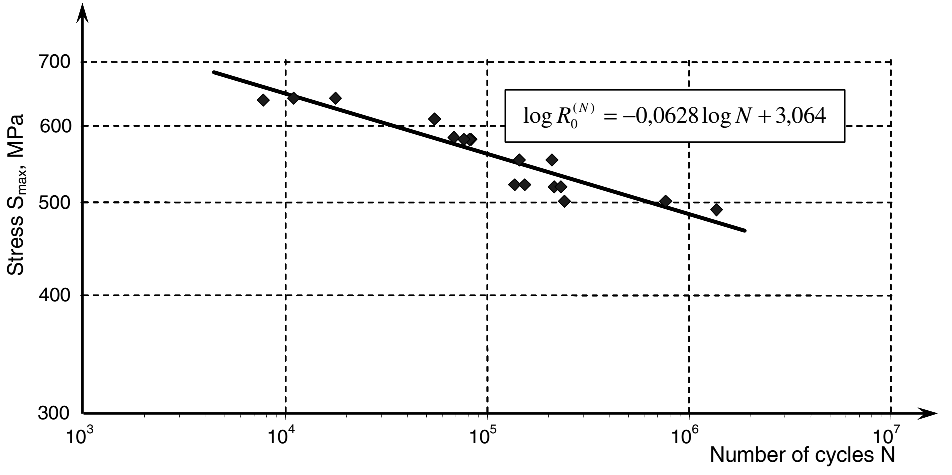

Fig. 4. The S-N curve for S355JO steel determined under fluctuating stress conditions (R = 0)

Table 1. Results of stress sensitivity coefficient of a material ψ_N for S355JO steel calculated according to Formula (6)

1_0	N	10^2	10^3	10^4	10^5	10^6	10^7
2_0	$\psi_{N\ ex}$	0.333	0.28	0.23	0.175	0.126	0.08

3. Analysis of the results of the research

Results from Table 1 marked on the diagram in logarithm graph are shown in Fig. 5. By analysing the diagram printed in Fig. 2, it can be assumed that, with the decrease of the number of cycles N→1 angle value → 45°, the stress sensitivity coefficient of a material ψ_N → 1,0.

Taking into account above the assumption and analysing the result distribution $\psi_{N\ ex}$ in Fig. 5 the empirical form of the formula for determining $\psi_{N\ cal}$ can be accepted.

$$\psi_{N\ cal} = N^{-k} \qquad (10)$$

The special case, from which the value of index exponent k is calculated, is the following:

$$\psi = N_0^{-k} \qquad (10a)$$

Value ψ, for fatigue limit ($N_0 = 10^7$) can be found in references, e.g. in works [1] and [4]. This value depends on the kind of material and its strength ($R_{m)}$. In Fig. 6, the appropriate diagrams are developed on the basis of work [3] and private research results have been given.

For S355JO steel, assuming the statements given above index exponent value k = 0.1583.

In Table 2, Line 2, the results of calculations of the stress sensitivity coefficient of S355JO steel in cycle function N (Line 1, Table 2) are depicted.

Empirical formula (10) facilitates the fatigue calculation for different loads than alternating stress (R = -1), changing loads, and particularly for fluctuating stress (R = 0). It does not require the determination of ψ_N coefficient value, and the knowledge of full forms of Formulas (8) and (9), which, as it was assumed, requires considerable amount of financial outlay and time.

Knowing the S-N curve for R = -1, the determination of the S-N curve for R = -0 can be easily done with the use of a well-known Gerber's formula [1, 4].

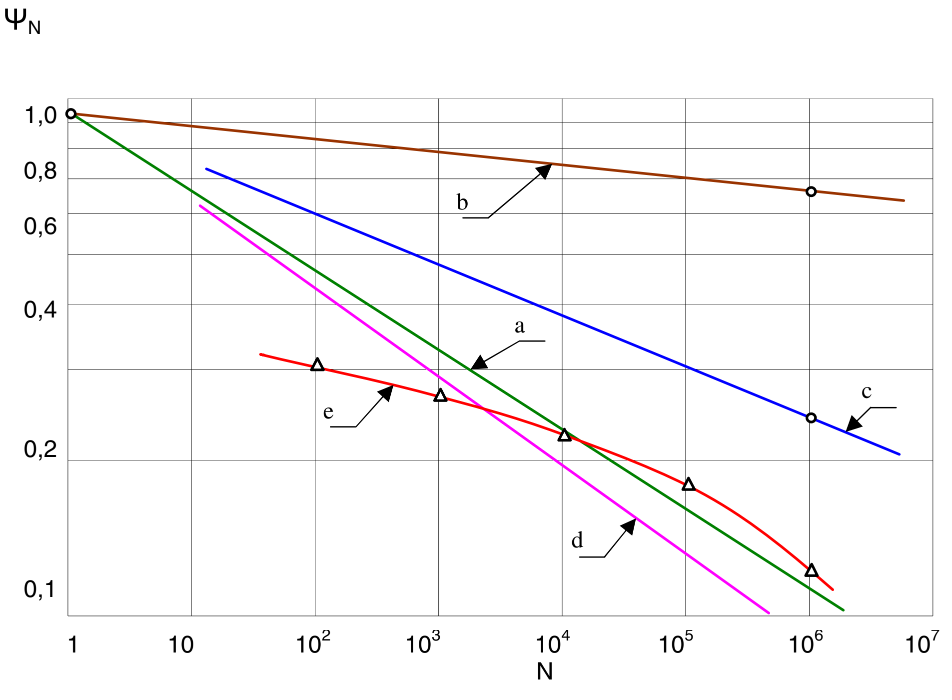

Fig. 5. Diagrams showing stress sensitivity coefficient of a material a) for S355JO steel (private research), b) for NiCoMn cast steel, c) for 42CrMo4 steel, d) for st37 steel, e) empirical data for S355JO steel. (Diagrams b, c and d prepared on the base of data from work [3] after [1])

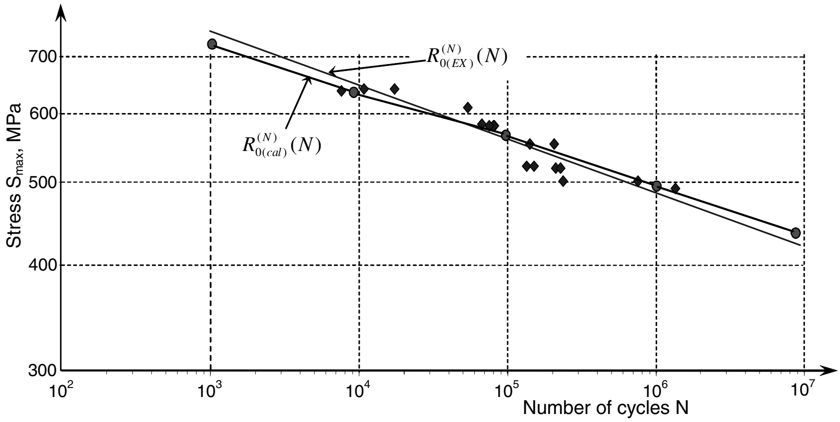

Fig. 6. The S-N curve for S355JO steel and R = 0: determined empirically $R^{(N)}_{(EX)}(N)$ and calculated according to the Formula (12) – $R^{(N)}_{(cal)}(N)$

$$S_{aekw} = S_a + \psi_N \; S_m \tag{11}$$

Where:

S_{aekw} – equivalent amplitude (for R = -1),
S_a – asymmetric stress cycle,
S_m – average value of cycle stress (for R ≠ -1).

By using the relation above (11) for scientific symbols accepted in this study and after transformation we receive the following:

$$R_0^N = \frac{2R_{-1}^N}{1+\psi_N} = \frac{2R_{-1}^N}{1+N^{-k}} \tag{12}$$

In Table 2, Line 3, $R_{-1}^{(N)}$ values enabling the calculation of the $R_0^{(N)}$ value according to Formula (12) have been given. The results of these calculations have been correlated in Line 5 of Table 2.

For comparison, in Line 4 of Table 2 the empirical data is given.

The calculation and empirical results are listed in the form of diagrams shown in Fig. 6. From the comparison between the results of calculations and examination shown in the table and in the Fig. 6, it turns out that the differences between the calculated and empirical values are relatively small, taking the considerable dispersion of the results of the fatigue research into account. The relative values of these differences calculated according to the formula (13) in the extreme case equals σ = 8.9%.

$$\delta = \frac{R^N_{0,ex} - R^N_{0,obl}}{R^N_{o,ex}} \cdot 100\% \tag{13}$$

This case for S355JO steel lies in the range of low-cycle fatigue and requires a different calculation method.

Table 2. The results of calculation and fatigue research of S355J0 steel specimens for stress ratio R = 0

1.	N		10^2	10^3	10^4	10^5	10^6	10^7	Calculations according to the formula
2.	ψ *N cal.*		0.465	0.35	0.23	0.162	0.115	0.08	(9)
3.	$R^{(N)}_{-1}$	E_x	578.7	480.2	398.4	330.5	274.2	227.5	(12)
4.	$\frac{R^{(N)}_0}{2}$	E_x	433.5	375.5	324.9	281.2	234.3	210.6	(8)
5.		cal.	394.7	355.4	323.6	284.2	245.7	210.5	(10)
6.	δ%		8.9	5.3	0.4	-1.0	-1.0	0.0	(13)

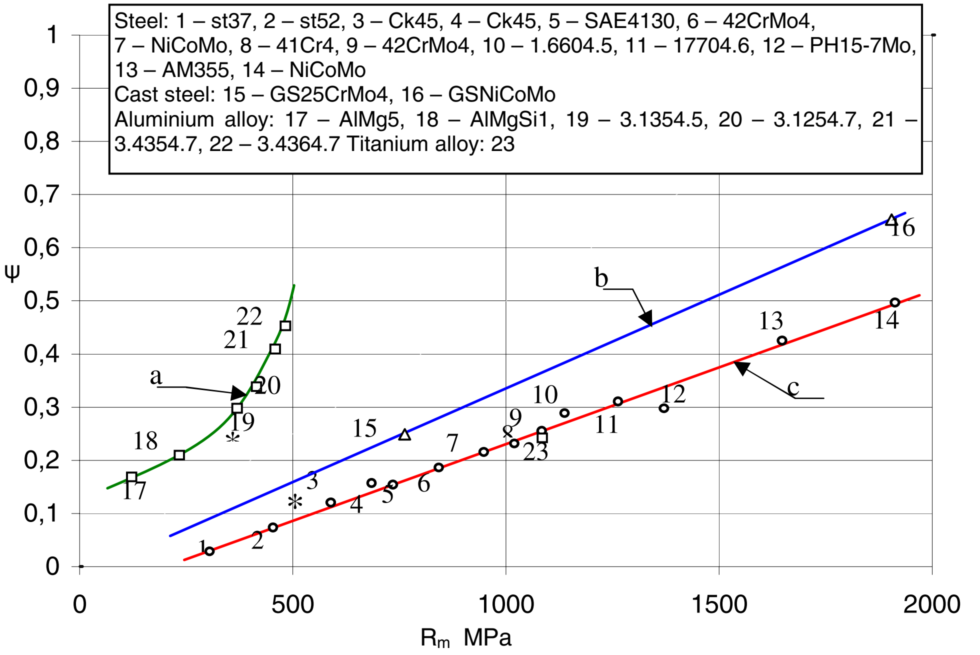

Fig. 7. Diagrams (prepared on the basis of work [3] and my own research results (*)) depicting the dependence of stress sensitivity coefficient of a material ψ in range of fatigue limit on tensile strength R_m for: a) aluminium alloys, b) cast steel, c) steel

4. Conclusions

1. Experimental verification of the empirical formula (10) for calculating stress sensitivity coefficient of a material value ψ_N has shown good conformity of calculation results to empirical results (Table 1, Line 2; Table 2, Line 2; Diagram – Fig. 5).
2. Knowledge of the ψ_N coefficient value and the S-N curve for R = -1 enables easy determination of the fatigue diagrams for any stress ratio value R from the range of its variability. In this case, it turns out that it is possible to determine two-parametric fatigue characteristics N ($S_{a,}$ S_m). Good conformity of calculation results of limited fatigue strength value $R_0^{(N)}$ to empirical results arises from data in Table 2, Line 6.
3. The significance of the knowledge of stress sensitivity coefficient of a material value results from the methods of calculations of fatigue life of structural components which have been subjected in service conditions to random load, especially broad-bans. The load spectrum applied in calculations contains sinusoidal cycle of various, within the broad margin, stress ratios R.

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Reviewer:
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Współczynnik wrażliwości naprężenia materiału w zakresie zmęczenia wysokocyklicznego

Słowa kluczowe

Odporność na zmęczenie, charakterystyki dwuparametryczne, stal typu S355J0.

Streszczenie

W pomiarach odporności na zmęczenie elemtnów konstrukcyjnych, które w warunkach eksploatacyjnych poddane zostały stochastycznym obciążeniom, występują widma pod postacią cykli sinusoidalnych o różnej asymetrii. Asymetria cyklu różni się współczynnikiem naprężenia definiowanym jako $R = S_{min}/S_{max}$. W przypadku podanego obciążenia, współczynnik R jest zmienny w zakresie $-\infty$ do $+\infty$. W artykule przedstawiono analizę wrażliwości naprężenia materiału na asymetrię cyklu wybranych obciążeń. Wrażliwość ta jest opisana odpowiednim współczynnikiem. W odniesieniach podano analityczne formy obliczania siły zmęczenia. W artykule powołano się na formułę HCF oraz empiryczną formułę współczynnika wrażliwości naprężenia dla materiałów poddanych wybranym obciążeniom.