# SINGULAR SPECTRUM ANALYSIS AS A SMOOTHING METHOD OF LOAD VARIABILITY

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Summary

Application of SVD to fault extraction from the machine symptom observation matrix (**SOM**) seems to be validated enough, especially by data taken from many real diagnostic cases. However, frequently we have situation of varying machine load during the production process, where by observed primary symptoms are influenced greatly. This concerns generalized symptoms too, so decision making process and forecasting is disturbed. But we can apply some new data smoothing procedure called singular spectrum analysis (SSA), to eliminate load influenced symptom fluctuation, and obtain the machine wear trend only. This seems to be true, as it was shown in the paper, but special care should be taken to choose smoothing approximation order properly.

Keywords: condition monitoring, singular value decomposition, singular spectrum analysis.

## ANALIZA WIDMA SZCZEGÓLNEGO JAKO METODA WYGŁADZANIA ZMIENNOŚCI OBCIĄŻENIA

Streszczenie

Zastosowanie rozkładu SVD do wydobycia informacji o uszkodzeniu z symptomowej macierzy obserwacji (ang. **SOM**) wydaje się być wystarczająco uzasadnione, szczególnie dla danych pochodzących z wielu rzeczywistych przypadków diagnostycznych. Jednakże w wielu przypadkach mamy do czynienia z sytuacją zmiennych obciążeń maszyny podczas procesu produkcji, silnie wpływających na obserwowane symptomy. Dotyczy to także symptomów uogólnionych, co utrudnia proces podejmowania decyzji i prognozowania. Możemy jednak zastosować pewną nową procedurę wygładzania nazywaną analizą widma szczególnego (ang. SSA), aby wyeliminować obciążenia wpływające na fluktuacje symptomu i otrzymać tylko trend zużycia maszyny. Wydaje się to być prawdą, jak zostało pokazane w pracy, jednak z zachowaniem szczególnej uwagi w poprawnym wyborze rzędu przybliżenia w procedurze wygładzania.

Słowa kluczowe: diagnostyka, rozkład na wartości szczególne, analiza widma szczególnego.

## 1. INTRODUCTION

The idea of symptom observation matrix (**SOM**) in multidimensional condition monitoring of machines is well established and brings several advantages [1]. Usually it is basing on $p > r$ rectangular matrix, with ($r$) symptoms $\boldsymbol{S}_r$ in columns, measured along the system life $\theta$, what gives $p$ symptom readings (matrix rows). It allows placing all physically different symptoms[1] measured in a phenomenal field of the machine in a one **SOM**, and to process them in order to obtain projection of observation space to the fault space of machine. Of course, at the beginning we usually observe more symptoms (columns of **SOM**), than there is expected number of faults in a machine.

The preprocessing of **SOM** may be different (see for example [2]), but for condition monitoring it was found that column normalization and extraction by symptom initial value is the best solution, bringing all symptoms to their dimensionless form. Then, the application of SVD to the dimensionless form of **SOM** gives needed projection of observation space to the fault space. The resultant matrices of SVD decomposition allow calculating of two important matrices. The first is **SD** matrix, which give us generalized fault symptoms $\boldsymbol{SD}_i$ of machine, and in theory they are independent each other. From this matrix we can calculate so called total damage symptom, as the sum of all $\boldsymbol{SD}_i$ generalized fault symptoms. This is mainly in order to calculate the symptom limit value $S_l$, or to make the forecast of the total damage symptom. The second **AL** matrix allows us to assess the contribution of primary measured symptoms $\boldsymbol{S}_r$ to a newly formed generalized fault symptoms $\boldsymbol{SD}_i$. In this way we can just say which of primary symptom is redundant, giving no substantial information contribution, and as such can be rejected from further calculations

[1] Symptom, measurable quantity covariable (*or assumed to be*) with the system condition

and/or future measurements [3]. But should we only reduce the dimensionality of **SOM** by rejecting redundant symptoms? Maybe the addition to **SOM** some information which is inherent in observed machine will give much better results, or we should do both operations on every **SOM**? Some years ago, the present author has added intuitively life time symptom of the machine, as the first symptom before **SOM** processing, increasing this way the rank of the matrix and amount of its information asset. As it was shown in the last paper of first author [4] this increase of information asset allow us to detect earlier the evolving second fault in a machine, and in some cases it can be done on an automatic way.

However basing on the condition monitoring data of real objects we are encountering frequently the variability of machine load, which influences on the readings of almost all symptoms, in some cases. This concerns the generalized symptoms $\boldsymbol{SD}_i$ as well, what disturbs our fault detection ability and assessment of its severity too. But it seems, even in this case of load variability, there is possibility to smooth out the chosen generalized symptom in order to base diagnostic decision on the data showing stable machine wear trend. Such an opportunity brings the application of special method called Singular Spectrum Analysis (SSA), used lately with a success in physics [5] and economic forecasting [6]. We will try to adopt this method to our diagnostic needs in this paper, in a similar way as it was already shown in our last paper [7].

Concerning **SOM** decomposition, in reality there is no big choice of decomposition method; principal components analysis (PCA), which uses SVD as it can be shown [8], and both are well diagnostically interpretable [9], [10], [11]. The well known QR decomposition seems to be not usable in multidimensional diagnostics[2], according to unpublished study of the first author. Here only the main diagonal of the upper triangular matrix **R** of this decomposition can be compared to the first generalized symptom $\boldsymbol{SD}_1$, the higher upper diagonals are shortened and do not carry readable diagnostic information.

## 2. OPTIMIZATION OF MULTI SYMPTOM MACHINE OBSERVATION

It was described earlier, our information about machine condition evolution is contained in $p$ x $r$ **SOM**, where in $r$ columns and $p$ rows of the successive readings of each symptom are presented. Usually they are made at equidistant system life time moments $\theta_n$, $n$=1,2,...$p$. In pre-processing operations the columns of **SOM** are centred and normalized to the three point average of initial readings of every symptom. This is in order to make the **SOM** dimensionless, and to diminish starting disturbances of symptoms. This allows also to present the evolution range of every symptom from zero up to few times of the initial symptom value $S_{0r}$, (measured in the vicinity of $\theta = 0$).

After such preprocessing we obtain the dimensionless **SOM** in the form [1]:

$$\mathbf{SOM} = \mathbf{O}_{pr} = \left[S'_{nm}\right], \quad S'_{nm} = \frac{S_{nm}}{S_{0m}} - 1. \tag{1}$$

This is the way of **SOM** preprocessing when we do not include life symptom (***LS***). As it is seen from above this new symptom should be normalized and also adjusted to given form and values of observed symptoms. This additional symptom can not have to small values or to large values, because in this way it will, or will not, influence our calculation and final result. If machine observation starts from its good condition, than usually symptoms starts also from small values, and at the end of life we have maximal symptom values. Hence one way of scaling life symptom ***LS*** may include multiplying by the average of last readings of all symptoms. Let the counting of symptom readings in **SOM** will be $i$ = 1,2 … $n$, and for $r$ symptoms one can write:

$$\boldsymbol{LS} = \frac{i}{rn}\sum_{m=1}^{r} S_{n,m} \tag{1a}$$

where $i$ =1,2, … $n$, $S_{n,m}$ means the last readings of symptom number $m$.

Now, adding ***LS*** symptom as a first column to the old **SOM** we have a new $\mathbf{SOM}_L$, which includes explicit machine life information to our diagnostic calculations and decision. Having this we can apply the Singular Value Decomposition (SVD) [8][13][17] to our dimensionless **SOM** (1) and (1a), to obtain singular components (vectors) and singular values (numbers) of **SOM**, in the form:

$$\mathbf{O}_{pr} = \mathbf{U}_{pp} \cdot \mathbf{\Sigma}_{pr} \cdot \mathbf{V}_{rr}{}^{T}, \tag{2}$$

where: $T$ is matrix transposition, $\mathbf{U}_{pp}$ is $p$ dimensional orthonormal matrix of left hand side singular vectors $\mathbf{V}_{rr}$ is $r$ dimensional orthonormal matrix of right hand side singular vectors, and the diagonal matrix of singular values $\mathbf{\Sigma}_{pr}$ is defined as below:

$$\begin{aligned} &\mathbf{\Sigma}_{pr} = diag(\sigma_1, \sigma_2, ... \sigma_l), \\ &\sigma_1 > \sigma_2 > ... > \sigma_u > 0, \\ &\quad and \\ &\sigma_{u+1} = \sigma_l = 0, \\ &l = \max(p, r), \ \ u \le \min\ (p, r), \ \ u < r < p. \end{aligned} \tag{3}$$

Going back to SVD itself it is worthwhile to say, that every non square matrix has such decomposition, and it may be interpreted also as the product of three matrices [13], namely:

$$\boldsymbol{O}_{pr} = (\boldsymbol{Hanger}) \times (\boldsymbol{Stretcher}) \times (\boldsymbol{Aligner}^T). \tag{4}$$

This is a very metaphorical description of SVD transformation, but it seems to be an useful analogy

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[2] Hence, the QR method maybe the base for quick looking of main fault only.

for the inference and decision making in condition monitoring. The diagnostic interpretation of formulae (4) can be obtained very easily. Namely, using its left hand side part, we are stretching our **SOM** over the life (observations) dimension, obtaining the matrix of generalized symptoms **SD** as the columns of the matrix. And using the right hand side part of (4) we are stretching **SOM** over the observed (primary) symptoms dimension in the form of matrix **AL**, assessing in this way the contribution of each primary measured symptom to the generalized fault symptom $\boldsymbol{SD}_i$ , $i$=1,2,...$u$.

$$\mathbf{SD} = \mathbf{O}_{pr} \cdot \mathbf{V}_{rr} = \mathbf{U}_{pp} \cdot \mathbf{\Sigma}_{rr} ; \quad \text{and} \quad \mathbf{AL} = \mathbf{U}^T_{pp} \cdot \mathbf{O}_{pr} = \mathbf{\Sigma}_{rr} \cdot \mathbf{V}^T_{rr} \tag{5}$$

We will calculate the above matrices and use them for better interpretation of monitoring results (**SD**) and optimization of the dimension of the observation space (**AL**).

As the rows of **SOM** matrix were formed along the machine lifetime, so the columns of **SD** matrix have the discrete argument of life time $\theta$, and we can write their fault space interpretation as below:

$$\boldsymbol{SD}_t(\theta) \propto \boldsymbol{F}_t(\theta), \tag{6}$$

$$Norm\ (\boldsymbol{SD}_t) \equiv ||\boldsymbol{SD}_t|| = \sigma_t,\ t = 1,2 \ldots,u$$

For the assessment of total machine damage we can calculate the sum of all generalized symptoms:

$$\boldsymbol{SumSD}_i(\theta) = \sum_{i=1}^{z} \boldsymbol{SD}_i(\theta) = = \sum_{i=1}^{z} \sigma_i(\theta) \cdot \boldsymbol{u}_i(\theta) \propto \boldsymbol{F}(\theta) , \tag{7}$$

where; $\boldsymbol{u}_i$ is a column of $\mathbf{U}_{pp}$.

This concept of diagnostic inference, for individual fault $\boldsymbol{F}_t(\theta)$ (eq. 6), and total fault damage $\boldsymbol{F}(\theta)$ (eq. 7) has been proven in several papers [1][14] and we will use it here in further consideration.

The above results, based on generalized fault symptoms, have been obtained only from the first matrix **SD** of (5). And the second matrix **AL** gives us the relative measure of information contribution to each generalized symptom, as given by particular primary symptom measured during the **SOM** gathering. This is one way of assessment of the primary symptom redundancy, but we need some other global indicators of eventual rejection of the redundant symptom. We can use modified Frobenius norm of **SOM** and the generalized volume of the fault space created by **SOM** [3]. What is important here, these two measures are based on singular values of **SOM**, which in turn can be treated as the faults advancement measure (see (6-7)). Hence we can write the **SOM** measures [3]:

$$Frob1 = \Sigma \sigma_i ; \quad \text{and} \quad Vol1 = \Pi \sigma_i , \tag{8}$$

where: $i$ = 1,2,...$u$.

Looking for the way of value creation method of the above, one can say that if some primary symptom will be really redundant (small $\sigma_i$) its rejection should change *Frob1* measure only a little, and in contrary it should increase much the fault space volume *Vol1*. We will notice how it behaves with real examples of symptom rejection for **SOM** of diagnosed machines.

## 3. MULTIDIMENSIONAL CONDITION DETECTION OF MACHINE WITH VARYING LOAD

As a first example of application of our idea we will take a hard diagnostic case - a huge fan for coal milling working at one of Polish thermo power station. Here the root mean square vibration velocity ($V_{rms}$) has been used as a symptom of condition, and initially altogether 11 symptoms at different places of fan mill aggregate structure were constantly monitored, over 60 weeks of a lifetime $\theta$. We will process this case twice, first with inclusion of life time symptom to $\mathbf{SOM}_L$ (1a) and secondly **SOM** without life time symptom.

How unstable and noisy the fan running environment is, one can notice from the left top picture of the fig.1. It is seen further (middle left picture), that the symptom normalization and addition of life time symptom ***LS*** (straight line) do not change much the noisy behaviour of primary and generalized symptoms after **SVD** (bottom left picture).

Looking at the middle right picture of fig.1, where matrix **AL** is presented, one can notice that symptoms No 8,9,10, do not give substantial contribution to the two dominating generalized symptoms, and probably can be rejected as redundant at the second approach. With this respect please note the value of Frobenius modified measure *Frob1=33.82* and the volume of the fault space *Vol1=0.10*, at upper right picture. One can also note here, that there are two generalized symptoms with high information contents (picture top right), and due to that two symptom limit values are assessed: namely $S_{lc}$ for the total damage symptom, and $S_{l1}$ for the first generalized symptom (bottom pictures).

Using the same software we can make forecast the future symptom value of generalized symptom ***SD1*** by means of new method called grey system theory created by Deng [15], and adopted to condition monitoring by us in a previous paper [12]. As it seen from fig. 2 due to varying load of the fan the error of forecast is high, of 46% order, and the application of rolling window (picture bottom left) does not improve the forecast greatly, as it usually does. Hence we need some smoothing method which can reject the varying load undulations, and preserve the trend of machine wear only.

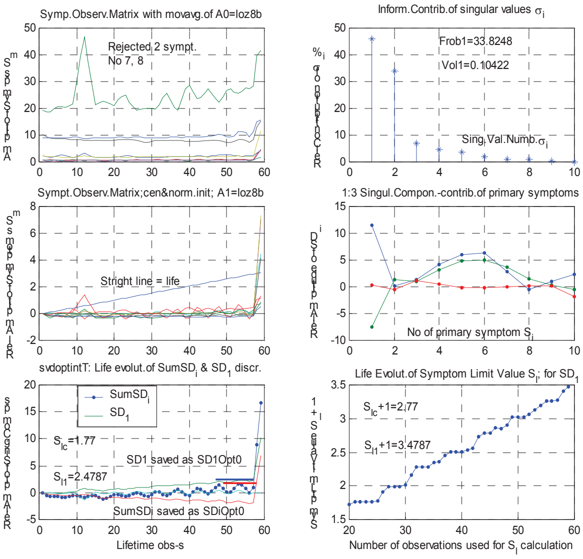


Fig. 1. Vibration condition monitoring ($V_{rms}$) of the coal mill fan observed at bearings of fan and electric motor

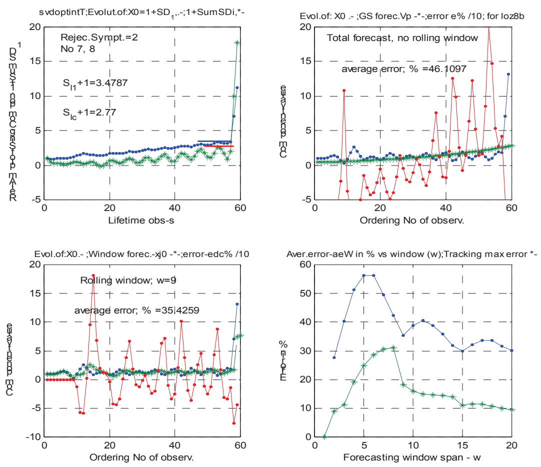


Fig. 2. Grey system theory forecast with and without rolling window for the fan mill data with varying load as on fig. 1

There are some cases of condition monitoring that are much worse, as it is shown on the fig.3 for the case of rolling bearing testing with pulsating load (loz12r). There was here 19 symptoms measured altogether, and after rejection of 3 most redundant at the first approach, the situation was not much improved, as it can be seen from the fig. 3. When approaching to obtain the forecast of next symptom value by means of GST method of rolling window (see fig. 4), the obtained result is similar to obtained previously, when forecasting the future symptom value 46% error without rolling window, and 35% with rolling window applied. Again one can see we need some method to filter varying load effects in a symptom reading. But it seems, even in this case of load variability, there is possibility to smooth out the chosen generalized symptom in order to base diagnostic decision on the data showing stable machine wear trend. Such an opportunity brings the application of SSA.

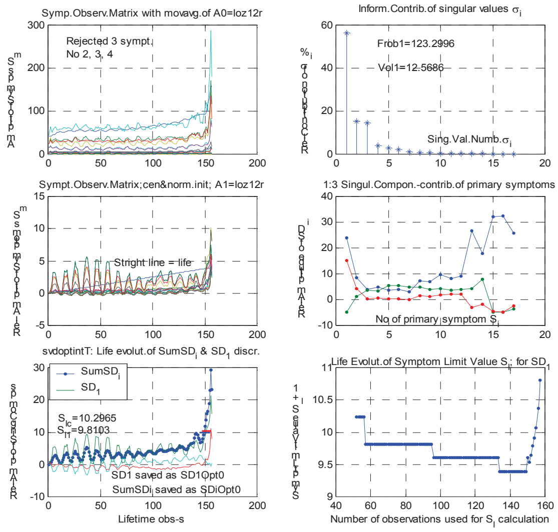


Fig. 3. Condition monitoring of a rolling bearing at the testing rig with the varying load

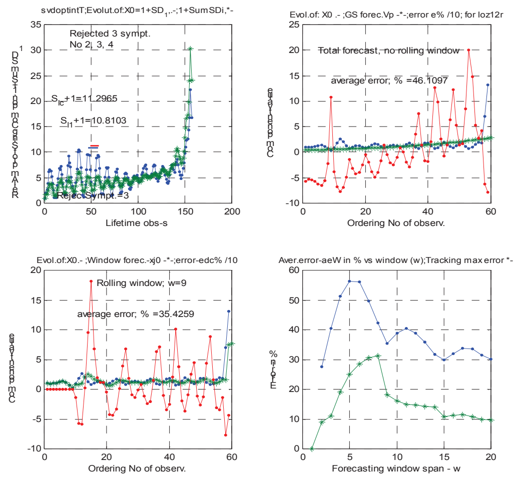


Fig. 4. GST forecast of a symptom value (first generalized symptom), with and without rolling window, for the ball bearing test rig (see Fig. 3)

## 4. TRAJECTORY MATRIX OF GENERALIZED SYMPTOM AND ITS DECOMPOSITION

The SSA method itself consists of four steps [16], [6]. In the first step, called embedding, the one dimensional time series denoted here by **S** is recast into $L$ dimensional time series composed in trajectory matrix **X**, (**TM** see (9)). As a second step the **TM** is decomposed by SVD into a sum of orthogonal matrices of rank one. This gives us a new set of time subseries, where each component can be identified; as a trend, quasi periodic component, or noise. By truncating the unwanted components in the third step we can preserve trendlike component into matrix **D** (see (10)) for further processing. The last step is reconstruction of de-noised time series by a special procedure called diagonal summation over matrix **D** (see (11)).

Let us denote the original total damage symptom (***TDS*** vector) by $\boldsymbol{x} = [x_1, x_2, \ldots x_N]$, and for a chosen embedding dimension $L$ we obtain the trajectory matrix, as below:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_{N-L} \\ x_2 & x_3 & x_4 & \ldots & x_{N-L+1} \\ x_3 & x_4 & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots. \\ x_L & \ldots & \ldots & \ldots & \ldots\ldots x_N \end{bmatrix} \qquad (9)$$

Such a matrix, is by definition, a Hankel matrix, as it is $X_{i,j} = x_{i+j-1}$, so all elements of a particular left hand diagonals are equal. This can be seen even from the definition, as above (9).

The subsequent SVD of trajectory matrix **X** allows as to decompose it into a sum of orthogonal matrices (subseries) which can be arbitrarily grouped as two sums; the trend matrix **D** and the noise as below:

$$\mathbf{X} = \sum_{i=1}^{\mathbf{q}} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T + \sum_{i=q+1}^{\mathbf{N}} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T = \mathbf{D} + \mathbf{Noise} \qquad (10)$$

where, as before, $\sigma_i$ is the singular value, and vectors $\boldsymbol{u}_i$, $\boldsymbol{v}_i$ are elements of secondary SVD of **X** matrix, similarly as it was in relation (2-3). The level of approximation $q$ of trajectory matrix (*de-noising*) should be chosen arbitrary, usually $q$=1 or 2 is enough, depending on the structure of the data.

The fourth step of SSA is a diagonal averaging over the first matrix **D** of (10), it means summation of all elements in given diagonal and division over the number of these elements. And in this way we obtain a low order approximation of input vector denoted here as $\hat{\boldsymbol{S}}$ with the same number of elements N as input series $\boldsymbol{x}$. This can be done as below:

$$\begin{cases} \hat{S}_r = \dfrac{1}{r} \displaystyle\sum_{i=1}^{r} D_{i,r-i+1} & \text{for } r \le k \\ \hat{S}_r = \dfrac{1}{k} \displaystyle\sum_{i=1}^{k} D_{i,r-i+1} & \text{for } k < r \le L \\ \hat{S}_r = \dfrac{1}{k-r+L} \displaystyle\sum_{i=1}^{k-r+L} D_{r-L-i,L-i+1} & \text{for } r > L \end{cases} \qquad (11)$$

where $k$ – number of rows of matrix **D**, and $r$ = 1,2, … $L$+$k$-1

Starting from this de-noised symptom and using the already elaborated Matlab program **gsago.m** for GST forecasting we can make de-noised series $\hat{\boldsymbol{S}}$ with much better forecast than before, but the amplitude scaling may be a little different, due to the rejection of some number of noisy components of input vector (approximation order $q$).

Let us return now to the total damage symptom of ball bearing on a testing rig (Loz12r) shown on Fig.3 (see bottom left picture) and saved as *SDiOpt0* after SVD decomposition. Next figure 5 presents us the results of SSA processing outlined above (9-11) and calculated by special program **singspectrel.m**, applied already in a modified version in a published paper [7].

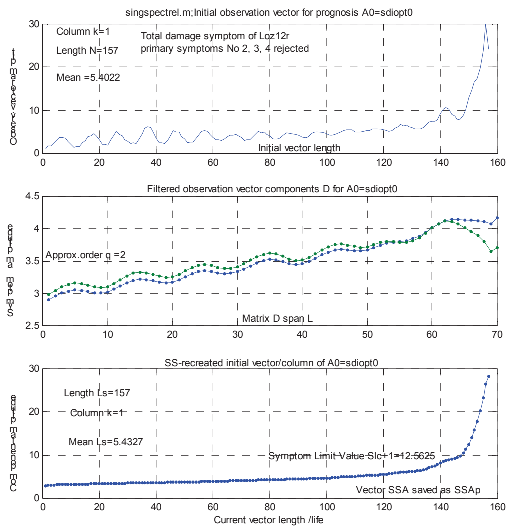


Fig. 5. Denoised total damage symptom of a rolling bearing testing rig with pulsating load

The order of approximation taken in de-noising operation was taken as $q$=2, and this seems to be good when we take a look for a lower picture of Fig.5. Here no significant symptom oscillation can be observed, and also no significant reduction of symptom amplitude after reconstruction is observed, when we compare upper and lower pictures.

Going to obtain the forecast of total damage symptom we will apply again the GST procedure and calculate also the symptom limit value $S_l$. The results of such calculations are shown on Fig.6, which seems to be self explaining. As one can notice there, the forecast of total damage symptom is amazingly good having the average error of order of 0.6%, which seems to be very small in comparison to previous result of 35% error, as it is seen on figure 4. Hence, the application of Singular Spectrum Analysis, as the smoothing method seems to be good validated in this case.

But the same operation made for generalized symptom ***SD1*** gives much worse smoothing and prognostic result with $q$=2, one should apply here the lower approximation order $q$=1, what in turn makes the amplitude of symptom more than twice smaller as the input amplitude of ***SD1***. This seems to be not good for the condition inference and needs special rescaling of symptom after its recreation by SSA procedure.

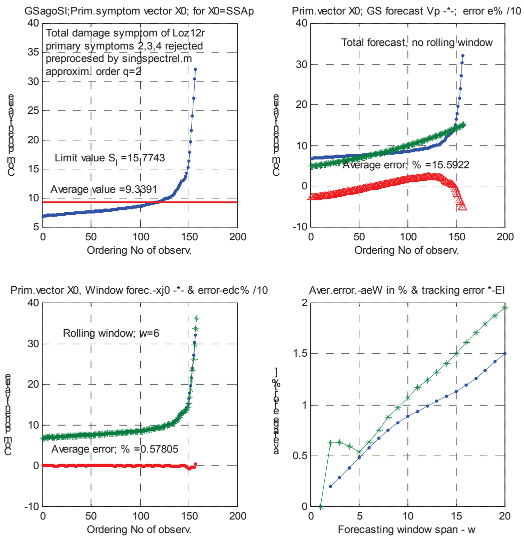


Fig. 6. GST forecast of de-noised total damage symptom for loz12r

## 5. CONCLUSIONS

Varying load of machines with condition monitoring is a frequent case, and that was the reason in one of our paper [14] we have proposed rescaling of observed symptoms. This time we propose not to rescale symptoms, but to filter or smooth, the SVD decomposition results by a special method called SSA, used with a success in physics and econometrics. Special diagnostic oriented software have been prepared for that purpose, and it has been shown that using it and choosing the approximation order *q* carefully we can obtain wear trend like behaviour of generalized symptoms of monitored machine.

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