POZNAN UNIVERSITY OF TECHNOLOGY ACADEMIC JOURNALS
No 85 Electrical Engineering 2016

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ELECTRONIC REALIZATIONS OF FRACTIONAL-ORDER ELEMENTS: I. SYNTHESIS OF THE ARBITRARY ORDER ELEMENTS

The paper presents a synthesis method of new fractional-order elements, using classic fractional-order components, such as supercapacitors and real coils with ferromagnetic cores. These new fractional-order elements have been realized using a generalized impedance converter GIC. Two cases have been analyzed in the paper: first, with only one fractional-order element being a load impedance of the generalized impedance converter, and second, with two fractional-order elements - one being a part of the structure of the GIC and the second being the load impedance. The derived relations are illustrated by simulation examples for the circuit. The impact of different values of the parameter α on the solution of the considered problem has been analyzed too.

KEYWORDS: fractional-order inductance L_β and capacitance C_α, generalized impedance converter (GIC)

1. INTRODUCTION

Many scientific researches on the systems containing fractional-order reactance elements have been carries out in recent years. These elements can be models of different physical systems or components, including supercapacitors [1], batteries [2], fuel cells [3] and many others. They have also been used in various electronic systems [4, 5]. The simplest models of the capacitor C_α and the inductor L_β of fractional-orders *α, β* are described in frequency domain by formulae:

$$Z_C(s)=\frac{U(s)}{I(s)}=\frac{1}{s^\alpha C}, \tag{1}$$

$$Z_L(s)=\frac{U(s)}{I(s)}=s^\beta L, \qquad \alpha,\beta\in \mathrm{R}. \tag{2}$$

The realization of fractional-order elements is conducted with three methods. First of them uses the specific physicochemical properties of the materials:

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– electrolytes and dielectrics in case of supercapacitors [6, 7],
– soft ferromagnetic materials, in case of coils [8].

It should be noted, that the fractional exponents α, β (see formulae (1), (2)) are not arbitrary and their values result from the physical features of the materials used for the construction of the fractional-order elements. these exponents generally have the values placed in the range of $\langle 0,1 \rangle$.

The second method of the fractional-order element realization involves their frequency or time-domain models approximation to the *RC*-ladder systems [9].

The third method is based on the application of electronic systems, implementing transformations of the elements C_α, L_β impedances, which are obtained using first of the mentioned methods.

The generalized impedance converter (GIC) seems the most useful tool for that. Such converters can be constructed using transistors [10], classic operational amplifiers [11], transconductance amplifiers [12], mirrored operational amplifiers [13], conveyors [14] as well as digital systems [15].

The paper concerns a synthesis of fractional-order reactance elements C_α, L_β, which exponents can take much wider range of values, than in naturally existing elements for which $\alpha, \beta \in \langle 0,1 \rangle$.

The results presented below are a continuation and development of the results shown in [16]. The presented idea is based on the impedance of the natural elements (1), (2) transformation using the generalized impedance converter.

2. FORMALIZATION OF THE PROBLEM

The generalized impedance converter (GIC) constructed as shown in Fig. 1, has been used for the realization of the fractional-order impedance given by (1) and (2).

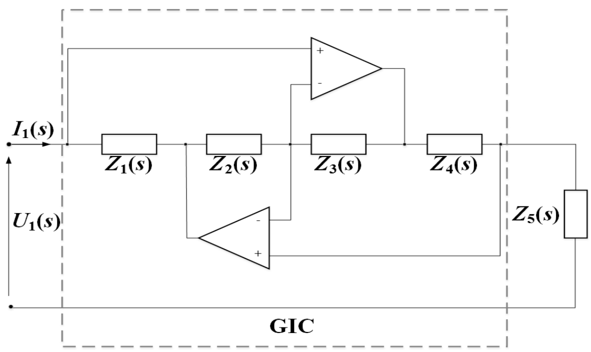

Fig. 1. Realization of the fractional-order impedance using the generalized impedance converter (GIC)

Impedances $Z_1(s)$, $Z_2(s)$, $Z_3(s)$, $Z_4(s)$ are the elements of the converter structure and $Z_5(s)$ is its load impedance. Assuming that the operational amplifiers are ideal (see Fig. 1), it can be proved that the input impedance is defined by formula:

$$Z_{in}(s)=\frac{U_1(s)}{I_1(s)}=\frac{Z_1(s)Z_3(s)}{Z_2(s)Z_4(s)}Z_5(s). \tag{3}$$

Further analysis takes into account, that the system from Fig. 1 consists of elements, such as:

– classic inductances and capacitances,
– resistances,
– at least one fractional-order element, for which the fractional exponent of impedance (α or β) belongs to the range $\langle 0,1\rangle$.

With adopted assumptions, formula (2) takes the form:

$$Z_{in}(s)=Ks^{\gamma}, \tag{4}$$

where: K – coefficient interpreted as the equivalent fractional-order inductance ($\gamma > 0$) or the equivalent fractional-order capacitance ($\gamma < 0$).

The coefficient K depends on the value of parameters describing impedances appearing in the formula (2). Various cases of the presented idea have been described next. The analysis has been limited to cases, where the fractional-order parameter γ of the impedance $Z_{in}(s)$ is different from the exponents of the fractional-order impedances included in the system structure from Fig. 1.

3. ANALYSIS OF SPECIFIC CASES

3.1. System with one original fractional-order element

The results of analysis, based on formula (3), have been presented in Table 1.

Table 1. Obtained fractional-order input impedance, for one fractional-order impedance in (3)

$Z_1(s)$	$Z_2(s)$	$Z_3(s)$	$Z_4(s)$	$Z_5(s)$	$Z_{in}(s)$ Ω	
					K	γ
Ω	Ω	Ω	Ω	Ω		
sL_1	R_2	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{L_1R_3L_{\beta}}{R_2R_4}$	β+1
R_1	sL_2	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{R_1R_3L_{\beta}}{L_2R_4}$	β-1

Table 1 cont. Obtained fractional-order input impedance, for one fractional-order impedance in (3)

sL_1	R_2	sL_3	R_4	$s^{\beta}L_{\beta}$	$\frac{L_1L_3L_\beta}{R_2R_4}$	β+2
R_1	sL_2	R_3	sL_4	$s^{\beta}L_{\beta}$	$\frac{R_1R_3L_\beta}{L_2L_4}$	β-2
$\frac{1}{sC_1}$	R_2	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{R_3L_\beta}{C_1R_2R_4}$	β-1
R_1	$\frac{1}{sC_2}$	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{R_1R_3C_2L_\beta}{R_4}$	β+1
$\frac{1}{sC_1}$	R_2	$\frac{1}{sC_3}$	R_4	$s^{\beta}L_{\beta}$	$\frac{L_\beta}{C_1C_3R_2R_4}$	β-2
R_1	$\frac{1}{sC_2}$	R_3	$\frac{1}{sC_4}$	$s^{\beta}L_{\beta}$	$C_1C_4R_1R_3L_\beta$	β+2
sL_1	$\frac{1}{sC_2}$	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{L_1R_3C_2L_\beta}{R_4}$	β+2
$\frac{1}{sC_1}$	sL_2	R_3	R_4	$s^{\beta}L_{\beta}$	$\frac{R_3L_\beta}{L_2C_1R_4}$	β-2
$\frac{1}{sC_1}$	sL_2	$\frac{1}{sC_3}$	R_4	$s^{\beta}L_{\beta}$	$\frac{L_\beta}{C_1C_3L_2R_4}$	β-3
sL_1	$\frac{1}{sC_2}$	sL_3	R_4	$s^{\beta}L_{\beta}$	$\frac{L_1L_3C_2L_\beta}{R_4}$	β+3
sL_1	$\frac{1}{sC_2}$	sL_3	$\frac{1}{sC_4}$	$s^{\beta}L_{\beta}$	$C_2C_4L_1L_3L_\beta$	β+4
$\frac{1}{sC_1}$	sL_2	$\frac{1}{sC_3}$	sL_4	$s^{\beta}L_{\beta}$	$\frac{L_\beta}{C_1C_3L_2L_4}$	β-4
sL_1	R_2	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{L_1R_3}{R_2R_4C_\alpha}$	-α+1
R_1	sL_2	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{R_1R_3}{L_2R_4C_\alpha}$	-α-1
sL_1	R_2	sL_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{L_1L_3}{R_2R_4C_\alpha}$	-α+2

Table 1 cont. Obtained fractional-order input impedance, for one fractional-order impedance in (3)

R_1	sL_2	R_3	sL_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{R_1R_3}{L_2L_4C_{\alpha}}$	$-\alpha-2$
$\frac{1}{sC_1}$	R_2	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{R_3}{C_1R_2R_4C_{\alpha}}$	$-\alpha-1$
R_1	$\frac{1}{sC_2}$	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{R_1R_3C_2}{R_4C_{\alpha}}$	$-\alpha+1$
$\frac{1}{sC_1}$	R_2	$\frac{1}{sC_3}$	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{1}{C_1C_3R_2R_4C_{\alpha}}$	$-\alpha-2$
R_1	$\frac{1}{sC_2}$	R_3	$\frac{1}{sC_4}$	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{C_1C_4R_1R_3}{C_{\alpha}}$	$-\alpha+2$
sL_1	$\frac{1}{sC_2}$	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{L_1R_3C_2}{R_4C_{\alpha}}$	$-\alpha+2$
$\frac{1}{sC_1}$	sL_2	R_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{R_3}{L_2C_1R_4C_{\alpha}}$	$-\alpha-2$
$\frac{1}{sC_1}$	sL_2	$\frac{1}{sC_3}$	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{1}{C_1C_3L_2R_4C_{\alpha}}$	$-\alpha-3$
sL_1	$\frac{1}{sC_2}$	sL_3	R_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{L_1L_3C_2}{R_4C_{\alpha}}$	$-\alpha+3$
sL_1	$\frac{1}{sC_2}$	sL_3	$\frac{1}{sC_4}$	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{C_2C_4L_1L_3}{C_{\alpha}}$	$-\alpha+4$
$\frac{1}{sC_1}$	sL_2	$\frac{1}{sC_3}$	sL_4	$\frac{1}{s^{\alpha}C_{\alpha}}$	$\frac{1}{C_1C_3L_2L_4C_{\alpha}}$	$-\alpha-4$

Table 1 does not include cases, which does not introduce a significant change in the parameter α and β, eg. replacing sL_1 by sL_3 and R_3 by R_1 in the second line of the analyzed cases would not change the value of the parameter β, therefore such cases have been skipped in the Table. As it can be seen from the Table 1, the highest order of a new fractional-order element, which can be obtained, is 5. It occurs when the parameter α or β is close to 1 ($\alpha, \beta \approx 1$) and there are four reactance elements in the GIC structure The lowest value of the order of the new fractional-order element equals -5, when parameters α or β are close to 0 ($\alpha, \beta \approx 0$) with four reactance elements. In other cases, the order of the new fractional-order element gains the value between the range of $\langle -5, 5 \rangle$.

3.1. System with two original fractional-order elements

The results of analysis with two fractional-order elements, based on formula (3), have been presented in Table 2.

Table 2. Obtained fractional-order input impedance, for two fractional-order impedances in (3)

$Z_1(s)$	$Z_2(s)$	$Z_3(s)$	$Z_4(s)$	$Z_5(s)$	$Z_{in}(s)$ Ω	
					K	γ
Ω	Ω	Ω	Ω	Ω		
$s^{\beta 1}L_{\beta 1}$	R_2	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_3 L_{\beta 1} L_{\beta 5}}{R_2 R_4}$	$\beta_5+\beta_1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_1 R_3 L_{\beta 5}}{L_2 R_4}$	$\beta_5-\beta_2$
$s^{\beta 1}L_{\beta 1}$	sL_2	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1} R_3 L_{\beta 5}}{L_2 R_4}$	$\beta_5+\beta_1-1$
sL_1	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1 R_3 L_{\beta 5}}{L_{\beta 2} R_4}$	$\beta_5-\beta_2+1$
$s^{\beta 1}L_{\beta 1}$	R_2	sL_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1} L_3 L_{\beta 5}}{R_2 R_4}$	$\beta_5+\beta_1+1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	sL_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_1 R_3 L_{\beta 5}}{L_2 L_4}$	$\beta_5-\beta_1+1$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1} R_3 L_{\beta 5} C_2}{R_4}$	$\beta_5+\beta_1+1$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_3 L_{\beta 5}}{C_1 L_{\beta 2} R_4}$	$\beta_5-\beta_2-1$
$s^{\beta 1}L_{\beta 1}$	R_2	$\frac{1}{sC_3}$	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1} L_{\beta 5}}{C_3 R_2 R_4}$	$\beta_5+\beta_1-1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	$\frac{1}{sC_4}$	$s^{\beta 5}L_{\beta 5}$	$\frac{R_1 R_3 C_4 L_{\beta 5}}{L_{\beta 2}}$	$\beta_5-\beta_2+1$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	sL_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1} L_3 L_{\beta 5} C_2}{R_4}$	$\beta_5+\beta_1+2$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	R_3	sL_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_3}{C_1 L_{\beta 2} L_4}$	$\beta_5+\beta_2-2$

Table 2 cont. Obtained fractional-order input impedance, for two fractional-order impedances in (3)

sL_1	$s^{\beta 2}L_{\beta 2}$	R_3	$\frac{1}{sC_4}$	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1R_3C_4L_{\beta 5}}{R_4}$	$\beta_5 - \beta_2 - 1$
$s^{\beta 1}L_{\beta 1}$	sL_2	$\frac{1}{sC_3}$	sL_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 1}R_3C_4L_{\beta 5}}{L_{\beta 2}}$	$\beta_5 + \beta_1 - 2$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	sL_3	$\frac{1}{sC_4}$	$s^{\beta 5}L_{\beta 5}$	$L_{\beta 1}R_3C_2C_4L_{\beta 5}$	$\beta_5 + \beta_1 + 2$
sL_1	$s^{\beta 2}L_{\beta 2}$	sL_3	$\frac{1}{sC_4}$	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1L_3L_{\beta 5}C_4}{L_{\beta 2}}$	$\beta_5 - \beta_2 + 2$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	$\frac{1}{sC_3}$	sL_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_{\beta 5}}{C_1C_3L_{\beta 2}L_4}$	$\beta_5 - \beta_2 - 3$
R_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{R_1R_3C_{\alpha 2}L_{\beta 5}}{R_4}$	$\beta_5 + \alpha_2$
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1R_3C_{\alpha 2}L_{\beta 5}}{R_4}$	$\beta_5 + \alpha_2 + 1$
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	sL_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1L_3C_{\alpha 2}L_{\beta 5}}{R_4}$	$\beta_5 + \alpha_2 + 2$
$\frac{1}{sC_1}$	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{L_1L_3C_{\alpha 2}L_{\beta 5}}{R_4}$	$\beta_5 + \alpha_2 - 1$
$\frac{1}{sC_1}$	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	$\frac{1}{sC_3}$	R_4	$s^{\beta 5}L_{\beta 5}$	$\frac{C_{\alpha 2}L_{\beta 5}}{C_1C_3R_4}$	$\beta_5 + \alpha_2 - 2$
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	sL_3	$\frac{1}{sC_4}$	$s^{\beta 5}L_{\beta 5}$	$L_1L_3C_{\alpha 2}C_4L_{\beta 5}$	$\beta_5 + \alpha_2 + 3$
$\frac{1}{sC_1}$	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	$\frac{1}{sC_3}$	sL_4	$s^{\beta 5}L_{\beta 5}$	$\frac{C_{\alpha 2}L_{\beta 5}}{C_1C_3L_4}$	$\beta_5 + \alpha_2 - 3$
$s^{\beta 1}L_{\beta 1}$	R_2	R_3	R_4	$\frac{1}{s^{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}R_3}{R_2R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$\frac{1}{s^{\alpha 5}C_{\alpha 5}}$	$\frac{R_1R_3}{L_{\beta 2}R_4C_{\alpha 5}}$	$-\alpha_5 - \beta_2$
$s^{\beta 1}L_{\beta 1}$	sL_2	R_3	R_4	$\frac{1}{s^{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}R_3}{L_2R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 - 1$
$s^{\beta 1}L_{\beta 1}$	R_2	sL_3	R_4	$\frac{1}{s^{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}R_3}{R_2R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 + 1$

Table 2 cont. Obtained fractional-order input impedance, for two fractional-order impedances in (3)

sL_1	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1R_3}{L_{\beta 2}R_4C_{\alpha 5}}$	$-\alpha_5 - \beta_2 + 1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	sL_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_1R_3}{L_{\beta 2}L_4C_{\alpha 5}}$	$-\alpha_5 - \beta_2 - 1$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}R_3C_2}{R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 - 1$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_3}{C_1L_{\beta 2}R_4C_{\alpha 5}}$	$-\alpha_5 - \beta_2 - 1$
$s^{\beta 1}L_{\beta 1}$	R_2	$\frac{1}{sC_3}$	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}}{C_3R_2R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 - 1$
R_1	$s^{\beta 2}L_{\beta 2}$	R_3	$\frac{1}{sC_4}$	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_1R_3C_4}{L_{\beta 2}C_{\alpha 5}}$	$-\alpha_5 - \beta_2 + 1$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	sL_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}L_3C_2}{R_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 + 2$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	R_3	sL_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_3}{C_1L_{\beta 2}L_4C_{\alpha 5}}$	$-\alpha_5 - \beta_2 - 2$
sL_1	$s^{\beta 2}L_{\beta 2}$	R_3	$\frac{1}{sC_4}$	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1R_3C_4}{L_{\beta 2}C_{\alpha 5}}$	$-\alpha_5 - \beta_2 - 2$
$s^{\beta 1}L_{\beta 1}$	sL_2	$\frac{1}{sC_3}$	sL_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}}{C_3L_2L_4C_{\alpha 5}}$	$-\alpha_5 + \beta_1 - 3$
$s^{\beta 1}L_{\beta 1}$	$\frac{1}{sC_2}$	sL_3	$\frac{1}{sC_4}$	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_{\beta 1}L_3C_2C_4}{C_{\alpha 5}}$	$-\alpha_5 + \beta_1 + 3$
sL_1	$s^{\beta 2}L_{\beta 2}$	sL_3	$\frac{1}{sC_4}$	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1L_3C_4}{L_{\beta 2}C_{\alpha 5}}$	$-\alpha_5 - \beta_2 + 3$
$\frac{1}{sC_1}$	$s^{\beta 2}L_{\beta 2}$	$\frac{1}{sC_3}$	sL_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{1}{L_{\beta 2}L_4C_1C_3C_{\alpha 5}}$	$-\alpha_5 - \beta_2 - 3$
R_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_1R_3C_{\alpha 2}}{R_4C_{\alpha 5}}$	$-\alpha_5 + \alpha_2$
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1R_3C_{\alpha 2}}{R_4C_{\alpha 5}}$	$-\alpha_5 + \alpha_2 + 1$
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	sL_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1L_3C_{\alpha 2}}{R_4C_{\alpha 5}}$	$-\alpha_5 + \alpha_2 + 2$

Table 2 cont. Obtained fractional-order input impedance, for two fractional-order impedances in (3)

$\frac{1}{sC_1}$	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	R_3	R_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{R_3C_{\alpha 2}}{C_1R_4C_{\alpha 5}}$	$-\alpha_5+\alpha_2$ -1
sL_1	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	sL_3	$\frac{1}{sC_4}$	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{L_1L_3C_{\alpha 2}C_4}{C_{\alpha 5}}$	$-\alpha_5+\alpha_2$ +3
$\frac{1}{sC_1}$	$\frac{1}{s^{\alpha 2}C_{\alpha 2}}$	$\frac{1}{sC_3}$	sL_4	$\frac{1}{s_{\alpha 5}C_{\alpha 5}}$	$\frac{C_{\alpha 2}}{C_1C_3L_4C_{\alpha 5}}$	$-\alpha_5+\alpha_2$ -3

Table 2 does not include cases, which does not introduce a significant change in the parameter α and β, similarly as in has been included in Table 1. As it can be noted from the Table 2, the highest order of a new fractional-order element can be 5, similarly to the previous case. It occurs when the parameters α_1, α_2, α_5 or β_1, β_2, β_5 are close to 1 ($\alpha, \beta \approx 1$). This situation can be obtained by including four integer-order reactance elements in the GIC structure. By introducing two fractional-order elements, the input impedance can be modeled in a lot more ways than in the previous case and the fractional-order exponent can be selected in a more arbitrary way than in the case of four classic integer-order elements.

4. EXAMPLE

Based on the previous studies, simulations of the exemplary fractional-order input fractional-order impedance, using the generalized impedance converter and an ideal supercapacitor (fractional-order capacitor) have been performed. There were assumed the following parameters of the circuit elements: ideal supercapacitor of the nominal capacitance $C_{\alpha 5}$ = 15 mF/s$^{1-\alpha}$, resistances R_1, R_2, R_4= 10 Ω, and the inductance L_3 = 15 mH. Simulations of the input impedance, its RMS value and phase (Bode diagrams), were conducted in Mathematica program. The new fractional-order element has the impedance, written in Laplace domain as:

$$Z_{in}(s)=\frac{U_1(s)}{I_1(s)}=\frac{R_1L_3}{R_2R_4C_\alpha}s^\gamma, \tag{5}$$

where:

$$\gamma=-\alpha+1. \tag{6}$$

Writing the impedance (5) in frequency domain, as the module $|Z_{in}(\omega)|$ and phase $\varphi(\omega)$ (Bode characteristics), it can be written in the form:

$$|Z_{in}(\omega)|=\frac{R_1L_3}{R_2R_4C_\alpha}\frac{1}{\omega^{\alpha-1}}, \tag{7}$$

and:

$$\varphi = (1-\alpha)\pi/2\,. \tag{8}$$

Illustrations of these characteristics are shown in Figs. 2-3.

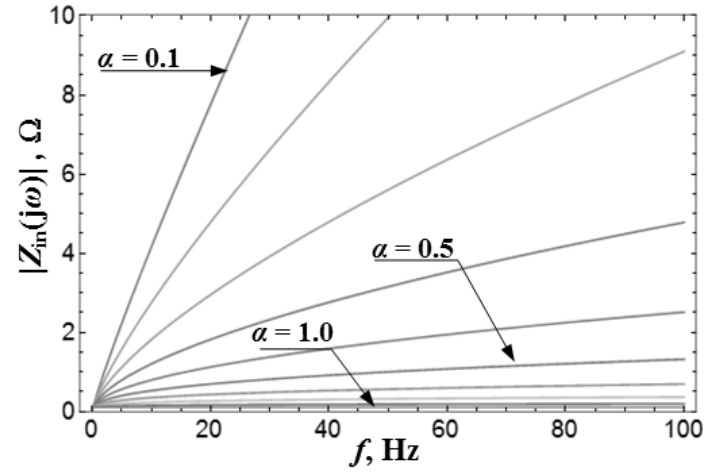

Fig. 2. Characteristics of the input impedance $|Z_{in}(\omega)|$ module, obtained in Mathematica

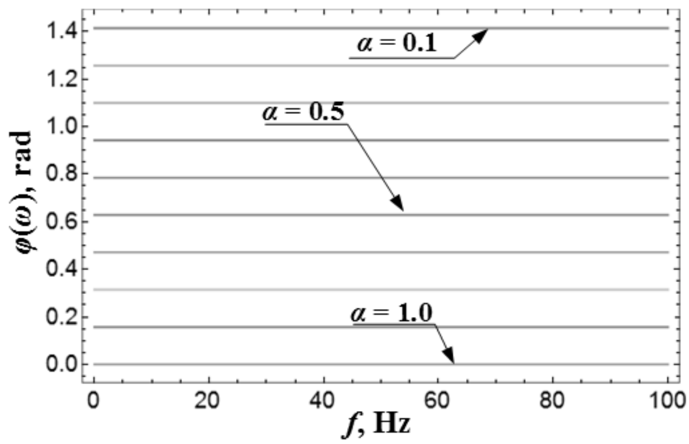

Fig. 3. Characteristics of the input impedance $\varphi(\omega)$ phase, obtained in Mathematica

The presented example consists of: one fractional-order reactance element (a capacitor) as the load impedance, the classic coil and three resistors in the structure of the generalized impedance converter GIC. Such configuration has given a new fractional-order element, with constant phase and rising RMS value. The RMS value rises almost linearly, when the coefficient $\alpha \rightarrow 0$, and tends to constant value, when $\alpha \rightarrow 1$.

5. SUMMARY

The paper presents an analysis of the possibility of realization of new fractional-order elements, using classic fractional-order components, such as supercapacitors and real coils with ferromagnetic cores. The realizations of new fractional-order elements have been obtained by using a generalized impedance converter GIC. Two cases of the system configurations have been analyzed in the paper: first, when the load impedance is the only one fractional-order, and second, with two fractional-order elements - one being a part of the structure of the GIC and the second being the load impedance. Possible configurations with one and two fractional-order elements in the GIC structure have been presented in Tables 1 and 2. More fractional-order elements in the GIC structure allows to select more diversified value of the order of the new fractional-order element. Tables 1 and 2 show that in case of two fractional-order impedances, there are a lot more possible system configurations.The derived relations have been illustrated by simulation example for the considered circuit. The impact of different values of the parameter α on the solution has been analyzed too.

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(Received: 8. 02. 2016, revised: 7. 03. 2016)