DOI: 10.5604/20830157.1212261

METHOD COMPARISON OF STATISTICAL AVERAGING ON THE HELMHOLTZ COILS CALIBRATION EXAMPLE

Mariusz Janeczko[1,2]

[1] Central Office of Measures, Electric Department, [2] Warsaw University of Technology, Faculty of Physics

Abstract: *During a calibration of Helmholtz coils, in which more than one parameter is measured directly, there are various approaches to statistical averaging. In this paper will be discussed two of them: the averaging at the beginning of directly measured magnitude and the averaging of the final value. In order to compare the methods they will be referenced to the Monte Carlo method, having regard to the uncertainty of type A.*

Keywords: Calibration of Helmholtz coils, statistical averaging, Monte Carlo method

PORÓWNANIE METOD UŚREDNIANIA STATYSTYCZNEGO NA PRZYKŁADZIE WZORCOWANIA CEWEK HELMHOLTZA

Streszczenie*. Podczas wzorcowania cewek Helmholtza podczas których mierzy się bezpośrednio więcej niż jeden parametr możliwe są różne podejścia do uśredniania statystycznego. W pracy omówione będą dwa z nich: uśrednianie na początku bezpośrednio zmierzonych wielkości oraz uśrednianie wielkości końcowej. W celu porównania metod zostaną one odniesione do Metody Monte Carlo z uwzględnieniem niepewności typu A.*

Słowa kluczowe: wzorcowanie cewek Helmholtza, uśrednianie statystyczne, metoda Monte Carlo

Introduction

One calibration method of the magnetic field meters consists in generating the reference magnetic field, and reading of a calibrated meter. This field is usually obtained by a set of two Helmholtz coils (commonly referred to as "Helmholtz coil" or simply "coil") with a coil constant *K*. The field strength is then equal to the product of the constant *K* and the current intensity *I*. This constant can be calculated from the geometry of the coil [3], however the accuracy of this method is low. The reason for this is the necessity to make many complex measurements and approximations which can then be used in deriving the utility equation for a coil constant *K*.

The solution to this problem is the determination of a constant coil through its calibration using a calibrated magnetic field meter. Such a method is in fact the reverse procedure used for the calibration of field meters. The field strength generated by the coil, at the determined current intensity, is measured with a previously calibrated meter of the field strength *H*. For this purpose is used a Hall-effect meter called a transfer meter. In this paper we will use the current intensity measurement method for measuring the voltage on the calibrated resistor.

In order to reduce the uncertainty of determining the constant coil *K*, the temperature of both, the resistor and the Hall-effect probe transfer meter should be controlled (however, for technical reasons, in this paper we will present an approach based on the temperature resistor only).

The parameters which are directly measured is the temperature *T* and the voltage *V* on the resistor. For a constant coil the equation is:

$$K = H_{WZ} I^{-1} = H_{WZ} R V^{-1} = H_{WZ} R_0 \left[1 + \alpha (T - T_0)\right] V^{-1} \quad (1)$$

where *H* is the selected magnetic field intensity, R_0 - the resistance of the standard resistor in a temperature T_0 and α - the temperature coefficient of the first order.

In all equations in this paper only a first order coefficient is used, but for the calculations also the coefficient of the second order was employed.

The calibration procedure of the coil is as follows: when adjusting the current generated by the power supply (or a generator) set the current value to the value indicating the intensity of the magnetic field on the transfer meter, which should be exactly *H,* according to the calibration value on the meter's certificate. Then we read the voltage *V* value on the standard resistor *T*. This measurement we repeat *M* times for one selected field strength *H* indicated by the transfer meter. Then we select the next value of the field strength and then we make the *M* measurements of the voltage *V* and the temperature *T*. If the number of set values of the field strength was *N*, then we get a total of *N · M* measured values *V* and *T*.

The coil constant *K* can be determined in two different ways. The first method consists of averaging the temperature and voltage separately and then inserting the averages for equation (1) and calculating a constant *K*. This approach is described by equation (2). The second way is to calculate value *M* of the constant K_i for fractional values of *T* and *V* by using equation (1), and by averaging K_i values only, what was done in equation (3).

The coil constants calculated by using two methods we denote as the K_1 and the K_2 respectively.

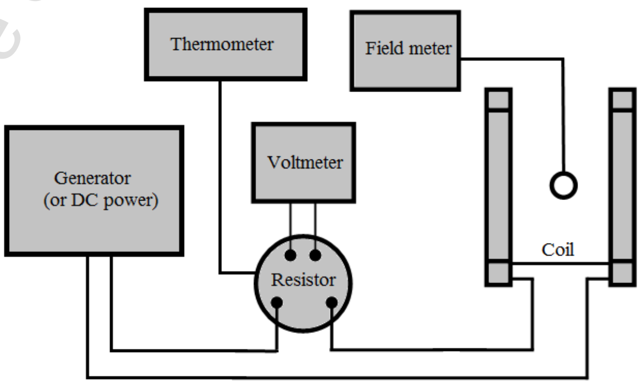


Fig. 1. Scheme of the measuring system

In this paper the results of calculations a coil constant *K* are compared and the uncertainty of analytical methods (the first and the second) of the Monte Carlo method as well.

The Monte Carlo method is required to take into account the uncertainty of the systematic and the non-systematic separately [5]. The systematic components of uncertainty budget are more than 40 independent factors. In order to compare the calculation results with the abovementioned analytical methods, the expanded uncertainties have been calculated separately; they are containing only components *A* denoted as U_A, and the total expanded uncertainties, including components of both A and B marked as U_{A+B}.

1. Presentation of the methods

Let us first consider the situation in which we set the *M*-fold coil current, so as to obtain the value of field strength *H* (measured with the transfer meter). By inserting into equation (1) an average value of the voltage and temperature, we thus obtain the equation for the coil constant K for the first method.

$$K_1 = HR_0\left[1+\frac{\alpha}{M}\sum_{j=1}^{M}\left(T_j - T_0\right)\right]\left(\frac{1}{M}\sum_{j=1}^{M}V_j\right)^{-1} \tag{2}$$

where T_j is j-th value of the temperature measurement and V_j - j-th value of the voltage measurement. By inserting into the equation (1) the values of T_j and V_j and then averaging the partial results K_j only, we get the average value of the coil constant K for the second method.

$$K_2 = \frac{1}{M}\sum_{j=1}^{M}K_j = \frac{1}{M}\sum_{j=1}^{M}HR_0\left[1+\alpha\left(T_j - T_0\right)\right]V_j^{-1} \tag{3}$$

When calculating the constant K for a larger number of field strength H value, we obtain N values of K_i, given the equation (2) or (3). Averaging these values we obtain finally the following equations. For the first method:

$$K_1 = \frac{1}{N}\sum_{i=1}^{N}H_i R_0\left[1+\frac{\alpha}{M_i}\sum_{j=1}^{M_i}(T_{ij} - T_0)\right]\left(\frac{1}{M_i}\sum_{j=1}^{M_i}V_{ij}\right)^{-1} \tag{4}$$

with the standard uncertainty:

$$u(K_1) = \sqrt{\left(\frac{t(\nu_{K,i})}{1{,}96}\frac{\sigma(K_i)}{\sqrt{N}}\right)^2 + \sum_{i=1}^{N}\left(\begin{array}{c}\left(\frac{\partial K_i}{\partial T}\frac{t(\nu_{T,i})}{1{,}96}\frac{\sigma(T_i)}{\sqrt{M_i}}\right)^2 + \\ \left(\frac{\partial K_i}{\partial V}\frac{t(\nu_{V,i})}{1{,}96}\frac{\sigma(V_i)}{\sqrt{M_i}}\right)^2 + \\ \sum_{q=1}^{Q}\sum_{j=1}^{M_i}\left(\frac{\partial K_{ij}}{\partial X_q}\right)^2 u^2(X_q)\end{array}\right)} \tag{5}$$

For the second method we have:

$$K_2 = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{M_i}\sum_{j=1}^{M_i}K_{ij} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{M_i}\sum_{j=1}^{M_i}H_i R_0\left[1+\alpha\left(T_{ij} - T_0\right)\right]V_{ij}^{-1} \tag{6}$$

with the standard uncertainty:

$$u(K_2) = \sqrt{\left(\frac{t(\nu_{ij})}{1{,}96}\left(\sum_{i=1}^{N}M_i\right)^{-\frac{1}{2}}\sigma(K_{ij})\right)^2 + \sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{q=1}^{Q}\left(\frac{\partial K_{ij}}{\partial X_q}\right)^2 u^2(X_q)} \tag{7}$$

where: K_i is the partial value of the coil constant for each value of the field strength H_i, $\sigma(X)$ is the standard deviation of a physical quantity X_q, $u(X_q)$ is the standard uncertainty magnitude X_q estimated by the B method (ie. non-statistical method in accordance with the Guide) [4], X_q represents all the values from the equation (1), Q is the quantity of these values, $t(\nu)$ is the quantile of the t-distribution with ν degrees of freedom, inserted here for the reasons explained in [1].

The expanded uncertainty was calculated using the method described in [1]. This method comes down to the assumption that mezurand is described by a flat-normal distribution, then consists of reading the coverage factor from the table given also in [1]. The coverage factor is therefore a discrete function dependent on parameter r on the formula (8). This parameter determines the percentage component of the uniform distribution of mezurand's combined uncertainty.

$$r = \frac{c_B \cdot Max\{|\, c_l u_l \,|\cdot k_{UP,l}\}}{\sqrt{u_k^2 - c_B^2 \cdot Max\{|\, c_l u_l \,|\cdot k_{UP,l}\}^2}} \tag{8}$$

where u_k is the complex overall uncertainty, c_l is the corresponding sensitivity coefficient, ul is the partial uncertainty, l is an indicator ordering all components of the partial uncertainty which are occurring in the uncertainty budget (a total of which is more than 40).

c_B is equal to 1 divided by $\sqrt{MN}$ and k_{UP} is a percentage share of components of the uniform distribution of fractional uncertainty u_l and is given by:

$$k_{UP} = \frac{10\cdot(1{,}96 - k_X)}{3{,}3} \in \langle 0,1\rangle \tag{9}$$

where k_X is the coefficient of extension taken from the calibration certificate of the physical magnitude X, which corresponds to the uncertainty ul. If the value of this parameter is not given, or if it is greater than 1.96, it should be converted to 1.96, which is equivalent to the assignment of a physical magnitude X of normal distribution.

To use the Monte Carlo method, which takes into account only the uncertainty of type A, in equation (1) the random variables T and V must be submitted with the help of a random variable D_n, containing information about the distribution of the variables T and V with a preset mean value and standard deviation. For this purpose we use the substitution $T = \overline{T} + D_n \cdot \sigma\left(\overline{T}\right)$ and $V_{ZM} = \overline{V} + D_n \cdot \sigma\left(\overline{V}\right)$. The random variable D_n was carried out using the RAND function used in MS EXCEL for the purpose of implementation of the Monte Carlo method. The RAND function simulates the white noise from the value range [-1,1], which means it performs the uniform distribution. When we want to get a normal distribution, then we have to use the RAND function 12 times for the one embodiment of the random variable. We use the fact that the average value from the N random variables with uniform distribution tends to a normal distribution. $\sigma(\overline{X})$ is the standard deviation of the mean value of the random variables X_i, averaged over the M partial measurements. The number of steps in the Monte Carlo method is $L \approx 20.1$ thousand for each value of the field strength H. Finally, the value of a coil constant, determined now by K_{NUM}, and the corresponding expanded uncertainty of type A, are given by equations:

$$\begin{gathered} K_{NUM} = \frac{1}{N\cdot L}\sum_{i=1}^{N}\sum_{n=1}^{L}K_{i,n} = \\ \frac{R_0}{N\cdot L}\sum_{i=1}^{N}\sum_{n=1}^{L}H_i[1+\alpha(\overline{T}_i + D_n\sigma(\overline{T}_i) - T_0)]\left(\overline{V}_i + D_n\sigma(\overline{V}_i)\right)^{-1} \end{gathered} \tag{10}$$

The expanded uncertainty is expressed by the order statistics:

$$U_A(K_{NUM}) = \frac{1}{2\sqrt{N}}\left(K_{0.975L} - K_{0.025L}\right) \tag{11}$$

2. Comparison results

The measurements of the constant K were carried out for a commercial coil of NFH63,4 type, using a transfer meter of Rx21 type with the temperature-compensated Hall probe. In order to omit the impact of the current frequency on a constant coil [2] the measurements were performed for DC. The full description of the calibration procedure and the calculation of uncertainty is given in [6]. The results averaged over $N = 9$ values of the field strength H_i, measured each time $M = 6$ times, with the corresponding uncertainties, are shown in Table 1.

Table 1. The comparison of results of the coil constant K calculated for the various methods for N = 9 and M = 6 = const (with nominal value of α)

	$\overline{K}$ [A/m/A]	U_A [A/m/A]	U_{A+B} [A/m/A]
K_1	18749,4	31	41
K_2	18749,5	14	30
K_{NUM}	18749,9	23	-

The values of the coil constants with the same number of measurement repetitions for the same field intensity (with the same M), obtained by different methods, do not vary within the limits of uncertainty. We observe a greater difference between constants K when the number of measurement repetitions per field value changes. The results of coil constants K calculations for $N = 13$ of the field strength values, with the number of repetitions for each field value M_i = {5, 5, 5, 6, 6, 1, 6, 1, 5, 1, 6, 1, 6} together with the corresponding uncertainties, are presented in Table 2. Adopted here is the assumption that when $M_i = 1$ then for the calculations an expression $\sigma(X) = 0$ has been inserted.

Table 2. The results of the comparison coil constant K calculated for the different methods for N = 13 and different values Mi (with nominal value of α)

	$\overline{K}$ [A/m/A]	U_A [A/m/A]	U_{A+B} [A/m/A]
K_1	18736	27	38
K_2	18747	14	30

The significant differences in the results of different methods of the averages calculations were observed in case of uncertainty only. This means that the basic difference between the abovementioned methods of calculating the coil constant is the estimation of the type A uncertainty. The uncertainty of type B for both methods is almost the same.

This difference becomes obvious if we look at the number of degrees of freedom, which for both methods is different. In the first case we perform averaging 9 times at different values of the field strength measured 6 times, which each time gives the number of degrees of freedom 6 only. In the latter case, we are averaging over all elements, of which a total is 9 · 6, which gives the number of degrees of freedom equal to 54.

3. Influence of resistor temperature

The temperature of the resistor has little importance for the calculation results due to the low values of temperature coefficients ($\alpha = 1.9\cdot10^{-5}$ K^{-1} and $\beta = -3.9\cdot10^{-5}$ K^{-1}) as well as small changes in temperature during the measurements. The standard deviations of the voltage and the temperature were approximately of $\sigma(V) = 0.31$ mV, and of $\sigma(T) = 0.4$°C respectively.

However, by artificially increasing the value of the temperature coefficients, one can enhance this effect. Table 3 shows the difference between the calculated coil constants for a thousand-fold increased value of this parameter.

Table 3. Coil constants K for N = 9 and M = 6 = const with resistor temperature coefficient increased thousand times

α·103	$\overline{K}$ [A/m/A]	UA [A/m/A]	UA+B [A/m/A]
K1	18556,0	283	305
K2	18556,1	38	120
KNUM	18556,8	42	-

As can be seen, for the highly sensitive thermistor the values of the calculated constants are not significantly different. However, the uncertainty of type A, calculated using the second method, is almost 7.5 times lower, and the total uncertainty of type A + B is almost 2.5 times lower. The uncertainty achieved by the second method is consistent with the result obtained by the Monte Carlo method.

As shown in Table 4, the reduction of the temperature coefficient does not cause noticeable changes in comparison to the results in Table 1.

Table 4. The coil constant K determined for N = 9 and = 6 = const with the temperature coefficient thousandfold reduced

α·10-3	$\overline{K}$ [A/m/A]	UA [A/m/A]	UA+B [A/m/A]
K1	18749,6	31	41
K2	18749,7	14	30
KNUM	18749,6	22	

4. Analysis results of Monte Carlo method

The uncertainty calculated using the Monte Carlo method is different from both of the methods defining a coil constant K_1 and K_2. The histogram obtained from all 181 000 calculated partial K shows that the reason for this difference is an asymmetrical and bimodal distribution, which is contrary to the assumptions of both methods. The probable cause of this anomaly is the dependency of the coil constant K_i on the field strength H.

Fig. 3 shows this dependence with the expanded uncertainty given by the equation where $U_{NUM},_i$ is the expanded uncertainty of type A using the Monte Carlo method for each value of the field strength H and k is the coverage factor calculated using the method discussed in Chapter 1.

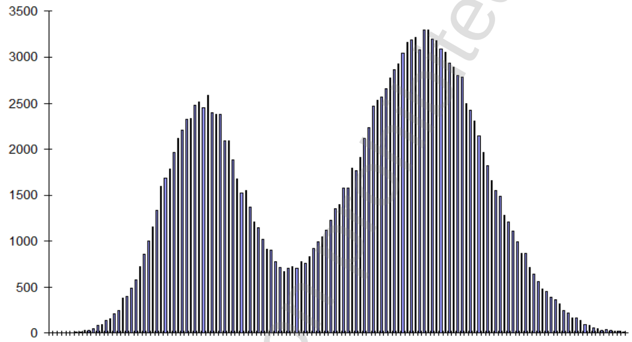


Fig. 2. Total histogram of 9 times 21 thousand samples of Kij (with nominal value α)

The causes of the coil constant dependency on the filed strength, and the occurrence of two modes in the constant K distribution require further research. It was only stated that the histogram of the first 4 values of the field strength (Fig. 3) made for the last few field strength values are monomode and symmetrical.

The shape of the curve of the constant K dependency on the field strength H (Fig. 3) is maintained also in the case of the increased number field strength points. Also, when we perform the measurements for the decreasing values of the field strength (with the opposite direction of change of the field strength), the shape of the curve shown in Fig. 3 does not change, thus providing the basis for elimination a potential cause of the observed non-linearity, which is the heating of the coil.

The observed effect should be studied further and included in the uncertainty budget.

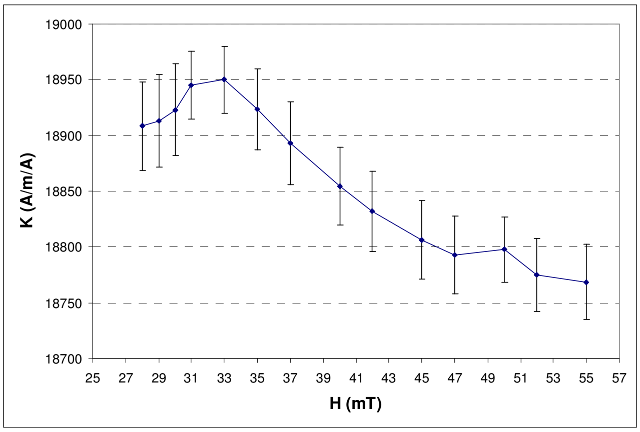


Fig. 3. The coil constant K dependence on the field strength H as a function (proportional to the current flowing through the coil)

There were also made calculations of the constant K histograms for a resistor with thermal coefficient thousand times smaller than the nominal and a thousand times greater than the nominal. In the case of a thousandfold lower thermal sensitivity of the standard measuring resistor the histogram is bimodal, but for a thousand times more thermally sensitive one, the resulting distribution becomes symmetric and monomodal.

In this case the dependency of the partial coil constants K_i on the field strength H is flatter and therefore more corresponds to the expectation for the coil constant to be less dependent on field strength (Fig. 4) but uncertainty is so high that nothing can be excluded.

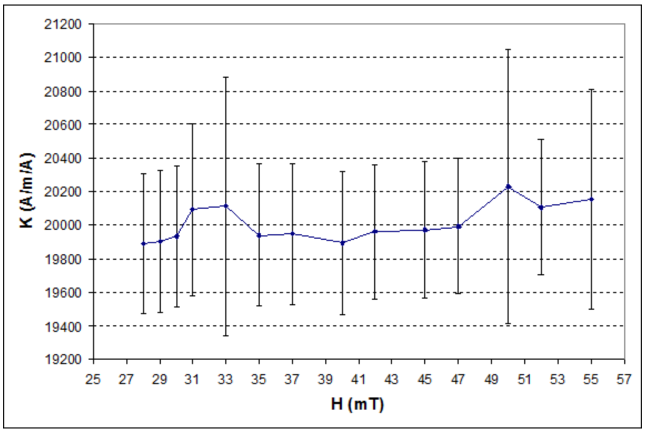


Fig. 4. The dependence of the coil constant K on the field strength H with a housandfold increased temperature coefficient of the standard resistor R

5. Summary

In this paper two methods of averaging the results of coil constant calculations are compared: the first is based on averaging a series of measurements of the current intensity and temperature and on inserting the averages to the formula (1) and the second consists of averaging the constants K values calculated for the partial results of the current intensity and temperature measurements.

The main difference between those abovementioned methods of the coil constant calculating is the difference in the estimation of the uncertainty type A.

The results were compared with the Monte Carlo method; also a better compatibility with the second method was achieved, that is, in averaging the partial results of the constant K. This method gives more degrees of freedom and is easier to implement analysis. Where the number of measurements for a single field strength value is small (M_i < 3), this method overcomes the problem of calculating the partial standard deviation. Moreover, when the measured values are correlated or dependent, there is no need to calculate the correlation functions.

Using the Monte Carlo method allows to examine the distribution of composed random variable.

In this case is shown that the distribution of a coil constant K is asymmetric and bimodal, what may be associated with the dependence of the coil constant on field strength.

Also the simulation of the impact of increasing and decreasing the temperature coefficient of the standard resistor was carried out.

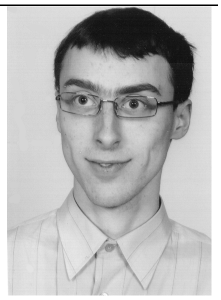
The reduction of the temperature coefficient thousand times does not alter the distribution and the assigned uncertainties, while increasing it thousandfold causes the constant K distribution become monomodal.

Bibliography

[1] Fotowicz P.: Using of flat-normal distribution for calculating the uncertainty of measurement. PAK 6/2011, 595-598.
[2] Gersak G., Humar J., Fefer D.: Calibration of a reference field coil by means of the NMR magnetometer and induction coils, Elektrotehniski vestnik 68(5)/2001, 294–299.
[3] IEEE Standard for Calibration of Electromagnetic Field Sensors and Probes (Excluding Antennas) from 9 kHz to 40 GHz – IEEE Std 1309-2013, eq A33.
[4] JCGM 100:2008 Guide to the Expression of Uncertainty in Measurement.
[5] JCGM 101:2008 Evaluation of measurement data — Supplement 1 to the “Guide to the expression of uncertainty in measurement” — Propagation of distributions using a Monte Carlo method.
[6] Measurement instructions of Microwave, Electromagnetic Field and Electromagnetic Compatibility laboratory in the Electrical Department of Central Office of Measures in Poland.

M.Sc. Mariusz Janeczko
e-mail: m.janeczko@gum.gov.pl

Graduated from the Faculty of Physics, Warsaw University of Technology, in 2013. After obtaining a Masters began doctoral studies at the same department under the guidance of prof. M. Urbanski and has been an employee of the Central Office of Measures, taking care of electromagnetic field laboratory. Interested in the theoretical metrology, especially in the mathematical development of measurement results and their uncertainties.



otrzymano/received: 22.04.2016 *przyjęto do druku/accepted: 01.07.2016*