OCENA CZASU OCZEKIWANIA NA PRZYSTANKACH DLA LOSOWYCH ZGŁOSZEŃ PASAŻERÓW[1]

Petro Horbachov
prof., Kharkiv National Automobile and Highway University, ul. Petrovskogo 25, 61002Kharkiv, Ukraine, tel.: +38 057 707 3783, e-mail: gpf@khadi.kharkov.ua

Vitaliy Chyghyk
Kharkiv National Automobile and Highway University, ul. Petrovskogo 25, 61002 Kharkiv, Ukraine, tel.: +38 057 707 3783, e-mail: chyzhyk_v@mail.ru

Streszczenie. *Wartykule zdefiniowano problem i przedstawiono ujęcie analityczne modelowania obliczeń czasu oczekiwania na przystanku, gdy zgłoszenie pasażera na przystanek na charakter losowy. Rozważany jest szeroki zakres zmienności przebiegów tras linii – poczynając od przyjęcia pełnej zgodności interwałów kursowania linii w rozkładach jazdy do całkowicie losowych odjazdów pojazdów z przystanków pośrednich. Zostały również porównane doświadczalne i teoretyczne oszacowania czasów oczekiwania na przystankach dla podstawowych linii.*

Słowa kluczowe: *miejski system przesiadkowy, czas oczekiwania, teoria prawdopodobieństwa*

1. Introduction

The transport modelling is the main decision-making tool in the field of urban transit systems. Modern modelling software allows making assessment of many characteristics of the transportation process quantitatively. The most important of the transport characteristics are the travel costs. When studying the transit systems operation in the total travel time it is necessary to take into account the waiting time of the passengers at the stop. Unfortunately, this element of the travel cost is not yet properly reflected in software products of transport modelling.

Only two extreme cases are considered in modelling software:the complete knowledge of the transit route schedules(schedule-based service) or absolute ignorance of it (headway-based service). Estimates of waiting time all too simple in both cases:zero in the first case and a half of the meanheadway in the second one. Transportation models do not estimate the route schedule quality and the level of interaction between the routes.They do not allow developing effective proposals to improve the quality of passenger service due tothe waiting time reduction.

The adequate assessment of the waiting time is especially necessary in Ukraine. It is caused by the low level of the transit system operation in Ukrainian cities. The

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timetables are absent in most cities. It leads to the situationwhen the most passengers are not aware of the departure time of transport units (TU) from the stops.

In fact the municipal transportation agencies do not control of the operation of transit companies. It often leads TU drivers to neglect schedule and promotes an excessive freedom in a choice of TU movement mode in urban transit system. As a consequence, for many of transit routes the self-regulated relationship between drivers was formed.It led to the emergence of new modes of TU movement on the transit routes that claim to make adequate models for the calculation of the waiting time.

2. Literature review

The first model for a waiting time assessment in transit system was published in 1932 by the Soviet scientist A.H. Zilbertal [1]:

$$E(t) = \frac{H}{2} + \frac{\sigma_h^2}{2H} \tag{1}$$

where:
E(t) – the expected waiting time for an arrival at stop, min,
H – mean headway (service headway), min,
σ_h – standard deviation ofthe headway, min.

This model was derived from the case, when passenger arrivesat stop randomlyand independently of the transit routeschedules.Later, the equation (1) was obtained by many authors, presented in [2]. This model is mentioned very often in articles devoted to the assessment of a passengers waiting time at transit stops, but we do not know any examples of its application in software packages for the transport modelling. It is mainly used to compare new models createdto account the factors which define a passenger conscious choice of an arrival time ata stop point. A large number of the scientific papers are devoted to such models.Psarros et al provide good example and their wide review [3].

Another area of the modelling is the research of the passenger relation to waiting time, for example the paper [4]. It is usually assumed that passengers consciously choose an arrival time ata stop point. In this direction, a large number of investigations are also conducted.The general goal of investigations is to find out how the waiting time affects the probability of a transit system usage.

The results of many investigations are very interesting, but not one of them has been implemented in modern software products for transport planning. The particular nature of the results can raise the problem. The models typically assume that passengers know the transit routeschedule.So, theuse of thesemodels is always based on some model of passenger behaviour and on the specified level of the transit system reliability. The common theoretical basis for the passenger arrival time ata stop point for a known schedule cannot be created as it is the individual decisions under the influence of many factors. The accumulated factual material is not enough to develop the general empirical function. Itstrongly complicates the use of waiting time models in practical tasks of the transport planning.

3. Theoretical modelling of waiting time

The real transport situation cannot be described by one of the extreme cases of the passenger awareness about the schedule on transit routes. In the modern transit systems it is difficult to find the examples of a complete ignorance of the schedule or full awareness of all passengers about the schedule. Improvementof the reliability of the transit system operation leads to increase in the number of passengers, who pay attention to the route schedule. The number of such passengers can only be determined by selective surveys that cannot be considered as simple and accurate solution method for this issue. After that passenger behaviour for the known route schedules is still needed to be investigated. The simple and precise assessment of the actual passenger waiting time at the transit stop is not possible in such circumstances.

The variant of the waiting time assessment for a random arrival of the passengers into a stop point looks much more perspective.The information in the transportation model about the transit system operation is quite sufficient for a correct assessment of the passenger waiting time. The transport designer needs only to specify the level of the reliability of the route schedules. Using quite correct assumptions the equation (1) is general form of the waiting time dependenceon the expectation and dispersion of route headway. It is the basis to create new models of waiting time for the random arrival of the passengers at the stop and new modes of the TU operation on the transit routes.

An upper estimate of the average waiting time isderived from using such models of waiting time and the transport designer can directly evaluate the quality of the routeschedules. In addition, it allows us to estimate the waiting time for a variety of technologies of the TU operation that is very important for Ukraine. The lack of the line control led to the emergence of such variants of the TU operationon the transitroutes, in which drivers can deviateseriously from the route schedules. Moreover, route timetables may not be followed at all, bothin the trip number andthe departure time from the initial stop point of the route.

The goalof this study is to formalize all known forms of the TU operationson transit routes with a random passenger arrival at a stop. It is assumed that the passenger arrival at a stop is the simplest flow of events and has the properties of the stationary and a lack of the ordinary after-effect [7] and the passenger has no information about the schedule of the transport routes.Another general limitation in the formation of waiting time models is the impossibility to simultaneous arrival and departure of the several TU at the same stop.

In the best case of the TU operation on the transit routes, when the factual headway is the constant $\sigma_h = 0$, the equation (1) becomes a standard form for software products of transport modelling:

$$E(t) = \frac{H}{2} \tag{2}$$

In the reality there may be some deviations from the scheduled arrival time. It is reasonable to assume that these deviations are distributed normally with the expectation value 0. A standard deviation of thedistribution is taken to follow the condition of the separate TU operation on the route. At the average headway on the basis of three-sigma rule, can be obtained:

$$\sigma_d = \frac{H}{6} \tag{3}$$

where:
σ_d – a standard deviation in the TU arrival divergences, min.

The general equation of the waiting time calculation correspondingly [7]written as follows:

$$E(t) = \frac{E(h^2)}{2E(h)} \tag{4}$$

with

$$E(h^2) = H^2 + \sigma_h^2 \tag{5}$$

where:
h – the factual headway, min.

In that case, when $H = const$, from the condition of the two dispersions addition of the independent random variables can be derived:

$$\sigma_h^2 = 2\sigma_d^2 = \frac{H^2}{18} \tag{6}$$

Substituting (6) in the equation (1) defines the expectation value of the waiting time:

$$E(t) = \frac{H}{2}(1 + \frac{1}{18}) \tag{7}$$

Such variant of the TU operation provide a sufficient level of the passenger service quality which insignificantly (< 6 %) differs from the best case of the TU operation on the transit routes. The estimation can be received for any level of the schedule implementation it is only necessary to set the level. It is possible to set it in a convenient form viathe standard deviation in the TU arrival divergence.

The next variant of the TU operation occurs when there are different headwaysin the route schedule. The analytic equation of the waiting time is based on the assumption that the headway can take only two values: H_n, H_x – minimum and maximum headway correspondingly, min. At the same time the sum of the minimum and maximum headwayof the transit routeis equalto two middle headways:

$$H_n + H_x = 2 \cdot H \tag{8}$$

On the basis of the equation (2) is written:

$$E(t) = \frac{H_n^2 + H_x^2}{2(H_n + H_x)} \tag{9}$$

Auxiliary variable $S \geq 0$, which indicates how many times the maximum headway greater than minimum one is introduced to characterize the headway variation. Then the maximum headwayis calculated as:

$$H_x = S \cdot H_n \tag{10}$$

Substituting (10) in the equation (9) gives the expectation value of the waiting time as follows:

$$E(t) = \frac{H_n}{2} \; \frac{1 + S^2}{1 + S} \tag{11}$$

On the other hand, taking into account (8), the minimum headway:

$$H_n = \frac{2H}{S + 1} \tag{12}$$

then

$$E(t) = \frac{1 + S^2}{(1 + S)^2} \tag{13}$$

In a case when $S = 2$:

$$E(t) = \frac{1 + 2^2}{(1 + 2)^2} H = 0.55 \cdot H \tag{14}$$

When $S = 3$ $E(t) = 0.625 \cdot H$. That is for such route schedule the waiting time is directly proportional to value of S that should be considered while the efficiency of route schedules is being estimated.

Both equations (7) and (14) are particular cases of general equation (1) that can also be used for an assessment of the waiting time with such schedule variants. Only the waiting time dispersion on the route should be estimated at the beginning. For this purpose the route schedules and any assessment of the operation punctuality are enough.

The waiting time can be estimated not only for one route, every pair of the origin – destination can get average waiting time. However equation (1) are not widely used in modern software products for transport planning and the purpose of development of the equations (7) and (14)is to show how much the waiting time can change dependence on the headway uniformity and the operation punctuality.

During our observations of the transit routes operation in Ukrainian cities the operation variants when the bus drivers do not follow the route schedulewere identified. Very often the TU drivers determine the time of the trip startingon their own according to some rules that have formed as a result of their collective decisions. One of the main cases of the self-regulation is the operation variant when the TU departs from the initial stopping after the previous vehicle while the passenger quantity come up to some critical level. The value of the critical level can equal to the number of available seatsin the TU, for example.

The time of the passenger cabin filling is not significant and does not influence the headway when the intensity of the passenger approach to the transit stop is high and the TU number is small. Such situation can be observed on the initial stop of the transit route or on intermediate stops with a big number of passengers (near metro stations, markets, etc.) where there is an intensive accumulation of people. Then, at the moment of the TU arrival at the stop such number of passengers accumulates that provides sufficient loading and the TU does not wait for the other passengers. In the other cases, the departure time directly depend on the level of the passenger cabin filling.

Let TU departs from the initial stopping after the previous vehicle while the passenger quantity come up to some level $l \geq 0$. When the passenger arrival at a stop is the simplest flow of events the frequency distribution of the time interval between the sequential departs of the TU is determined based on the Poisson distribution:

$$f_l(x) = \lambda \cdot \frac{(\lambda x)^{l-1}}{l-1\ !} \cdot e^{-\lambda x} \qquad (15)$$

where:

l – the given level of the passenger cabin load, pas.;

λ – an intensity of the passenger flow, pas./min.

The time interval between the sequential departs of the TU equal to the total time of the l passengers approach to the stop. With the simplest passenger arrival at a stop the time between two sequential arrivals of passengers corresponds to the exponential distribution. From this condition it follows that at such variants of the TU operation on the transit routes the time between two consecutive departures corresponds to the Erlang distribution of l degree. In addition, it should be noted that headway does not depend on the TU quantity and the idle TUs stand at the initial stop.

On the basis of the equation (15) the expectation value of the time between the sequential departs of the TU equal:

$$H = E(h) = \frac{l}{\lambda} \tag{16}$$

with

$$E(h^2) = \int_0^{\infty} x^2 f_h(x)dx = \frac{l \cdot (l+1)}{\lambda^2} \tag{17}$$

The stationary expectation of the waiting time on the basis of (4) is calculated as follows:

$$E(t) = \frac{H}{2}E(t) = \frac{H}{2} \cdot \left(1 + \frac{1}{l}\right) + \frac{\sigma_h^2}{2H} \tag{18}$$

Resulting from the equations (16) and (18), such technology leads the expectation of the waiting time to the linear dependency of the given level of the passenger cabin filling. When the level $l = 10$ and the intensity of the passenger flow to the stop $\lambda = 60$ pas/hthe expectation of the time between the sequential departs of the TU equal $H = l/\lambda = 10$ min.Then the expectation of the waiting time equals:

$$E(t) = \frac{H}{2} \cdot \left(1 + \frac{1}{10}\right) = 0.55 \cdot H$$

The technology has a serious impact on a passenger waiting time at the stop that has to be accounted at planning decisions.

The next variant of the operation technology emergesfrom the basis of the previous onewhen the departure time is limited by the given time interval.Such situation is typical when there are too many TUs on the route. Then the departure time can be limited by others drivers who are on the initial stop of the route.

Let the departure time be limited by constant T. Then TU stand on the initial stop the time of the l passengers filling, but not longer interval T since departure of the previous TU. The process of the passenger approach to the stop is described by the simplest flow with the parameter λ as in the previous variant of operation technology. Then the interval of the TU departurehasthe distribution density as follows:

$$f_l(x,t) = \begin{cases} \lambda \cdot \frac{(\lambda \cdot x)^{l-1}}{(l-1)!} \cdot \frac{e^{-\lambda x}}{A_l(T)}; x \leq T \\ 0; \qquad x < T \end{cases} \tag{19}$$

where:

$A_l(T) = 1 - e^{-\lambda T} - \lambda \cdot T \cdot e^{-\lambda T} - \dots - \frac{(\lambda \cdot T)^{l-1}}{(l-1)!} \cdot e^{-\lambda T}$ – a constant is derived from the normalizing condition of the probability density.

The expectation of the time between the sequential departs of the TU is determined after the integration of the next equation:

$$H = E(X) = \int_{0}^{\infty} x \cdot f_l(x;T) \cdot dx = \frac{l}{\lambda} \cdot \frac{A_{l+1}(T)}{A_l(T)} \tag{20}$$

The second moment of the random headway value looks as follows:

$$E(h^2) = \int_{0}^{\infty} x^2 \cdot f_l(x;T) \cdot dx = \frac{l \cdot (l+1)}{\lambda^2} \cdot \frac{A_{l+2}(T)}{A_l(T)} \tag{21}$$

Substitution of the equations (20) and (21) into equation (4) gives the expectation of the waiting time as follows:

$$E(t) = \frac{l+1}{2 \cdot \lambda} \cdot \frac{A_{l+2}(T)}{A_{l+1}(T)} \tag{22}$$

or

$$E(t) = \frac{H}{2} \cdot \left(\frac{A_l(T) \cdot A_{l+2}(T)}{(A_{l+1}(T))^2} \right) \cdot \left(1 + \frac{1}{l}\right) \tag{23}$$

Because of a monotonically increasing character of the function $A_l(T)$ the limit of the TU departure time decreases the passenger waiting time.

The worst variant of the operation technology can be when the TU depart from the initial stopping in a completely random sequence. Certainly, such variant of the technology is unlikely present in the real transit systems. However, it has a certain theoretical value as it gives a notion of the maximum waiting time with a headway-based service in the transit system. The simultaneous arrival and departure of the several TU at the same stop is strictly impossible in a random sequence of TU.

Let the passenger approachesat the stop at the moment t. TU are uniformly distributed on a transit route, fig. 1.

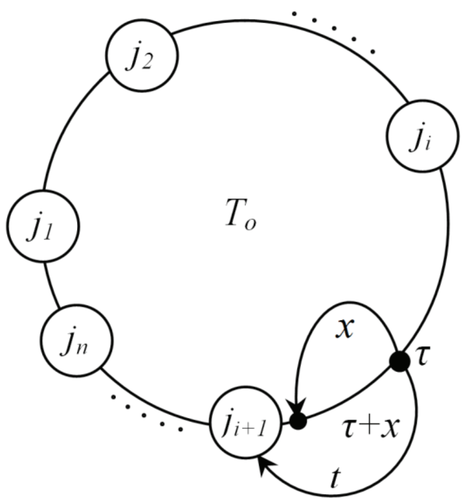

Fig. 1 – Distribution of the TU

Notation conventions on Fig. 1:
$j_i \dots j_n$ – time points, on which are found transport units;
τ – the time point of the one passenger approach to the stop;
T_o – the identical to all TU interval of the time between two consecutive departures TU from start point of route (cycle time of the route);
x – nonnegative auxiliary variable.

If the passenger has appeared on the stop point at the time t and within the time interval $t + x$ does not appear any TU, then this means that all n TU are located within the time interval $T_o - x$. The probability of suchcondition for each TU equals to $\frac{T_o - x}{T_o}$. Witha condition the uniform and independent TUdistribution on the routecan be found the probability that the waiting time is greater than x. For the n TU, this probability is calculated as follows:

$$P\{t > x\} = \left(1 - \frac{x}{T_o}\right)^n \tag{24}$$

The average waiting time is found by integrating the expression (23)

$$E(t) = \int_0^{T_o} \left(1 - \frac{x}{T_o}\right)^n \cdot dx = \frac{T_o}{n+1} \tag{25}$$

The substitution in equation (25) the value of the average waiting time in dependence of the TU quantity T_o/n gives this equation as follows:

$$E(t) = H \cdot \frac{n}{n+1} \tag{26}$$

The average waiting time increases from $H/2$ at $n = 1$ with of the TU quantity growth asymptotically coming nearer to a mean headway of the route H at $n = \infty$. This tendency can be expanded for other operation technologies in the transit system.That is the importance of the correct organization of the transit TU operationgrows with increase in the TU quantity on the route.

4. Experimental test of waiting time models

The theoretical models described in this paper are derived from the condition of the simplest flow of the passengers. Such conditions by consideration of the real transit routesare quite seldom. It is caused by the requirement of stationarity and lack of the after-effect in the simplest flow. The real passenger flow depends both on the time period and the number of the passengers who have already arrived to the stop.

This situation requires an experimental verification of the analytical models. Unfortunately, it is difficult to find a stop that is served by only one transit routein

Ukrainian cities. At the stop with more than one transit route the waiting time of one route cannot be found using observation of the passenger behaviour. Furthermore an observer cannot determine whether the passenger knows the route schedule or not.

Therefore, the only available way to verify the theoretical values of the waiting time is a simulation experiment. In order to do that a special simulation model is developed, in which the TU arrival time at the stop is formed in accordance with the above described methods.

The model of the passenger arrival time at the transit stop is based on the results of a questionnaire survey of home based work trips observed in Kharkov. The early time of an exit from home is determined once for each passenger. The early exit time, in accordance with the survey results are normally distributed with the expectation equal to 7:20 and the standard deviation equals 45 min.

The actual time of each exit from home is determined in the experiment using the deviation from the early time. The actual time deviation is exponentially distributed with parameter |= 0.137 according to the same survey.

The experiment is planned as follows: for each headway values is carried out 300 calculations. Every calculationgivesone random value of the passenger waiting time. The results are compared with the theoretical expectation of the waiting time and the divergences of the experimental values from the theoretical expectationare calculated. The expectation and standard deviation for the obtained divergences is estimated, as well as their compliance withthe normal distribution. The hypothesis of the analytical model compliance withthe experiment results is rejected in two cases:

- if the absolute value of the average divergence exceeds 5% of the theoretical value of the average waiting time;
- if the distribution of the obtained divergences is not normal.

The test of the simulation model on the best operation technology with equal headways and without deviation from the scheduled arrival time at the stops became the first step of the verification, tab. 1.

Tab. 1. The results of experimental verification of the analytical models of waiting time {min.}

The variant of the TU operation	Mean headway	Expectation value of waiting time		The experimental divergence from theoretical expectation	
		Analytical model	Experiment	Average	Relative, %
The factual headway is constant	1	1	0,998	-0,002	0,20
	2	2,5	2,504	0,004	0,16
	5	5	4,975	-0,025	0,50
The factual headway is constant,thereare random deviations from the scheduled arrival time	1	1,05	1,049	-0,001	0,10
	2	2,63	2,627	-0,003	0,11
	5	5,28	5,313	0,033	0,63
In the schedule there are two headways	1	1,03	1,022	-0,008	0,78
	2	2,6	2,606	0,006	0,23
	5	5,2	5,271	0,071	1,37

TU departs from the initial stop point while the passenger quantity come up to some level	1	1,1	1,098	-0,002	0,18
	2	2,75	2,747	-0,003	0,11
	5	5,5	5,443	-0,057	1,04
TU departs from the initial stop point while the passenger quantity come up to some level or certain time interval is elapsed	1	1,08	1,076	-0,004	0,37
	2	2,63	2,623	-0,007	0,27
	5	5,28	5,333	0,053	1,00
TU departs from the initial stop point in a random sequence	1	1,96	1,947	-0,013	0,66
	2	4,8	4,808	0,008	0,17
	5	9,23	9,261	0,031	0,34

The verification of the simulation model with the best operationtechnology showed that experimental model leads to the estimate of the mean waiting time which is similar to the analytical expectation and the divergence distribution is normal. It has created the basis for an experimental evaluation of other analytical models of waiting time, tab. 1.

The divergence distribution for all the operation technologies is normal. The average divergence of experimental mean waiting time from the analytical values in most cases did not exceed 1%.

The results indicate sufficient accuracy of the analytical models and their usefulness to assess passenger waiting time on the transit stops.

5. Summary

Analytical models of the waiting time expectation were derived for six variants of the TU operation on the transit routes.After experimental verification all models confirm a possibility of their application with the quality assessment of the line schedules in transit system. The models make an upper estimate of the average waiting time and provide a possibilityto assessthe line schedulesconsistency at each transit stop.

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