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ELECTRONIC REALIZATIONS OF FRACTIONAL-ORDER ELEMENTS: II. SIMULATION STUDIES

The paper presents simulation studies on the synthesis of new fractional-order elements, using classic fractional-order components, such as supercapacitors and real coils with ferromagnetic cores. These new fractional-order elements have been realized using a generalized impedance converter GIC. Firstly, a new method for fractional-order elements modeling in PSpice has been described, which involves the use of controlled voltage E or current G sources, defined in Analog Behavioral Modeling (ABM). An equivalence of the classic and proposed description of the fractional-order capacitor have been presented. For the GIC structure, two cases of the have been analyzed in the PSpice: first, with models of ideal operational amplifiers, modeled as controlled voltage sources, and the second with models of the real operational amplifiers, of type OPA549. The obtained frequency characteristics of an exemplary GIC structure with one fractional-order element show big similarity for both types of models. The impact of different values of the parameter α and the type of the used real amplifier on the solution has been analyzed too.

KEYWORDS: fractional-order inductance L_β and capacitance C_α , generalized impedance converter (GIC), fractional-order element

1. INTRODUCTION

The paper is a continuation of studies conducted in [1], on the possibilities and methods of synthesis of new fractional-order electronic components. Researches on the realization of fractional-order coils using the generalized impedance converter GIC have been carried out in [2]. It has been proved in the paper [2] that if one or two impedances in the GIC structure are of fractional order $\alpha, \beta \in \langle 0, 1 \rangle$, then the equivalent converter impedance seen from the output terminals is of fractional order too. The order is within the range $\langle -5, 5 \rangle$.

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It should be noted, that the fractional exponents α , β (see formulae (1), (2) from [1]) are not arbitrary and their values result from the physical features of the materials used for the construction of the fractional-order elements. These exponents generally have the values placed in the range of $\langle 0,1 \rangle$. Connections of such converters (e.g. in cascade) allow to obtain even higher orders of the equivalent impedances seen from the system terminals. However, this issue requires further studies. Simulation studies in Mathematica on the realization of fractional-order inductance using the GIC converter have been performed in [2] as well.

The paper is a continuation of the work [1] and it consists of:

- proposition of modeling the fractional-order elements using the generalized controlled sources (in ABM mode) in PSpice program,
- simulation studies of generalized impedance converters containing a single fractional-order element $\alpha \in \langle 0,1 \rangle$. These studies were conducted in two ways. First of them concerned the modeling of converter GIC using ideal operational amplifiers, and the other with the use of real amplifiers,
- conclusions resulting from simulation studies, which will provide guidance in the construction of physical model of the GIC converter. It can be used for construction of the two-terminal impedance of fractional order for a wide range of exponent changes.

2. MODEL OF THE FRACTIONAL-ORDER ELEMENT IN PSpICE

Fractional-order passive elements can be modeled in PSpice by using the Analog Behavioral Modeling mode. In order to check the feasibility of PSpice program, for modeling circuits with fractional-order elements, a simple comparative analysis for a series RC circuit has been performed. In the first case, the capacitor C is modeled as a classic capacitor, whereas in the second case, as a fractional-order capacitor C_α . Both analyzed models have been presented in Figs. 1–2. In the second case, the fractional-order capacitor C_α has been modeled as a controlled voltage source E1. In order to compare both circuits, a time-domain analysis .TRAN has been performed for the voltage waveforms across the classic capacitor C1 and the voltage of the controlled source E1. The source Vx with zero-voltage amplitude has been added to model the voltage-current transfer function. Simulations have been made for the following circuit parameters: the voltage sources $V_1, V_2 = 1$ V, the resistances $R_1, R_2 = 10^6$ Ω and a capacitor of the capacitance $C_1 = 1$ μF .

The source code in PSpice A/D simulating both circuits is presented as follows:

```

Simple RC circuit
c1 2 0 1u
r2 1 2 1meg
V2 1 0 dc 1
.TRAN 0.01 5 uic
.options opts
.probe
.end
xxxxx
E1 5 0 LAPLACE {i(v1)} = {(-1/s)*1meg}
r1 4 5 1meg
Vx 4 3 dc 0
V1 3 0 dc 1
.TRAN 0.01 5 uic
.options opts
.probe
.end

```

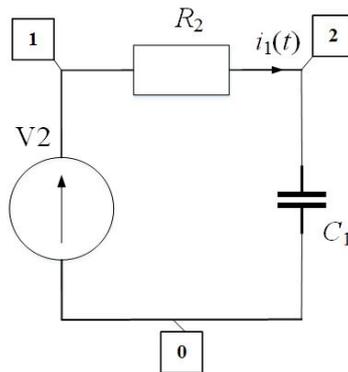


Fig. 1. Classic simple RC branch circuit modeled in PSpice

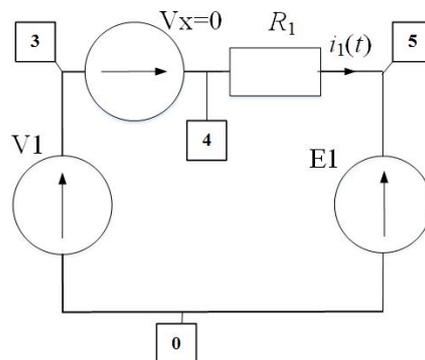


Fig. 2. Simple RC_α branch circuit with fractional-order capacitor modeled in PSpice

The waveforms of the voltage across the capacitor and voltage of the controlled source E1 have been shown in Fig. 3.

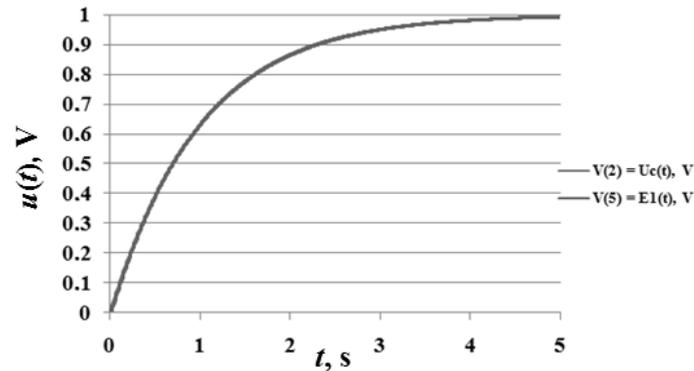


Fig. 3. Comparison of the voltage on the classic capacitor and the fractional-order element, modeled as the controlled source E1 (for $\alpha = 1$)

As it can be noted from Fig. 3, the waveforms of both voltages are identical, which means that circuits from Figs. 1 - 2 can be treated as equivalent. Similarly, by writing the voltage source E1, which is controlled by the current flowing in the branch where the considered source is located, a fractional-order coil can be modeled too. The schematic equivalent circuits in ABM mode of PSpice A/D program have been indicated in Fig. 4.

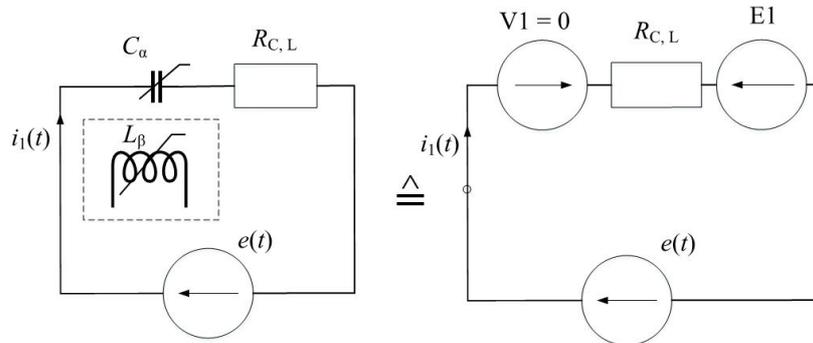


Fig. 4. Equivalent description of the circuits with reactance elements, which can be developed into the description of fractional-order elements

Justification for the use of this type of modeling of fractional-order elements in PSpice program is going to be explained in next sections of the paper. Modeled impedances of fractional-order capacitors and inductors in this way have been used for modeling of the new fractional-order elements, which α and

β coefficients may be placed beyond the scope $\langle 0,1 \rangle$. It has been obtained by connecting one or two original fractional-order elements to the generalized impedance converter GIC. The results of the conducted synthesis are going to be presented in the next sections of the paper.

3. MODEL OF THE SYSTEM WITH IDEAL OPERATIONAL AMPLIFIER

The model of the system realization of fractional-order input impedance $Z_{in}(s)$ using the generalized impedance converter is presented in Fig. 1. Its parameters values have been selected as in the previous example: $C_\alpha = 15 \text{ mF/s}^{(1-\alpha)}$, $L_3 = 15 \text{ mH}$, $R_1, R_2, R_4 = 10 \Omega$. The load impedance an ideal supercapacitor (excluding its internal series resistance ESR) and the operational amplifiers have been modeled in PSpice as ideal, controlled voltage sources, with the gain factor $\beta = 10^6$ and $R_5, R_6 = 10 \text{ M}\Omega$

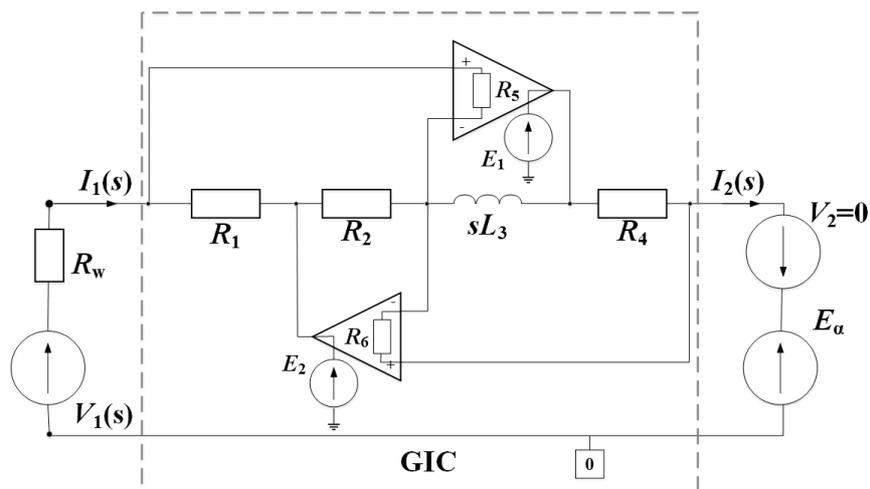


Fig. 5. Model of the generalized impedance converter GIC with one load fractional-order impedance

The results of simulation studies performed in PSpice of the input impedance are presented in Figs. 6–7.

Figs. 6–7 show that the shape of the input impedance module and phase are the same as the shapes of the characteristics obtained in Mathematica in [1]. Module and phase characteristics have the same values and shapes in both types of the simulations – the code and circuit-based simulations. For frequency characteristics studies, the difference for small values of the parameter α in case

of the current measurement does not appear, as it appears in case of simulations performed in time domain [3–4]. Results for $\alpha = 1$ coincide with the results for conventionally modeled integer-order capacitor of the same capacitance.

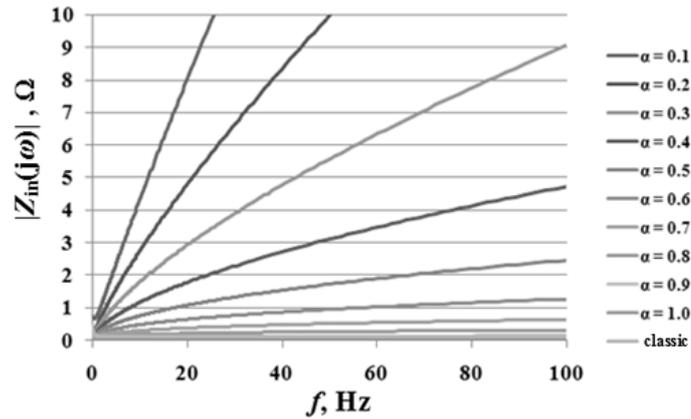


Fig. 6. Characteristics of the input impedance $|Z_{in}(j\omega)|$ module, obtained in PSpice A/D

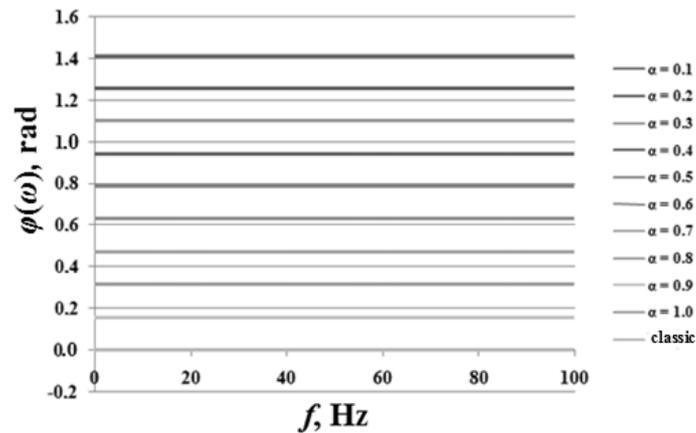


Fig. 7. Characteristics of the input impedance $\phi(\omega)$ phase, obtained in PSpice A/D

4. MODEL OF THE SYSTEM WITH REAL OPERATIONAL AMPLIFIER

Based on the previous studies, simulation studies have been performed with models of the real operational amplifiers. A model of the operational amplifier OPA549 [5] has been chosen for analysis. It is a high-voltage, high-current operational amplifier for a wide spectrum of use. The two amplifiers have been

supplied symmetrically by a DC voltage source $U = -15 \dots 15$ V. Simulations have been performed for the same elements included in the structure of the generalized impedance converter, as in the previous case and for the same load impedance. The analyzed system with real operational amplifiers is presented in Fig. 8.

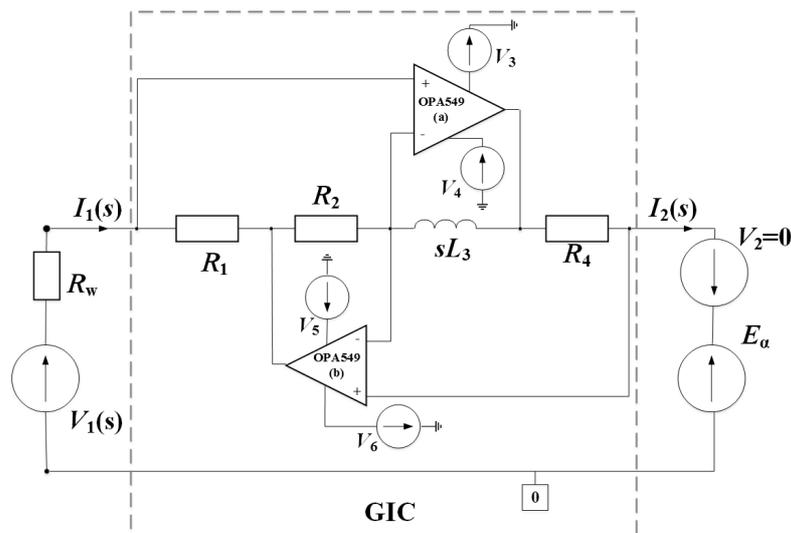


Fig. 8. Model of the generalized impedance converter GIC with real operational amplifiers modeled in PSpice

The results of the conducted simulation studies in PSpice of the input impedance are presented in Figs. 9–10.

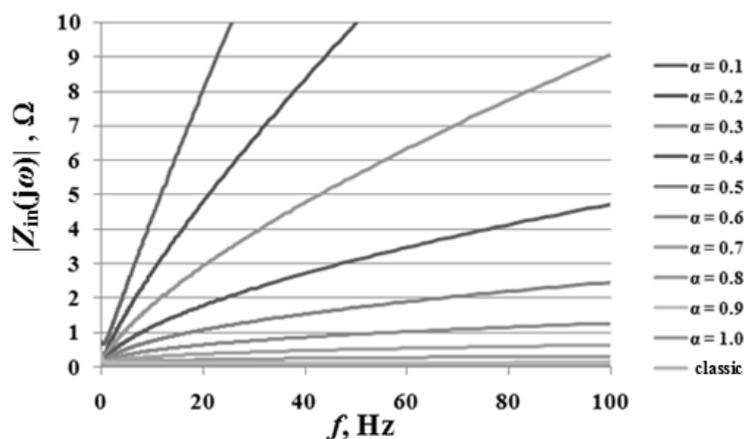


Fig. 9. Characteristics of the input impedance $|Z_{in}(j\omega)|$ module, obtained in PSpice A/D

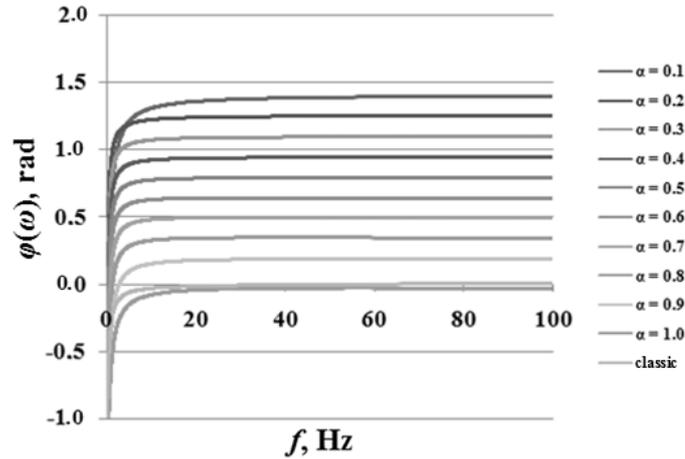


Fig. 10. Characteristics of the input impedance $\varphi(\omega)$ phase, obtained in PSpice A/D

Figs. 9–10 show that the shape of the input impedance module and phase are almost the same as the shapes of the characteristics obtained in previous section, for models of the ideal operational amplifiers. For very small frequencies, below about 1 Hz, the frequency characteristics in real system do not coincide with characteristics obtained for real amplifiers. It is due to the fact, that for low frequencies the offset voltage of the amplifier plays a significant role, because it does not depend on frequency. Small value of the reactance $Z_3(s)$ causes small voltage across the inductance for frequencies $\rightarrow 0$ Hz – less than the offset voltage - and consequently a relatively significant impact of the offset voltage in the system.

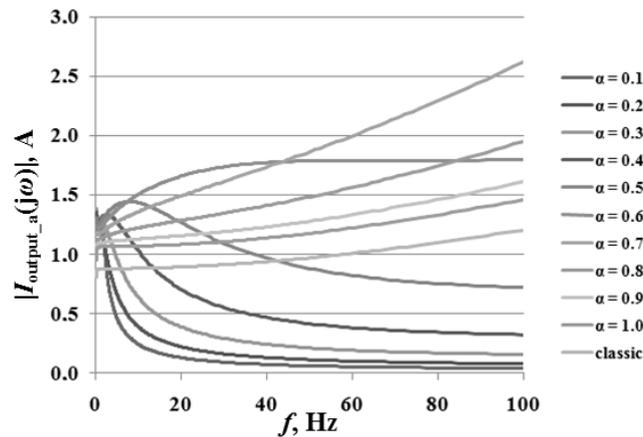


Fig. 11. Characteristics of the output current of the 1-st operational amplifier OPA549 (a)

For design purposes, the characteristics of the output currents and the active power issued by the two operational amplifiers OPA549 (a) and (b) have also been simulated. The frequency characteristics of the output currents have been presented in Figs. 11–12 and the characteristics of active power have been shown in Figs. 13–14.

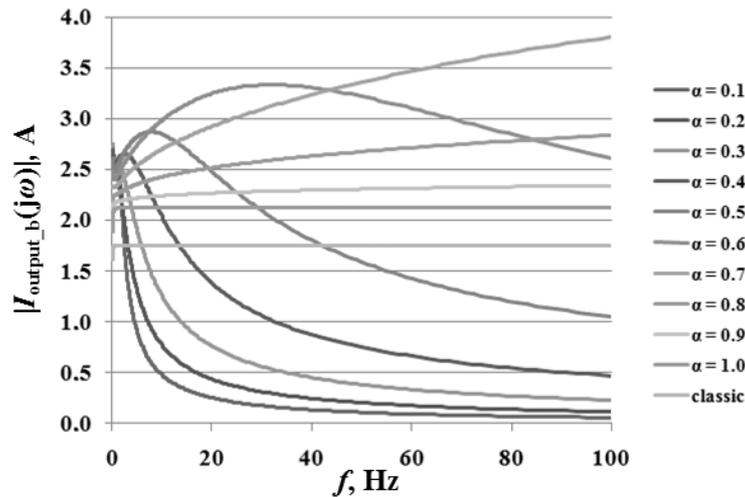


Fig. 12. Characteristics of the output current of the 2-nd operational amplifier OPA549 (b)

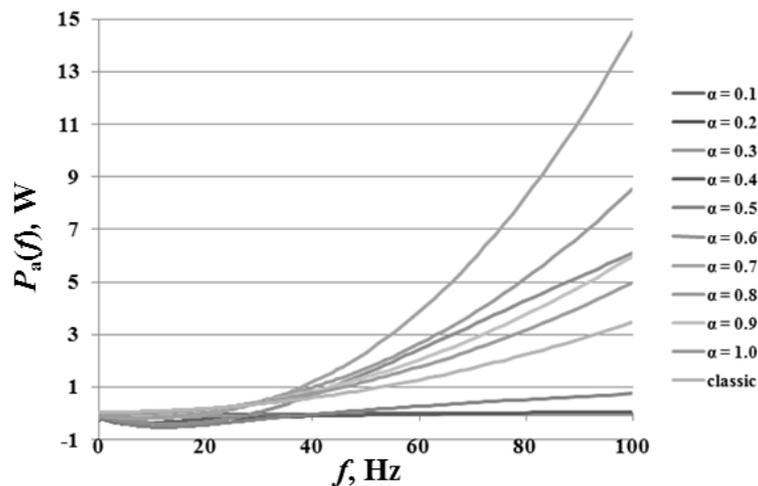


Fig. 13. Characteristics of the issued active power P_a by the 1-st operational amplifier OPA549 (a)

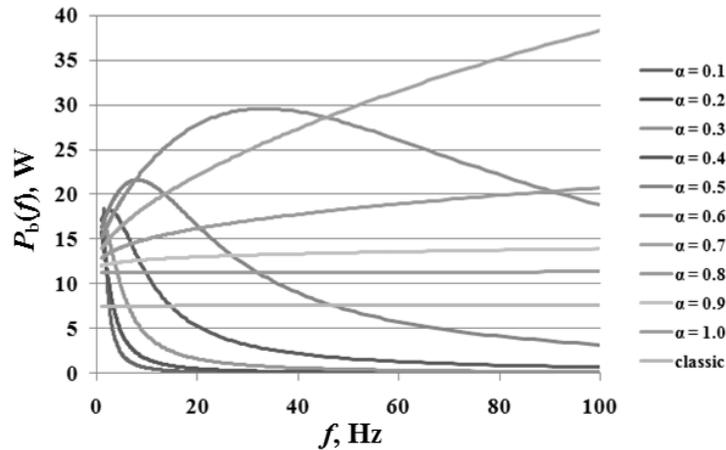


Fig. 14. Characteristics of the issued active power P_b by the 2-nd operational amplifier OPA549 (b)

It can be noted, that for both operational amplifiers, the output current does not exceed 5 A, so it has always the value under the maximum permitted value, of 10 A. Generally, the active power on both amplifiers does not exceed 40 W. The active power is bigger on the second operational amplifier.

5. SUMMARY

The paper presents simulation studies on the realization of new fractional-order elements, using classic, original fractional-order components, such as supercapacitors and real coils with ferromagnetic cores. For realization of the new fractional-order elements, an electronic active system has been used - the generalized impedance converter GIC. Simulation studies have been performed in PSpice A/D program. Different cases and types of modeling have been taken into consideration. The paper describes a method of fractional-order element modeling in PSpice program, writing in the Analog Behavioral Modeling (ABM) mode. It involves the definition of fractional-order element in the form of voltage E or current G controlled source, which is controlled by another voltage or current. These sources model the transfer function of the output and input signals. As it turned out, there is an equivalence of the waveforms and frequency characteristics for circuits with classic and fractional-order capacitors. The obtained input impedance characteristics for exemplary configuration of elements with one element of fractional order - the capacitor C_α , have been compared. Simulations have been performed for two types of models of the operational amplifiers - ideal and real. Firstly, the amplifiers have

been modeled as an ideal controlled voltage source. Real amplifier type OPA549 has been chosen for simulations, due to its higher current and voltage capacity than e.g. the classic popular amplifier μ A741. Small values of the circuit parameters have been selected, therefore as it turned out, currents flowing in the considered system are of several amperes (see Figs. 11-12). This explains the need for power amplifiers use in the system. Characteristics of both analysis are very similar, differing slightly for very low frequencies (below about 1 Hz). For such low frequency the internal offset voltage plays a role in a circuit with real amplifier, because of the small value of the selected reactance $Z_3(s)$. Two reactances in the circuit cause that it works as intended only for a limited range of frequencies. Next step of the considered problem is the experimental verification of the obtained results.

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