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ANALYTICAL AND NUMERICAL SOLVING OF LINEAR NON-HOMOGENEOUS DIFFERENTIAL EQUATIONS OF THE FIRST-ORDER WITH CONSTANT COEFFICIENTS BY USING CONSTANT VARIATION METHOD AND APPLICATION OF *MATHEMATICA* PROGRAM

Abstract

Introduction and aim: The paper presents the analytical and numerical algorithm of solving linear non-homogeneous equations of the first order with constant coefficients. The aim of the work is to show the algorithms for solving equations both analytically and numerically. The additional aim is to show numerical algorithms and graphical interpretation of solutions.
Material and methods: For selected equations, from the subject literature, constant variation method has been presented.
Results: The paper presents the selected linear non-homogeneous equations of the first order with constant coefficients containing exponential, polynomial and trigonometric functions.
Conclusion: Taking into account the constant variation method it is possible to solve the first order linear non-homogeneous differential equations. However, using the *Mathematica* program for numerical solution, you can quickly get a solution and create a graphical interpretation of solutions.

Keywords: Ordinary differential equations, linear, homogeneous, the first order, the constant coefficients, variation constant method, solutions, analytical and numerical, *Mathematica*.
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ROZWIĄZYWANIE ANALITYCZNO-NUMERYCZNE LINIOWYCH NIEJEDNORODNYCH RÓWNAŃ RÓŻNICZKOWYCH PIERWSZEGO RZĘDU O STAŁYCH WSPÓŁCZYNNIKACH PRZY UŻYCIU METODY WARIACJI STAŁEJ I ZASTOSOWANIEM PROGRAMU MATHEMATICA

Streszczenie

Wstęp i cel: *W pracy pokazano algorytmy analityczny i numeryczny rozwiązywania równań różniczkowych liniowych niejednorodnych pierwszego rzędu o stałych współczynnikach. Celem pracy jest pokazanie algorytmu rozwiązywania równań zarówno sposobem analitycznym jak i numerycznych. Ponadto również dodatkowym celem jest pokazanie algorytmów numerycznych oraz interpretacji graficznej rozwiązań.*
Materiał i metody: *Dla wybranych równań, z literatury przedmiotu, zastosowano metodę wariacji stałej.*
Wyniki: *W pracy opracowano wybrane równania różniczkowe liniowe niejednorodne pierwszego rzędu o stałych współczynnikach zawierających funkcje wykładnicze, wielomianowe i trygonometryczne.*
Wniosek: *Stosując metodę uzmienniania stałej jest możliwe rozwiązywanie równań różniczkowych liniowych niejednorodnych pierwszego rzędu o stałych współczynnikach. Natomiast wykorzystując do numerycznego rozwiązywania program Mathematica można szybko uzyskać rozwiązanie oraz sporządzić interpreatcję graficzną rozwiązań.*

Słowa kluczowe: *Równania różniczkowe zwyczajne, liniowe, niejednorodne, pierwszego rzędu, stałe współczynniki, metoda wariacji stałej, rozwiązania, analityczne i numeryczne, Mathematica.*
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1. Theoretical introduction

Definition 1.

The differential non-homogeneous linear equation of the first order with constant coefficients has the following form:

$$\frac{dy}{dx} + py = q(x) \tag{1}$$

where p is the real constant and q (x) is a continuous function in a certain interval (a, b) [2].

Definition 2.

The differential homogeneous linear equation of the first order with constant coefficients has the following form:

$$\frac{dy}{dx} + py = 0 \tag{2}$$

where p is the real constant [4].

Theorem 1.

The general solution of the non-homogeneous differential equation (1) is the sum of the general solution of the homogeneous differential equation (2) and the particular solution of the non-homogeneous differential equation (1) [6]-[8].

The general solution of the homogeneous equation (2), after variables separation, is obtained from the following equation [9]:

$$\frac{dy}{y} = -pdx. \tag{3}$$

The equation (3) we integrate on both sides respectively of the variables y and x:

$$\int \frac{dy}{y} = -p\int dx. \tag{4}$$

Hence, after integration

$$\ln\left|\frac{y}{C}\right| = -px. \tag{5}$$

Using the definition of a logarithm, we get:

$$\frac{y}{C} = e^{-px}. \tag{6}$$

Thus, the general solution of the homogeneous equation (2) has the form:

$$y(x) \equiv y_1(x) = Ce^{-px} \tag{7}$$

where C is the real constant [11].

The particular solution of the non-homogeneous equation (1) is found by the constant variation method. Therefore:

$$y(x) = C(x)e^{-px}. \tag{8}$$

Both sides of the above equality we differentiate relative to x variable:

$$\frac{dy}{dx}=\frac{dC}{dx}e^{-px}-pC(x)e^{-px}. \tag{9}$$

After substituting functions (9) and (8) in equation (1) we get:

$$\frac{dC}{dx}e^{-px}-pC(x)e^{-px}+pC(x)e^{-px}=g(x). \tag{10}$$

Hence

$$\frac{dC}{dx}=g(x)e^{px}. \tag{11}$$

After integration the equation (11) relative to x variable, we get:

$$C(x)=\int g(x)e^{px}dx. \tag{12}$$

Therefore, the particulary solution of non-homogeneous equation (1) has the form:

$$y(x)\equiv y_2(x)=e^{-px}\int g(x)e^{px}dx. \tag{13}$$

Finally, the general solution of the non-homogeneous equation (1) has the following form:

$$y(x)\equiv y_1(x)+y_2(x)=Ce^{-px}+e^{-px}\int g(x)e^{px}dx. \tag{14}$$

where C is the real constant [2], [12].

2. Analytical and numerical solving of the first order linear non-homogeneous differential equations using the constant variation method

<u>Example 1</u>.

Let us consider the following equation:

$$\frac{dy}{dx}+14y=30e^{x}. \tag{15}$$

The homogeneous equation has the form:

$$\frac{dy}{dx}+14y=0. \tag{16}$$

• Analytical solution

The general solution of the equation (16) is obtained from the following equation:

$$\frac{dy}{dx}=-14y. \tag{17}$$

The equation (17) we integrate on both sides respectively of the variables y and x:

$$\int\frac{dy}{y}=-14\int dx. \tag{18}$$

Hence, after integration we have:

$$\ln\left|\frac{y}{C}\right| = -14x. \tag{19}$$

Using the definition of a logarithm, we get:

$$\frac{y}{C} = e^{-14x}. \tag{20}$$

Thus, the general solution of the homogeneous equation (16) has the form:

$$y(x) = Ce^{-14x} \tag{21}$$

where C is the real constant.

The particular solution of the non-homogeneous equation (15) is found by the constant variation method. Therefore:

$$y(x) = C(x)e^{-14x}. \tag{22}$$

Both sides of the above equality we differentiate relative to x variable:

$$\frac{dy}{dx} = \frac{dC}{dx}e^{-14x} - pC(x)e^{-14x}. \tag{23}$$

After substituting functions (22) and (23) into equation (15) we get:

$$\frac{dC}{dx}e^{-14x} - pC(x)e^{-14x} + pC(x)e^{-14x} = 30e^{x}. \tag{24}$$

Hence

$$\frac{dC}{dx} = 30e^{15x}. \tag{25}$$

We integrate the equation (25) and we get:

$$\int\frac{dC}{dx}dx = 30\int e^{15x}dx. \tag{26}$$

After integration relative to x variable, we get:

$$C(x) = 2e^{15x}. \tag{27}$$

Therefore, the particulary solution of non-homogeneous equation (15) has the form:

$$y(x) = 2e^{x}. \tag{28}$$

Finally, the general solution of the non-homogeneous equation (15) has the following form:

$$y(x) = Ce^{-15x} + 2e^{x}. \tag{29}$$

where C is the real constant.

• Numerical solution

For numerical analysis we take into account the solution (29) where the constant C=3, [1]. Thus we have:

$$y(x) = 3e^{-15x} + 2e^{x}. \tag{30}$$

Mathematica 7.0 - Program 1 [1], [3], [5], [10]

In[1]:= DSolve[y'[x] == -14*y[x] + 30*Exp[x], y, x]

DSolve[y'[x] == -14*y[x] + 30*Exp[x], y, x] /. C[1] → 3

Plot[Evaluate[y[x] /. %], {x, -0.5, 2.5}, Background → RGBColor[0.9, 1, 1],

PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.009]}}, PlotRange → {0, 100},

AxesOrigin → {0,0}, AxesStyle → Thickness[0.004], AxesLabel → {"x","y"},

GridLines → Automatic, TextStyle → {FontFamily → "Arial", FonySize → 8}]

Out[1]= {{ y→Function[{x}, $2e^{x} + e^{-14x}$C[1]] }}

Out[2]= {{ y→Function[{x}, $2e^{x} + e^{-14x}$ 3] }}

Out[3]= = Graphics =

Fig. 1. The graph of the function (29) for constant C = 3 as a solution of the first order linear non-homogeneous differential equation (15)

Source: Program and graph elaborated by the Authors

<u>Example 2</u>. Let us consider the following equation:

$$\frac{dy}{dx} + 6y = xe^{x}. \tag{31}$$

The homogeneous equation has form:

$$\frac{dy}{dx} + 6y = 0. \tag{32}$$

• **Analytical solution**

The general solution of the equation (32) is obtained from the following equation:

$$\frac{dy}{dx} = -6y. \tag{33}$$

The equation (33) we integrate on both sides respectively of the variables y and x:

$$\int \frac{dy}{y} = -6\int dx. \tag{34}$$

Hence, after integration we have:

$$\ln\left|\frac{y}{C}\right| = -6x. \tag{35}$$

Using the definition of a logarithm, we get:

$$\frac{y}{C} = e^{-6x}. \tag{36}$$

Thus, the general solution of the homogeneous equation (32) has the form:

$$y(x) = Ce^{-6x} \tag{37}$$

where C is the real constant.

The particular solution of the non-homogeneous equation (31) is found by the constant variation method.

Therefore:

$$y(x) = C(x)e^{-6x}. \tag{38}$$

Both sides of the above equality we differentiate relative to x variable:

$$\frac{dy}{dx} = \frac{dC}{dx}e^{-6x} - pC(x)e^{-6x}. \tag{39}$$

After substituting functions (38) and (39) in equation (31) we get:

$$\frac{dC}{dx}e^{-6x} - pC(x)e^{-6x} + pC(x)e^{-6x} = xe^{x}. \tag{40}$$

Hence

$$\frac{dC}{dx} = xe^{7x}. \tag{41}$$

We integrate the equation (41) and we get:

$$\int \frac{dC}{dx}dx = \int xe^{7x}dx. \tag{42}$$

We integrate (42) the right-hand side of the equation using integration by parts method:

$$\int xe^{7x}dx = \left\langle \begin{matrix} u = x \\ v' = e^{7x} \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = \int e^{7x}dx = \frac{1}{7}e^{7x} \end{matrix} \right\rangle = \frac{x}{7}e^{7x} - \frac{1}{7}\int e^{7x}dx = \frac{x}{7}e^{7x} - \frac{1}{49}e^{7x}. \quad (43)$$

After integration relative to x variable, we get:

$$C(x) = \left(\frac{x}{7} - \frac{1}{49}\right)e^{7x}. \quad (44)$$

Therefore, the particulary solution of non-homogeneous equation (31) has the form:

$$y(x) = e^{x}\left(-\frac{1}{47} + \frac{x}{7}\right). \quad (45)$$

The general solution of the non-homogeneous equation (31) has the following form:

$$y(x) = Ce^{-6x} + e^{x}\left(-\frac{1}{47} + \frac{x}{7}\right). \quad (46)$$

where C is the real constant.

• Numerical solution

For numerical analysis we take into account both the solution (46) and the following equation where the constant C = –2, [3]:

$$y(x) = (-2)e^{-6x} + e^{x}\left(-\frac{1}{47} + \frac{x}{7}\right). \quad (47)$$

Mathematica 7.0 - Program 2 [1], [3], [5], [10]

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In[1]:= DSolve[y'[x] == -6*y[x] + x*Exp[x], y, x]
        DSolve[y'[x] == -6*y[x] + x*Exp[x], y, x] /. C[1] → -2
        Plot[Evaluate[y[x] /. %], {x, -0.5, 2}, Background → RGBColor[0.9, 1, 1],
        PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.009]}}, PlotRange → {-4, 2},
        AxesOrigin → {0,0}, AxesStyle → Thickness[0.004], AxesLabel → {"x","y"},
        GridLines → Automatic, TextStyle → {FontFamily → "Arial", FonySize → 8}]
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Out[1]= {{ y→Function[{x}, $e^{x}\left(-\frac{1}{49} + \frac{x}{7}\right) + e^{-6x}$C[1]] }}

Out[2]= {{ y→Function[{x}, $e^{x}\left(-\frac{1}{49} + \frac{x}{7}\right) + e^{-6x}(-2)$] }}

Out[3]= = Graphics =

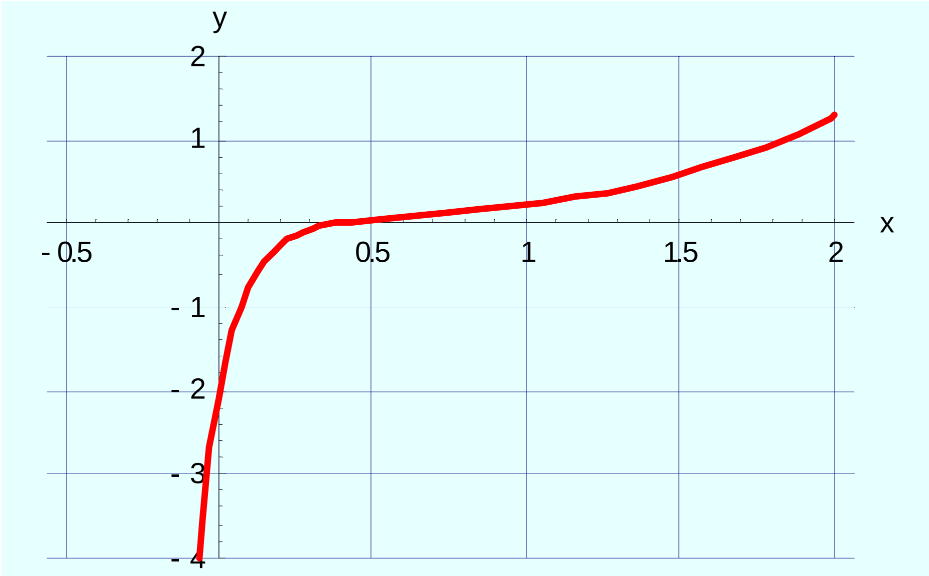

Fig. 2. The graph of the function (45) for constant C = –2 as a solution of the first order linear non-homogeneous differential equation (31)

Source: Program and graph elaborated by the Authors

Example 3. Let us consider the following equation:

$$\frac{dy}{dx} + 5y = x^3 + x. \tag{48}$$

The homogeneous equation has form:

$$\frac{dy}{dx} + 5y = 0. \tag{49}$$

- **Analytical solution**

The general solution of the equation (49) is obtained from the following equation:

$$\frac{dy}{dx} = -5y. \tag{50}$$

The equation (49) we integrate on both sides respectively of the variables y and x:

$$\int \frac{dy}{y} = -5\int dx. \tag{51}$$

Hence, after integration we have:

$$\ln\left|\frac{y}{C}\right| = -5x. \tag{52}$$

Using the definition of a logarithm, we get:

$$\frac{y}{C} = e^{-5x}. \tag{53}$$

Thus, the general solution of the homogeneous equation (49) has the form:

$$y(x) = Ce^{-5x} \tag{54}$$

where C is the real constant.

The particular solution of the non-homogeneous equation (48) is found by the constant variation method. Therefore:

$$y(x)=C(x)e^{-5x}. \tag{55}$$

Both sides of the above equality we differentiate relative to x variable:

$$\frac{dy}{dx}=\frac{dC}{dx}e^{-5x}-pC(x)e^{-5x}. \tag{56}$$

After substituting functions (55) and (56) in equation (48) we get:

$$\frac{dC}{dx}e^{-5x}-pC(x)e^{-5x}+pC(x)e^{-5x}=x^3+x. \tag{57}$$

Hence

$$\frac{dC}{dx}=(x^3+x)e^{5x}. \tag{58}$$

We integrate the equation (58) and we get:

$$\int\frac{dC}{dx}dx=\int(x^3+x)e^{5x}dx. \tag{59}$$

We integrate the right-hand side of the equation (59) using thrice integration by parts method:

$$\begin{aligned}
&\int(x^3+x)e^{5x}dx=\left\langle \begin{matrix} u=x^3+x \\ v'=e^{5x}\end{matrix}\middle| \begin{matrix} u'=3x^2+1 \\ v=\int e^{5x}dx=\frac{1}{5}e^{5x}\end{matrix}\right\rangle=\frac{1}{5}e^{5x}(x^3+x)-\frac{1}{5}\int(3x^2+1)e^{5x}dx= \\
&=\left\langle \begin{matrix} u=3x^2+1 \\ v'=e^{5x}\end{matrix}\middle| \begin{matrix} u'=6x \\ v=\int e^{5x}dx=\frac{1}{5}e^{5x}\end{matrix}\right\rangle=\frac{1}{5}e^{5x}(x+x^3)-\frac{1}{5}\left[\frac{1}{5}(3x^2+1)e^{5x}-\frac{6}{5}\int xe^{5x}dx\right]= \\
&=\frac{1}{5}e^{5x}(x+x^3)-\frac{1}{25}(3x^2+1)e^{5x}+\frac{6}{25}\int xe^{5x}dx=\left\langle \begin{matrix} u=x \\ v'=e^{5x}\end{matrix}\middle| \begin{matrix} u'=1 \\ v=\int e^{5x}dx=\frac{1}{5}e^{5x}\end{matrix}\right\rangle= \\
&=\frac{1}{5}e^{5x}(x+x^3)-\frac{1}{25}(3x^2+1)e^{5x}+\frac{6}{25}\left(\frac{1}{5}xe^{5x}-\frac{1}{5}\int e^{5x}dx\right)= \\
&=\frac{1}{5}e^{5x}(x+x^3)-\frac{1}{25}(3x^2+1)e^{5x}+\frac{6}{125}xe^{5x}-\frac{6}{125}\int e^{5x}dx= \\
&=\frac{1}{5}e^{5x}(x+x^3)-\frac{1}{25}(3x^2+1)e^{5x}+\frac{6}{125}xe^{5x}-\frac{6}{625}e^{5x}=\left(\frac{x^3}{5}-\frac{3x^2}{25}+\frac{31x}{125}-\frac{31}{625}\right)e^{5x}
\end{aligned} \tag{60}$$

After integration relative to x variable, we get:

$$C(x)=\left(\frac{x^3}{5}-\frac{3x^2}{25}+\frac{31x}{125}-\frac{31}{625}\right)e^{5x}. \tag{61}$$

Therefore, the particulary solution of non-homogeneous equation (48) has the form:

$$y(x)=\frac{x^3}{5}-\frac{3x^2}{25}+\frac{31x}{125}-\frac{31}{625}. \tag{62}$$

The general solution of the non-homogeneous equation (48) has the following form:

$$y(x) = Ce^{-5x} + \frac{x^3}{5} - \frac{3x^2}{25} + \frac{31x}{125} - \frac{31}{625}. \quad (63)$$

where C is the real constant.

• Numerical solution

For numerical analysis we take into account both the solution (63) and the following equation where the constant C = 30, [5]:

$$y(x) = 30e^{-5x} + \frac{x^3}{5} - \frac{3x^2}{25} + \frac{31x}{125} - \frac{31}{625}. \quad (64)$$

Mathematica 7.0 - Program 3 [1], [3], [5], [10]

In[1]:= DSolve[y'[x] == -5*y[x] + x+x^3, y, x]

DSolve[y'[x] == -5*y[x] + x+x^3, y, x] /. C[1] → 30

Plot[Evaluate[y[x] /. %], {x, 0, 3}, Background → RGBColor[0.9, 1, 1],

PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.009]}}, PlotRange → {0, 6},

AxesOrigin → {0,0}, AxesStyle → Thickness[0.004], AxesLabel → {"x","y"},

GridLines → Automatic, TextStyle → {FontFamily → "Arial", FonySize → 8}]

Out[1]= $\{\{ y \to \text{Function}[\{x\}, -\frac{31}{625} + \frac{31x}{125} - \frac{3x^2}{25} + \frac{x^3}{5} + e^{-5x}C[1]]\}\}$

Out[2]= $\{\{ y \to \text{Function}[\{x\}, -\frac{31}{625} + \frac{31x}{125} - \frac{3x^2}{25} + \frac{x^3}{5} + e^{-5x}30]\}\}$

Out[3]= = Graphics =

Fig. 3. The graph of the function (70) for constant C = 30 as a solution of the first order linear non-homogeneous differential equation (54)

Source: Program and graph elaborated by the Authors

Example 4. Let us consider the following equation:

$$\frac{dy}{dx} + 6y = 12\sin(x). \tag{65}$$

The homogeneous equation has form:

$$\frac{dy}{dx} + 6y = 0. \tag{66}$$

- **Analytical solution**

The general solution of the equation (66) is obtained from the following equation:

$$\frac{dy}{dx} = -6y. \tag{67}$$

The equation (66) we integrate on both sides respectively of the variables y and x:

$$\int \frac{dy}{y} = -6\int dx. \tag{68}$$

Hence, after integration we have:

$$\ln\left|\frac{y}{C}\right| = -6x. \tag{69}$$

Using the definition of a logarithm, we get:

$$\frac{y}{C} = e^{-6x}. \tag{70}$$

Thus, the general solution of the homogeneous equation (66) has the form:

$$y(x) = Ce^{-6x} \tag{71}$$

where C is the real constant.

The particular solution of the non-homogeneous equation (65) is found by the constant variation method.

Therefore:

$$y(x) = C(x)e^{-6x}. \tag{72}$$

Both sides of the above equality we differentiate relative to x variable:

$$\frac{dy}{dx} = \frac{dC}{dx}e^{-6x} - pC(x)e^{-6x}. \tag{73}$$

After substituting functions (72) and (73) in equation (65) we get:

$$\frac{dC}{dx}e^{-6x} - pC(x)e^{-6x} + pC(x)e^{-6x} = 12\sin(x). \tag{74}$$

Hence

$$\frac{dC}{dx} = 12e^{6x}\mathrm{Sin}(x). \tag{75}$$

We integrate the equation (75) and we get:

$$\int \frac{dC}{dx}dx = 12\int e^{6x}\sin(x)\,dx. \tag{76}$$

We integrate the right-hand side of the equation (76) using twice integration by parts method:

$$\begin{aligned}
&\int e^{6x}\sin(x)dx = \left\langle \begin{array}{c|c} u = e^{6x} & u' = 6e^{6x} \\ v' = \sin(x) & v = \int \sin(x)dx = -\cos(x) \end{array} \right\rangle = \\
&= -e^{6x}\cos(x) - 6\int e^{6x}\cos(x)dx = \\
&= \left\langle \begin{array}{c|c} u = e^{6x} & u' = 6e^{6x} \\ v' = \cos(x) & v = \int \cos(x)dx = \sin x \end{array} \right\rangle = \\
&= -e^{6x}\cos(x) - 6\left[e^{6x}\sin(x) + 6\int e^{6x}\sin(x)dx\right] = \\
&= -e^{6x}\cos(x) - 6e^{6x}\sin(x) - 36\int e^{6x}\sin(x)dx.
\end{aligned} \tag{77}$$

So we have:

$$\int e^{6x}\sin(x)dx = -\frac{12}{37}e^{6x}\cos(x) - \frac{72}{37}e^{6x}\sin(x). \tag{78}$$

The function C(x) has the following form:

$$C(x) = \left[-\frac{12}{37}\cos(x) - \frac{72}{37}\sin(x)\right]e^{6x}. \tag{79}$$

Therefore, the particulary solution of non-homogeneous equation (65) has the form:

$$y(x) = -\frac{12}{37}\cos(x) - \frac{72}{37}\sin(x). \tag{80}$$

The general solution of the non-homogeneous equation (65) has the following form:

$$y(x) = Ce^{-6x} - \frac{12}{37}\cos(x) - \frac{72}{37}\sin(x) \tag{81}$$

where C is the real constant.

• Numerical solution

For numerical analysia we take into account both the solution (81) and the following equation where the constant C = 4, [10]:

$$y(x) = 4e^{-6x} - \frac{12}{37}\cos(x) - \frac{72}{37}\sin(x). \tag{82}$$

Mathematica 7.0 - Program 4 [1], [3], [5], [10]

In[1]:= DSolve[y'[x] == -6*y[x] + 12*Sin[x], y, x]

DSolve[y'[x] == -6*y[x] + 12*Sin[x], y, x] /. C[1] → 4

Plot[Evaluate[y[x] /. %], {x, -0.5, 3}, Background → RGBColor[0.9, 1, 1],

PlotStyle → {{RGBColor[1, 0, 0], Thickness[0.009]}}, PlotRange → {0, 10},

AxesOrigin → {0,0}, AxesStyle → Thickness[0.004], AxesLabel → {"x","y"},

GridLines → Automatic, TextStyle → {FontFamily → "Arial", FonySize → 8}]

Out[1]= $\{\{\, y \to \text{Function}[\{x\},\ e^{-6x}C[1] + 12e^{-6x}\left(-\frac{1}{37}e^{6x}\text{Cos}[x] + \frac{6}{37}e^{6x}\text{Sin}[x]\right)]\,\}\}$

Out[2]= $\{\{\, y \to \text{Function}[\{x\},\ e^{-6x}4 + 12e^{-6x}\left(-\frac{1}{37}e^{6x}\text{Cos}[x] + \frac{6}{37}e^{6x}\text{Sin}[x]\right)]\,\}\}$

Out[3]= = Graphics =

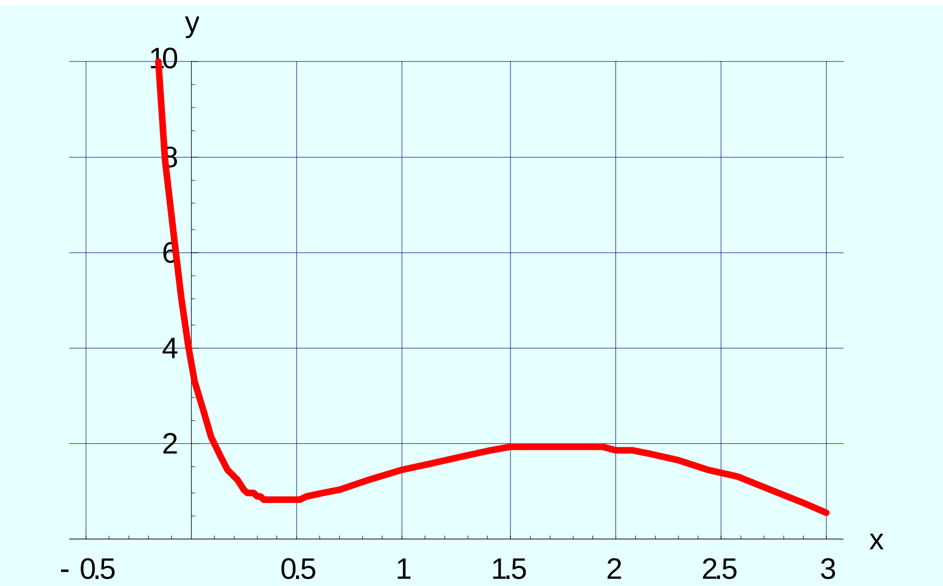

Fig. 4. The graph of the function (89) for constant C=4 as a solution of the first order linear non-homogeneous differential equation (72)

Source: Program and graph elaborated by the Authors

4. Conclusions

- Taking into account the constant variation method it is possible to solve the first order linear non-homogeneous differential equations. However, using the *Mathematica* program for numerical solution, you can quickly get a solution and create a graphical interpretation of solutions.
- Solving of the first order linear non-homogeneous differential equations requires knowledge of the integration of the function appearing on the right side of this equation.

- In particular, solving of the first order non-homogeneous differential equations requires knowledge of the integration of exponential, power, logarithmic, polynomial, trigonometric and irrational functions.

Literature

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